Consumer Search and the Long-Run Phillips Curve

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Abstract

We construct a continuous-time, monetary model with frictional goods and labor markets to revisit the long-run relationship between inflation and unemployment. The novelty relative to the literature (e.g., Berentsen et al., 2011) is the possibility given to consumers to search sequentially among different sellers to fulfill idiosyncratic consumption shocks. The value of consumers' outside options and firms' market power are endogenous and depend on the inflation rate. The long-run Phillips curve is generically ∪-shaped, i.e., at low inflation rates, an increase in anticipated inflation reduces the unemployment rate whereas at high inflation rates it raises it.

JEL Classification: D82, D83, E40, E50

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1 Introduction

The relationship between inflation and unemployment – the Phillips curve – is a cornerstone of macroeconomic models. The textbook view is that in the long run the Phillips curve is vertical, i.e., by virtue of the classical dichotomy, inflation does not affect the equilibrium or natural unemployment rate. This view has been embraced by canonical models of equilibrium unemployment, e.g., Lucas and Prescott (1978) and Mortensen and Pissarides (1994) – MP thereafter. However, it has long been recognized that the long-run Phillips curve might actually slope upward (e.g., Friedman, 1977), since anticipated inflation is a distortionary tax on monetary exchange and market activities. To the extent that money demand is downward sloping, inflation reduces consumers' payment capacity, which has an adverse effect on firms' profits and their incentives to hire workers. This idea has been formalized rigorously by Berentsen et al. (2011) – BMW thereafter – within a framework that combines the MP model of unemployment with the model of frictional goods markets with monetary exchange due to Lagos and Wright (2005).

While BMW provides an elegant description of the goods and labor markets as two frictional markets with pairwise meetings that open sequentially, they introduce an overlooked asymmetry regarding the treatment of agents' outside options across markets.[1] In the labor market, a worker who is negotiating her employment contract with a firm has the option to keep searching for an alternative firm. This outside option plays a critical role for the determination of the wage and the markdown relative to workers' productivity. In contrast, the goods market features random matching but no search: consumers wish to consume whenever they are matched with a producer but they do not have the option to search within each period. As a result, consumers face no opportunity cost from accepting a trade since their current decision does not affect their future opportunities. The lack of meaningful consumer outside options is not innocuous for firms' market power and markups. First, in models of sequential search (e.g. Mc Call, 1970), the effect of market frictions on market power, e.g., the time it takes for an agent to find an alternative trading partner, operates primarily through outside options - a channel that is shut down in BMW. Second, the lack of outside options prevents convergence to a perfect competition outcome where rents in pairwise meetings vanish, as shown, e.g., in Choi and Rocheteau (2021a).

Our paper has two contributions: one is methodological and the other is theoretical. In terms of the methodology, we construct a continuous-time monetary model of goods and labor markets with search frictions that provides a symmetric treatment of workers' and consumers' outside options and the determination of prices and wages. Specifically, we disentangle preference shocks from random matching shocks in the goods market. We assume that consumers receive infrequent but lasting preference shocks for the consumption of a good, allowing them to engage in a genuine search process. As a result of this formalization, the value of consumer outside options is endogenous and it affects prices and wages, markups and markdowns. More-

[1] Models with both frictional goods and labor markets also include Lehmann and Van der Linden (2010), and Petrosky-Nadeau and Wasmer (2015). See the literature review for additional references.

over, our model generates convergence to perfect competition in goods and labor markets as trading frictions vanish. Finally, our model is both tractable and general in that it encompasses BMW when we shut down consumers' outside options, and MP when we take a cashless limit.

Our theoretical contribution establishes the generic nonmonotonicity of the long-run Phillips curve. At low inflation rates, there is a negative relationship between inflation and unemployment whereas the opposite is true at high inflation rates. We explain this difference by showing that inflation has two opposite effects on equilibrium unemployment. First, there is a negative effect on consumers' real balances that tends to increase unemployment. This effect is the one identified in BMW. Second, there is a market power effect according to which inflation reduces the value of consumers' outside option, which consists in searching across different sellers. This search activity in monetary economies requires to hold real money balances at a cost that increases with inflation. This second effect, which raises firms' market power in the goods market and tends to reduce unemployment, dominates at low inflation rates whereas the first effect dominates at high inflation rates. Hence, the relationship between unemployment and inflation is generically U-shaped (in the inflation-unemployment space). We also establish that the relationship between the real wage and inflation is U-shaped: a small inflation pushes real wages up thereby contributing to a price-wage spiral.

In Figure 1, we illustrate our main finding by plotting the theoretical long-run Phillips curve. There is an inflation rate above the Friedman rule, denoted π^*, that minimizes unemployment. Below that inflation rate, the Phillips curve is downward-sloping, which generates a trade-off for policymakers whose dual mandate is to keep inflation and unemployment low. Above π^*, the long-run Phillips curve is upward-sloping, in which case there is no trade-off between inflation and unemployment. These results rationalize the choice of a positive inflation target between the Friedman rule and π^*. The unemployment-minimizing inflation depends on the search frictions in the goods market as well as persistence of preference shocks that determines the search horizon of consumers. As the frictions in the goods market subside, or as the frequency of preference shocks becomes very large, the market power of firms vanishes and the long-run Phillips curve becomes vertical. In a calibrated version of our model, π^* ranges from 0.5% to 4% as the frequency of preference shocks falls from a monthly to a quarterly basis.

Even though we emphasize the role of consumer search in the determination of firms' market power, in our baseline model, on the equilibrium path, consumers do not exercise their option to search once matched with a firm. Search is a threat that disciplines the demand of the firms during the negotiation for the terms of trade. In our last section, we explore a version of our model with horizontally differentiated products to induce search on the equilibrium path. We show that our main insight is robust, i.e. the long-run Phillips curve is downward sloping at low inflation rates. Numerical examples show that π^* drops when there is more product differentiation.

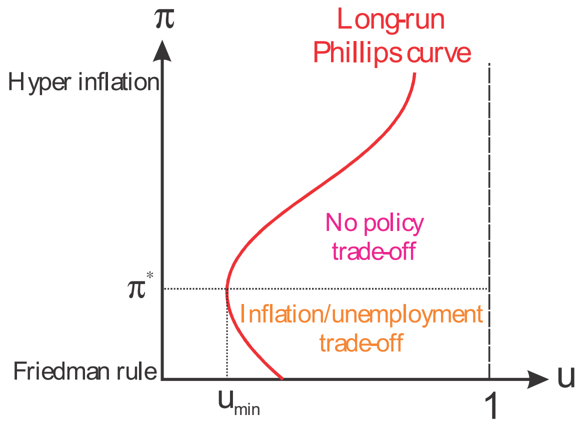

Figure 1: The ∪-shaped long-run Phillips curve

1.1 Literature review

Our model builds on Berentsen et al. (2011) that introduces a frictional labor market into the Lagos and Wright (2005) model.[2] Versions of the model with money and credit include Bethune et al. (2015) and Branch et al. (2016). A continuous-time version was constructed by Rocheteau and Rodriguez-Lopez (2014).

Other models with frictional goods and labor markets include Petrosky-Nadeau and Wasmer (2015, 2017) and Michaillat and Saez (2015). In the former, there is no money, i.e., agents pay with transferable utility. In the latter, money is introduced via a money-in-the-utility-function specification. In contrast, we formalize explicitly the role of money and the associated liquidity constraints that are critical for the formation of the terms of trade. In addition, we endogenize consumers' outside options and firms' market power in a dynamic setting.

There is a literature on search and inflation under menu costs that shows that inflation erodes firms' market power, e.g., Benabou (1988) and Diamond (1993). In those models, the economy is cashless, i.e., money has no transaction role and is a mere unit of account. Inflation reduces market power by preventing firms from maintaining their price at a monopoly level as in Diamond (1971). In contrast, in our model, prices are perfectly flexible and are determined through bilateral bargaining. Moreover, we formalize a monetary economy where consumers need to carry real money balances while searching for a producer. Therefore, contrary to Benabou (1988) and Diamond (1993), inflation makes it more costly for consumers to search, which reduces the value of their outside option and raises firms' market power.

Similar results are obtained in traditional New Keynesian models with sticky prices. Such models can feature a positive correlation between macroeconomic activity and inflation (see Walsh, 2017). For instance, in King and Wolman (1996), firms adjust prices infrequently as the inflation rate rises, resulting in a lower markup and a higher output. Devereux and Yetman (2002) show that, when the frequency of price adjustments is endogenized, the correlation between inflation and output becomes non-linear, and non-monotone.

[2] Other models of unemployment and inflation based on the Mortensen-Pissarides framework include Cooley and Quadrini (2004) and Lehmann (2012). A related approach is provided by Williamson (2015).

In their calibration, long-run inflation has a positive effect on output when the inflation rate is below 2%, and the effect becomes negative for higher inflation rates. Our theory also predicts that output is hump-shaped in the long-run inflation rate, but the main mechanism works through consumers' outside option of search and does not require sticky prices.

In our model the long-run Phillips curve slopes downward at low inflation rates. There are alternative explanations for why the long-run Phillips curve could be downward sloping. In Rocheteau et al. (2007), unemployment emerges due to indivisible labor and the long-run Phillips curve can be downward sloping, depending on the complementarity or substitutability between leisure and consumption.[3] Another mechanism to obtain a negative relationship between unemployment and inflation is through a Tobin effect according to which the rate of return on capital decreases, e.g., as in Rocheteau and Rodriguez-Lopez (2014).

In the consumer search literature, there is a long tradition of modeling consumers' outside options explicitly, i.e., the opportunity cost of trading includes the forgone benefit of searching for other sellers, e.g, McCall (1970), Wolinsky (1986), and Anderson and Renault (1999). These models generate equilibrium consumer search by assuming that products are horizontally differentiated. We follow this tradition in Section 5.

1.2 Empirical evidence

A key challenge in assessing the long-run effects of monetary policy is to identify monetary policy shocks. There are two approaches to this problem (Blanchard (2018)). One is to study recessions caused by intentional disinflations. These disinflations are large monetary shocks driven by changes in policies and are not reactions to other shocks, see, for example, Ball (2009, 2014) and Blanchard et al. (2015). Blanchard (2018) concludes that, according to cross country macroeconomic evidence, disinflation has a persistent positive effect on the unemployment rate and a negative effect on output.

Another approach, pioneered by King and Watson (1994), is to use a vector autoregression (VAR) methodology with identified monetary policy shocks. Using bivariate structural VARs, King and Watson (1994) find evidence of a negative long-run trade-off between inflation and unemployment in the US postwar, under the identifying assumption that business cycles are entirely due to demand shocks. But King and Watson (1997) show that the results are sensitive to the choice of identifying assumptions. More recently, Benati (2015) adopts both classical and Bayesian structural VARs, and shows that the US data is compatible with both positively and negatively sloped Phillips curves. The estimated long-run impact of a 1% increase in inflation on the unemployment rate ranges from -0.56% to 0.15% at a 90% confidence interval. Ascari et al. (2022) introduce stochastic trends into Bayesian VAR. They find that there is a threshold level of inflation below which potential output is independent of inflation, and above which potential output and inflation are negatively correlated. The threshold level of inflation is slightly below 4%. This finding is consistent with our theory which predicts that output rises in inflation when inflation is low and otherwise falls.

[3] Dong (2011) adopts the notion of competitive search equilibrium in this model to study the relationship between inflation and unemployment.

In terms of cross-country evidence, Bullard and Keating (1995) study a large sample of postwar economies using a structural VAR approach and find that inflation raised output in low-inflation countries, and either did not affect or reduced output in countries with higher inflation rate. A meta-study of VAR studies by De Grauwe and Costa Storti (2004) finds that the average effect of a 1% interest rate shock on output is -0.15% after five years. But the standard deviation of the distribution of estimates is 0.27%, so the Philips curve can have a positive or a negative slope. De Grauwe and Costa Storti conclude that the differences in results across different studies are mainly due to the differences in econometric methodologies.

2 Environment

The benchmark environment builds on Choi and Rocheteau (2021b).[4]

Time, agents, commodities Time is continuous and indexed by $t \in \mathbb{R}_+$. The economy is composed of three types of infinitely-lived agents: a unit measure of workers, a measure ω of consumers, and an endogenous measure of firms, n. There are two perishable goods, $y \in \mathbb{R}_+$ and $c \in \mathbb{R}$. Good c can be consumed and produced by all agents and is taken as the numéraire. It is traded continuously over time and competitively. Good y is produced exclusively by firms and valued by consumers only, which creates a need for interactions between consumers and firms outside of the firm/worker pairs. The flows of goods and payments between agents are summarized in Figure 2. The worker produces y units of goods for the consumer (labelled B as for buyer) who makes a payment p to the firm, who compensates the worker with a wage w.

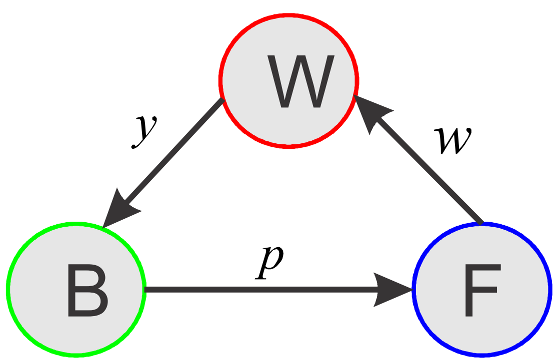

Figure 2: Agents: worker (W), firm (F), consumer (B)

[4]It can be interpreted as a continuous-time version of Lagos and Wright (2005) and Rocheteau and Wright (2005), except that centralized and decentralized markets do not alternate in discrete time but instead coexist in continuous time. For earlier versions, see Craig and Rocheteau (2008), Rocheteau and Rodriguez-Lopez (2014), and Rocheteau et al. (2018).

Preferences Preferences of consumers, workers, and firms are represented by the following utility functions:

$$\mathcal{U}^b = \mathbb{E}\left[\int_0^\infty e^{-\rho t} dC(t) + \sum_{n=1}^{+\infty} e^{-\rho T_n} y(T_n)\right] \tag{1}$$

$$\mathcal{U}^w = \mathbb{E}\left[\int_0^\infty e^{-\rho t} dC(t)\right] \tag{2}$$

$$\mathcal{U}^f = \mathbb{E}\left[\int_0^\infty e^{-\rho t} dC(t)\right], \tag{3}$$

where $C(t)$ is a measure of the cumulative net consumption of the numéraire good.[5] Negative consumption of the numéraire good is interpreted as production. So preferences of all agents are linear in the numéraire. The variable $y(t)$ represents the consumption of the y good produced by firms for consumers. In (1) the idiosyncratic stochastic process, $\{T_n\}$, indicates the times at which the consumer gets to consume good y, which requires that the consumer is hungry for good y and meets a worker-firm that produces it.

In the formulation of this environment, we separate agents according to their role (consumer, worker, firm) relative to the consumption or production of good y. It would be equivalent to consider a household composed of one worker and ω buyers with a single integrated budget constraint that incorporates firms' profits.

Consumer's idiosyncratic preference shocks Consumers are subject to two types of idiosyncratic shocks: shocks to preferences, and shocks arising from the random nature of the matching process. Relative to the former, we assume that consumers can be in two states: idle or active. An idle consumer has no desire to consume. The desire to consume arrives at Poisson rate $\lambda > 0$ in which case the consumer becomes active. This desire is fulfilled after consumption of any quantity $y > 0$ (e.g., consumers are satiated) or it disappears at Poisson rate $\gamma \geq 0$. In both events, the active consumer becomes idle. As an example, assume consumers wish to consume ice cream at some Poisson rate λ. Once they have consumed it, or if they have waited too long, they no longer desire ice cream for a while. The assumption $\lambda < +\infty$ according to which consumers remain temporarily idle (satiated) following consumption, $y_t > 0$, generates an opportunity cost from accepting a trade. This cost, together with bargaining shares, determines sellers' market power. In Berentsen et al. (2011), $\lambda = +\infty$, in which cases consumers are always active and accepting a trade has no opportunity cost. The measure of active consumers is denoted ω_1 and the measure of idle consumers is ω_0.

Technology Each worker-firm pair produces both types of output: a constant flow, $x > 0$, of the numéraire good and discrete quantities of good y in bilateral meetings with consumers. The production of y units of the decentralized-market good requires $\varphi(y)$ units of numéraire.[6] It is such that $\varphi(0) = \varphi'(0) = 0$, $\varphi'(h) > 0$,

[5] A similar cumulative consumption process is assumed in the continuous-time models of OTC trades of Duffie et al. (2005). If consumption (or production) of the numéraire happens in flows, then $C(t)$ admits a density, $dC(t) = c(t)dt$. If the buyer consumes or produces a discrete quantity of the numéraire good at some instant t, then $C(t^+) - C(t^-) \neq 0$.

[6] We also worked out a version where the variable cost is in terms of workers' labor or disutility. It does not affect the allocations or results, except for the expressions of the worker's compensation. We report these expressions in Appendix B.

and $\varphi''(h) > 0$. We denote y^* such that $\varphi'(y^*) = 1$.

Frictions in the goods market We denote $q \equiv n/\omega_1$ the measure of firms per active consumers where n is the measure of employment relationships composed of a firm and worker. The matching rate of a consumer in the decentralized goods market is $\alpha(q)$ where $\alpha(0) = 0$, $\alpha' > 0$, $\alpha'(0) = +\infty$, and $\alpha'' < 0$. The matching rate of a firm is $\alpha(q)/q$.

We distinguish two types of pairwise meetings according to the method of payment. A fraction χ_d of meetings are such that the consumer can produce the numéraire to pay for his consumption. We interpret these meetings as credit meetings. With complement probability, χ_m, the consumer cannot produce the numéraire in the meeting and is not trusted to repay his debt in the future. In such meetings the consumer needs a means of payment. There is an intrinsically useless object, called fiat money, that is perfectly storable and durable. The quantity of money at time t is denoted M_t. The constant money growth rate is $\pi \equiv \dot{M}_t/M_t$ and new money is injected in the economy through lump-sum transfers (or taxes if $\pi < 0$) to buyers. The price of money is terms of the numéraire is denoted ϕ_t and the lump-sum transfer is denoted $\tau_t = \phi_t \dot{M}_t$.

In pairwise meetings in the goods market, the quantities produced and consumed, and payments, are determined according to the proportional solution of Kalai (1977) where the share of the surplus received by sellers is $\mu \in [0, 1]$. The case for using the Kalai solution in the context of a quasi-linear environment with liquidity constraints is made in Aruoba et al. (2007), and Hu and Rocheteau (2020) who also provide strategic foundations based on variant of the Rubinstein game.[7]

Frictions in the labor market The labor market is also frictional. Labor market tightness is defined as the ratio of vacancies per unemployed worker, $\theta \equiv v/u$. The job finding rate of a worker is $f(\theta)$ with $f(0) = 0$, $f' > 0$, $f'(0) = +\infty$, and $f'' < 0$. The vacancy filling rate is $f(\theta)/\theta$. The flow cost of opening a vacancy is $k > 0$. We restrict our attention to contracts where workers are paid a constant wage, w, that is negotiated between the worker and the firm according to the Nash/Kalai solution. Unemployed workers receive a flow income b.

3 Equilibrium

3.1 Goods market

We denote $V^b(a)$ the value function of an active consumer with a units of real balances and $W^b(a)$ the value function of an idle consumer. Given the linearity of preferences with respect to the numéraire good, both value functions are linear with $V^b(a) = a + V^b$ and $W^b(a) = a + W^b$. We denote $Z \equiv V^b - W^b$ the opportunity cost from accepting a trade.

[7] The monotonicity property of the Kalai solution guarantees that it implements efficient quantities at the Friedman rule (Aruoba et al., 2007). This result is instrumental for some of our proofs.

The outcome of the negotiation between a consumer and a firm is a pair, (p, y), where p is the payment by the consumer expressed in the numéraire and y is the output produced by the firm. The surplus of the firm is the difference between the firm revenue expressed in terms of the numéraire and the disutility to produce y, $p - \varphi(y)$. The firm does not incur an opportunity cost since producing for its current consumer does not affect its ability to serve its future consumers. The surplus of the consumer is the difference between the utility from consuming y net of the payment and the opportunity cost of accepting a trade, $y - p - Z$. There are gains from trade if

$$\max_{y} \{y - \varphi(y) : \varphi(y) \leq a\} > Z. \tag{4}$$

For a monetary trade to be incentive feasible, the payment, which is bounded above by a_t, must at least cover the firm's variable cost, $\varphi(y)$. Moreover, the utility of consumption net of the variable cost must be greater than the consumer's opportunity cost.

A necessary condition for (4) to hold is that $Z \leq y^* - \varphi(y^*)$. So the opportunity cost of the consumer is bounded above by the first-best surplus. If $a \geq \varphi(y^*)$, the condition is also sufficient. If $a < \varphi(y^*)$, then (4) can be reexpressed as $\varphi^{-1}(a) - a > Z$. The existence of gains from trade requires that the consumer holds enough real balances.

The terms of trade, which are determined according to the Kalai proportional solution, solve

$$\max_{p,y} \{p - \varphi(y)\} \quad \text{s.t. } p - \varphi(y) = \mu [y - \varphi(y) - Z] \quad \text{and} \quad p \leq a. \tag{5}$$

The firm chooses (p, y) to maximize its profits subject to the condition that the profits are a fraction μ of the whole gains from trade and the consumer's payment does not exceed her real balances. The problem can be simplified as

$$\max_{p,y} \{y - \varphi(y)\} \quad \text{s.t. } p = \varphi(y) + \mu [y - \varphi(y) - Z] \leq a. \tag{6}$$

If the liquidity constraint does not bind, then $y = y^*$ and $p = (1-\mu)\varphi(y^*) + \mu y^* - \mu Z$. Otherwise, $p = a$ and $a = (1-\mu)\varphi(y) + \mu y - \mu Z$. We denote $y(a, Z)$ the outcome of the negotiation. It is nondecreasing in a and Z. In a credit trade, $y = y^*$.

In order to establish the connection between Z and firms' market power, we can define the markup associated with a transaction as

$$MKUP \equiv \frac{p - \varphi(y)}{\varphi(y)} = \frac{\mu [y - \varphi(y) - Z]}{\varphi(y)}. \tag{7}$$

The markup is the difference, in percentage term, between the payment and the variable production cost. For given y, the markup increases with μ but decreases with Z. Moreover, when $y = y(a, Z)$, the markup tends to 0 as Z tends to its upper bound, $y^* - \varphi(y^*)$, irrespective of μ.

We define the match surpluses in monetary and credit matches as

$$S^m(a, Z) \equiv y(a, Z) - \varphi[y(a, Z)] - Z \tag{8}$$

$$S^d(Z) \equiv y^* - \varphi(y^*) - Z. \tag{9}$$

The surplus in monetary matches given by (8) is increasing in a, and decreasing in Z. The surplus in credit matches given by (9) is decreasing in Z.

We now turn to the value functions of the consumer. The HJB equation for $V^b \equiv V^b(0)$ in a steady state (i.e. $\dot{V}^b = 0$) is

$$\rho V^b = \max_{a \geq 0} \left\{ -ia + \tau + \alpha(1-\mu) \left[\chi^m S^m(a,Z) + \chi^d S^d(Z) \right] - \gamma Z \right\}, \tag{10}$$

where $i \equiv \rho + \pi$ can be interpreted, by virtue of the Fisher equation, as the nominal interest rate on an illiquid bond. An active buyer incurs the cost of holding real balances, ia, receives a lump-sum transfer, τ, and enters a match with a firm at Poisson rate α. With probability χ^m, the match is monetary and the surplus is S^m. With complement probability, χ^d, the consumer can pay with the numéraire and the surplus is S^d. According to the last term on the right side of (10), if the preference shock is reversed and the consumer no longer wants to consume, at Poisson rate γ, he incurs a lifetime utility loss of $Z \equiv V^b - W^b$.

From the right side of (10), the optimal real balances are given by

$$a^* \in \arg\max_{a \geq 0} \left\{ -ia + \alpha(1-\mu)\chi^m S^m(a,Z) \right\}. \tag{11}$$

They maximize the expected consumer surplus in monetary matches net of the cost of holding real balances. From the first-order condition:

$$a^* = (1-\mu)\varphi(y) + \mu y - \mu Z \quad \text{where} \quad \frac{\alpha \chi^m (1-\mu) \left[1 - \varphi'(y)\right]}{\mu + (1-\mu)\varphi'(y)} = i, \tag{12}$$

if $-ia^* + \alpha(1-\mu)\chi^m S^m(a^*, Z) \geq 0$. Otherwise, $a^* = 0$. From (12), the output in monetary matches does not depend on the value of the consumers' outside options but the payment does. As Z increases, consumers purchase the same amount of goods but reduce their payment.

The HJB equation for the value function of an idle buyer, W^b, is:

$$\rho W^b = \tau + \lambda Z. \tag{13}$$

The idle buyer receives a preference shock with Poisson arrival rate λ, in which case he becomes active and enjoys a lifetime utility gain of $Z \equiv V^b - W^b$.

Substituting (13) from (10), the outside option of the consumer, Z, solves:

$$(\rho + \lambda + \gamma) Z = \max_{a \geq 0} \left\{ -ia + \alpha(1-\mu) \left[\chi^m S^m(a,Z) + \chi^d S^d(Z) \right] \right\}. \tag{14}$$

The left side of (14) can be interpreted as the opportunity cost of consumer search – the effective discount rate multiplied by the value of consumers' outside options – while the right side is the expected return from the search activity. The effective discount rate is composed of the rate of time preference, ρ, the rate at which idle consumers become active, and the rate at which active consumers become idle. The right side is decreasing in Z, is positive when $Z = 0$ provided that $\chi^d > 0$ and it approaches 0 as $Z \to y^* - \varphi(y^*)$. Hence, as shown in Figure 3, there is a unique $Z \in (0, y^* - \varphi(y^*))$ solution to (14). As the nominal interest rate increases, the curve representing the right side of (14) moves downward and Z decreases. If the cost of holding real balances increases, searching for better opportunities in the goods market becomes more costly, and the value of consumers' outside options decreases.

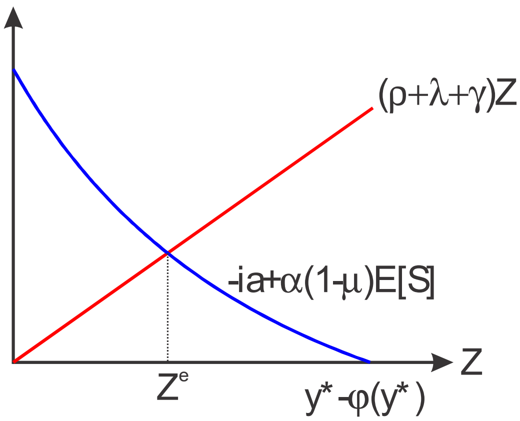

Figure 3: Determination of the value of consumers' outside options

Finally, we compute the measure of idle and active buyers. The steady-state measure of active consumers solves $(\gamma + \alpha)\omega_1 = \lambda(\omega - \omega_1)$, i.e.,

$$\omega_1 = \frac{\lambda}{\gamma + \alpha(q) + \lambda}\omega. \tag{15}$$

The measure of active consumers decreases with tightness in the goods market, q.

3.2 Labor market

The HJB equation for the value of a match composed of a worker and a firm is J that solves:

$$(\rho + \delta) J = \alpha^s \mu \left[\chi^m S^m(a^*, Z) + \chi^d S^d(Z)\right] + x - \rho U, \tag{16}$$

where δ is the job destruction rate, $\alpha^s \equiv \alpha(q)/q$ is the arrival rate of consumers, and U is the value function of an unemployed worker. The firm receives a share μ of the expected surplus generated by the trade. The second term on the right side is the flow of numéraire good produced by the worker-firm pair. The last term is the reservation wage of an unemployed worker. It solves

$$\rho U = b + f(\theta)\beta J, \tag{17}$$

where b is the income when unemployed, β is the worker's bargaining share, and $f(\theta)$ is the job finding rate.

The HJB equation for an employed worker is:

$$\rho E = w - \delta\beta J. \tag{18}$$

On the right side, the worker receives a wage, w, and at rate δ the match with the firm is destroyed, which generates a capital loss equal to βJ. We subtract ρU from both sides and use that $E - U = \beta J$ to obtain:

$$w = (\rho + \delta)\beta J + \rho U. \tag{19}$$

The first term on the right side is the fraction β of the value of the match that the worker can capture. The last term correspond to the reservation wage of the worker, which is the flow value of being unemployed. We substitute $(\rho+\delta)\,\beta J$ by its expression given by (16) to simplify the wage equation as follows:

$$w = \beta \left\{ \alpha^s \mu \left[\chi^m S^m(a^*, Z) + \chi^d S^d(Z) \right] + x \right\} + (1-\beta)\rho U. \tag{20}$$

The free-entry of firms in the labor market implies

$$k = \frac{f(\theta)}{\theta}(1-\beta)J. \tag{21}$$

The flow cost of posting a vacancy is equal to the vacancy filling rate multiplied by the value of a filled job, $(1-\beta)J$. Substituting $f(\theta)$ from (21) into (17), $\rho U = b + \beta k\theta/(1-\beta)$. We substitute this expression into (16) and replace J from (21) to obtain the equilibrium condition for market tightness at the steady state:

$$(\rho+\delta)\,\frac{k\theta}{f(\theta)} = (1-\beta)\left\{ \alpha^s \mu \left[\chi^m S^m(a^*, Z) + \chi^d S^d(Z) \right] + x - b \right\} - \beta k\theta. \tag{22}$$

There is a unique θ solution to (22). Relative to the Mortensen-Pissarides textbook model, the novelty is the term $\alpha^s \mu \left[\chi^m S^m(a^*, Z) + \chi^d S^d(Z) \right]$ that represents the sales in the frictional goods market. It is increasing in a^* and decreasing in q and Z. Firms are more profitable when consumers have a higher payment capacity but they are less profitable when consumers have better outside options.

We use $\rho U = b + \beta k\theta/(1-\beta)$ to rewrite the wage in (20) as

$$w = \beta \left\{ \alpha^s \mu \left[\chi^m S^m(a^*, Z) + \chi^d S^d(Z) \right] + x \right\} + (1-\beta)b + \beta k\theta. \tag{23}$$

Relative to both the MP and BMW models, the value of the consumers' outside option in the goods market affects the wage. As Z increases, the market power of the firm in the goods market decreases, which reduces wages. It will also have implications for how inflation affects wages. As a preview of the comparative statics, a higher i reduces consumers' payment capacity, a^*, which tends to lower firms' profits and wages. But a higher i also reduces Z, which affects firms' profits in the opposite direction.

We define the wage markdown in a symmetric fashion as the markup, that is:

$$MKDOWN \equiv \frac{\hat{x} - w}{\hat{x}}$$

where $\hat{x}$ is the net expected revenue generated by a worker, $\hat{x} = \mathbb{E}[p - \varphi(y)] + x$. It is also equal to

$$\hat{x} = \alpha^s \mu \left[\chi^m S^m(a^*, Z) + \chi^d S^d(Z) \right] + x. \tag{24}$$

which allows us to rewrite the wage markdown as

$$MKDOWN \equiv \frac{(1-\beta)\left\{ \alpha^s \mu \left[\chi^m S^m(a^*, Z) + \chi^d S^d(Z) \right] + x - b \right\} - \beta k\theta}{\alpha^s \mu \left[\chi^m S^m(a^*, Z) + \chi^d S^d(Z) \right] + x} \tag{25}$$

We can see that the markdown depends on bargaining powers in labor and goods markets as well as consumers' outside options in the goods market. From (22), it can be expressed as:

$$MKDOWN = \frac{(1-\beta)\,(\rho+\delta)\,k}{(\rho+\delta)\,k + \beta k f(\theta) + (1-\beta) b f(\theta)/\theta}.$$

The measures of employed and unemployed workers, at the steady state, are:

$$n = \frac{f(\theta)}{\delta + f(\theta)}, \quad u = \frac{\delta}{\delta + f(\theta)}. \tag{26}$$

Equation (26), which is analogous to the Beveridge curve, gives a positive relationship between employment and market tightness.

3.3 Definition of equilibrium

In order to simplify the definition of an equilibrium, we derive a relationship between the market tightness of the goods market, q, and the market tightness of the labor market, θ. From the definition $q\omega_1 \equiv n$, (15), and (26), the relationship between q and θ is given by:

$$\frac{\lambda \omega q}{\gamma + \lambda + \alpha(q)} = \frac{f(\theta)}{\delta + f(\theta)}. \tag{27}$$

The implicit solution, $q = Q(\theta)$, from (27) is an increasing function of θ with $Q(0) = 0$ and $Q'(\theta) > 0$. Using $Q(\theta)$, we can rewrite the matching rates in the goods market as functions of θ, i.e.,

$$\alpha^s(\theta) \equiv \frac{\alpha\left[Q(\theta)\right]}{Q(\theta)}, \quad \alpha^b(\theta) \equiv \alpha\left[Q(\theta)\right].$$

The rate at which firms sell their output decreases with θ. Indeed, as θ increases, the number of active firms increases, which raises tightness in the goods market and reduces firms' matching rate with consumers.

An equilibrium can be reduced to a 4-tuple, (θ, a^*, Z, w), solution to:

$$(\rho + \delta)\frac{k\theta}{f(\theta)} = (1-\beta)\left\{\alpha^s(\theta)\mu\left[\chi^m S^m(a^*, Z) + \chi^d S^d(Z)\right] + x - b\right\} - \beta k \theta \tag{28}$$

$$a^* \in \arg\max_{a \geq 0}\left\{-ia + \alpha^b(\theta)(1-\mu)\chi^m S^m(a, Z)\right\} \tag{29}$$

$$(\rho + \lambda + \gamma) Z = -ia^* + \alpha^b(\theta)(1-\mu)\left[\chi^m S^m(a^*, Z) + \chi^d S^d(Z)\right] \tag{30}$$

$$w = \beta\left\{\alpha^s(\theta)\mu\left[\chi^m S^m(a^*, Z) + \chi^d S^d(Z)\right] + x\right\} + (1-\beta)b + \beta k \theta. \tag{31}$$

Proposition 1 *If $\chi^d > 0$, then there exists an active steady-state equilibrium.*

The logic for the existence claim goes as follows. From (30) we express the value of the consumer's outside option, Z, as a function of labor market tightness, θ. It is an increasing function: as labor market tightness increases, more firms are created, and hence the measure of producers per consumer in the goods market increases, which improves consumers' outside options. From (29), we express consumers' real balances, a^*, as a function of θ. We obtain an increasing function, because as θ increases, the average time for an active consumer to find a producer decreases, which in turn reduces the average holding cost of real balances. We then substitute $Z(\theta)$ and $a^*(\theta)$ into (28) to obtain a single equilibrium condition in θ. We use the continuity of this equation and its values at $\theta = 0$ and $\theta = +\infty$ to establish that a positive solution exists. A sufficient condition for the existence of an active equilibrium is that a positive measure of transactions is conducted with credit, $\chi^d > 0$. Indeed, if θ becomes very small, the expected revenue of firms becomes very large because they can serve a large measure of consumers per firm. Hence, irrespective of b or x, there is always a sufficiently low θ so that firms' profits are positive and entry is profitable. Before we get to our main result, we consider some special cases that have been studied in the literature.

3.4 Pure credit economy

Suppose that all meetings in the decentralized goods market are credit meetings, $\chi^d = 1$.[8] An equilibrium can then be reduced to a triple (θ, Z, w) solution to

$$(\rho+\delta)\frac{k\theta}{f(\theta)} = (1-\beta)\left\{\alpha^s(\theta)\mu\left[y^* - \varphi(y^*) - Z\right] + x - b\right\} - \beta k\theta \tag{32}$$

$$(\rho+\lambda+\gamma)Z = \alpha^b(\theta)(1-\mu)\left[y^* - \varphi(y^*) - Z\right] \tag{33}$$

$$w = \beta\left\{\alpha^s(\theta)\mu\left[y^* - \varphi(y^*) - Z\right] + x\right\} + (1-\beta)b + \beta k\theta. \tag{34}$$

If firms have no bargaining power in the goods market, $\mu = 0$, θ is determined from (32), which is identical to the equilibrium condition of the MP model. In particular, it is independent of Z. The other polar case is when firms have all the bargaining power in the goods market, $\mu = 1$. From (33), $Z = 0$ and from (32) θ is uniquely determined by

$$(\rho+\delta)\frac{k\theta}{f(\theta)} = (1-\beta)\left\{\alpha^s(\theta)\left[y^* - \varphi(y^*)\right] + x - b\right\} - \beta k\theta.$$

The revenue of the firm depends negatively on θ through a competition/congestion effect in the goods market. More generally, the frictions in the goods market, as captured by α^s, have an impact on the labor market, i.e., as α^s decreases, θ goes down. As frictions vanish, $\alpha^s, \alpha^b \to +\infty$, and $Z \to y^* - \varphi(y^*)$.

Next, consider the interior case where the firm's bargaining power is $\mu \in (0,1)$. From (32), θ is a decreasing function of Z because as the value of consumers' outside option increases, the profits of the firm decrease. From (33), we can solve for Z in closed form and obtain:

$$Z = \frac{\alpha^b(\theta)(1-\mu)}{\rho+\lambda+\gamma+\alpha^b(\theta)(1-\mu)}\left[y^* - \varphi(y^*)\right]. \tag{35}$$

The value of consumers' outside option increases with θ and decreases with μ. We can substitute the expression for Z into (32) to reduce an equilibrium to a single equation in θ:

$$(\rho+\delta)\frac{k\theta}{f(\theta)} + \beta k\theta = (1-\beta)\left\{\frac{\alpha^s(\theta)\mu(\rho+\lambda+\gamma)}{\rho+\lambda+\gamma+\alpha^b(\theta)(1-\mu)}\left[y^* - \varphi(y^*)\right] + x - b\right\}. \tag{36}$$

It is easy to check that in a pure credit economy an equilibrium exists and is unique. The following proposition shows how the determinants of market power in the goods market, λ and γ, affect labor market outcomes.

Proposition 2 ***(Unemployment and consumer search in a pure credit economy)*** *Suppose $\chi_d = 1$ and $\mu \in (0,1)$. As λ or γ increases, the value of consumers' outside options (Z) decreases, labor market tightness (θ) increases, wages (w) increase, and unemployment (u) decreases.*

[8] For a review of models with frictional goods and labor markets where trades in the goods market are not subject to liquidity constraints, see Petrosky-Nadeau and Wasmer (2017).

If λ increases, i.e., consumers do not stay idle long, or γ increases, i.e., the desire to consume vanishes quickly, then the opportunity cost of accepting a trade decreases, which makes the search for an alternative producer less profitable and raises producers' market power. As a result, market tightness in the labor market increases, $\partial\theta/\partial\lambda > 0$ and $\partial\theta/\partial\gamma > 0$, wages increases, $\partial w/\partial\lambda > 0$ and $\partial w/\partial\gamma > 0$, and unemployment decreases, $\partial u/\partial\lambda < 0$ and $\partial u/\partial\gamma < 0$.

3.5 Pure currency economy without consumer search

The economy in BMW is a pure currency economy, $\chi_m = 1$, in which there is no opportunity cost for the consumer to complete a trade, $\lambda = +\infty$. From (15), $\omega_1 = \omega$, all buyers are active at all points in time. From (27) market tightness in the goods market is

$$q = \frac{f(\theta)}{\omega\left[\delta + f(\theta)\right]}. \tag{37}$$

From (30) $Z = 0$ and an equilibrium can be reduced to a pair (θ, y) solution to:

$$\begin{aligned}
(\rho+\delta)\frac{k\theta}{f(\theta)} &= (1-\beta)\left\{\alpha^s(\theta)\mu\left[y-\varphi(y)\right]+x-b\right\}-\beta k\theta && (38)\\
\frac{1-\varphi'(y)}{\mu+(1-\mu)\varphi'(y)} &= \frac{i}{(1-\mu)\alpha^b(\theta)} && (39)\\
w &= \beta\left\{\alpha^s\mu\left[y-\varphi(y)\right]+x\right\}+(1-\beta)b+\beta k\theta. && (40)
\end{aligned}$$

Suppose $x < b$. Equation (38) gives a positive relationship between θ and y with $\theta = 0$ when $y = 0$ and $\theta = \bar{\theta} > 0$ when $y = y^*$. Equation (39) gives a positive relationship between y and θ with $y = 0$ if $\theta < \underline{\theta}$ where $\underline{\theta}$ solves $\alpha^b(\underline{\theta}) = [i\mu/(1-\mu)]$ and $y \to y^*$ as $\theta \to +\infty$. There always exists a non-active equilibrium with $y = \theta = 0$ and, generically, if an active equilibrium exists, then the number of active equilibria is even.[9] In order to describe the effect of anticipated inflation on unemployment, we focus on the equilibrium with the highest labor market tightness.

Proposition 3 ***(Long-run Phillips curve in the absence of consumer search)*** *Consider a pure currency economy with $x < b$ and $\lambda = +\infty$. We focus on the equilibrium with the highest market tightness. An increase in π leads to a decrease in market tightness (θ), a decrease in wages (w), and an increase in unemployment (u).*

We only provide a graphical proof (see Figure 4) as the result is similar to the one in BMW. At the high equilibrium the MD curve representing (39) in the (θ, y) space intersects the JC curve representing (38) by above. As i increases, the MD curve (39) shifts downward. Hence, y and θ decrease. So the model predicts a positive relationship between unemployment and inflation in the long run, i.e., the long-run Phillips curve is upward sloping.

[9] These results mirror those of Rocheteau and Wright (2005) who describe a pure currency economy with free entry of sellers.

If we reintroduce credit trades, $\chi_d \in (0,1)$, then an active equilibrium always exists even though the equilibrium might not be monetary. To see this, note that (38) with credit trades becomes

$$(\rho+\delta)\,\frac{k\theta}{f(\theta)} = (1-\beta)\left\{\alpha^s(\theta)\chi_m\mu\left[y-\varphi(y)\right]+\alpha^s(\theta)\chi_d\mu\left[y^*-\varphi(y^*)\right]+x-b\right\}-\beta k\theta.$$

As θ goes to 0, $\alpha^s(\theta)$ goes to $+\infty$, so that firm's revenue is larger than b, which guarantees that an active equilibrium always exists. The same is true if $x > b$. Moreover, if χ_d is sufficiently large then the equilibrium is unique since market tightness is less sensitive to changes in consumers' real balances.

When $y = 0$ (real balances $a = 0$), the JC curve intersect the horizontal axis at a positive θ. There are still credit trades so that firms are willing to participate in the market.

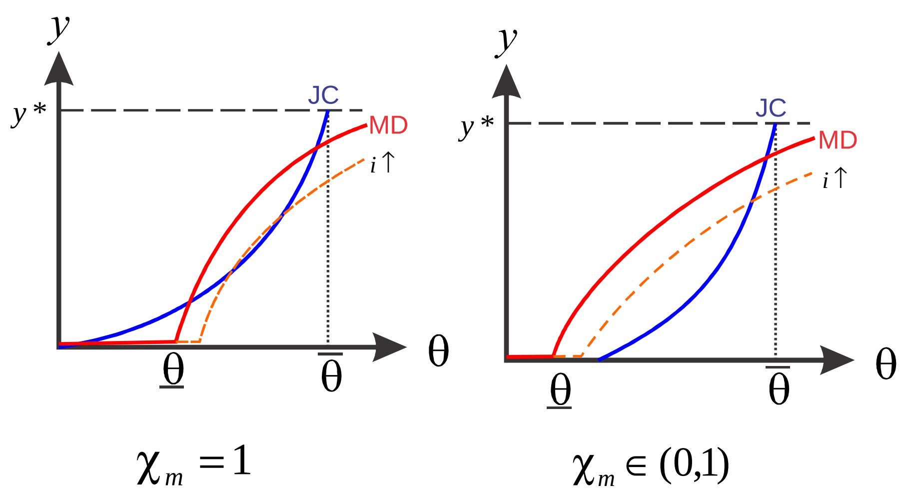

Figure 4: Left panel: Pure monetary economy. Right panel: Economy with money and credit.

3.6 The U-shaped long-run Phillips curve

We now turn to an economy with both money and credit, $\chi_m \in (0,1)$. We study two types of increases in the inflation rate: (i) a small increase in the neighborhood of the Friedman rule, $i = 0$; (ii) a large increase from $\pi = -\rho$ to $\pi = +\infty$ that reduces the value of money to zero.

Proposition 4 ***(The non-monotone long-run Phillips curve)*** *Suppose $\chi_m \in (0,1)$.*

1. *A small increase in i, starting from $i = 0^+$, leads to: an increase in labor market tightness (θ), a decrease in the unemployment rate (u) and an increase in wages (w).*

2. *A large increase in i from $i = 0^+$ to $i = +\infty$ leads to: a decrease in labor market tightness (θ), an increase of the unemployment rate (u) and a decrease in wages (w).*

An increase in π has two effects on the labor market. First, it reduces consumers' real balances, which tightens liquidity constraints and reduces the amount of goods firms can sell to consumers. This effect tends to reduce market tightness, raise unemployment and reduce wages. There is a second effect according to which an increase in i raises the cost for consumers to search for an alternative producer since they have to carry real money balances until they find a new opportunity to trade. When i is close to 0, y is close to y^*, and the first effect - the real balance effect of inflation - is negligible. Only the second effect on consumers' outside options - the market power effect of inflation - matters, i.e., inflation raises firms' market power by making it more costly for consumers to use their option to search.

When i is sufficiently large, the two effects described above are first order. We can show that when i is so high that consumers do not hold real balances, i.e., all trades are conducted with credit, then firms are worse-off compared to the equilibrium at the Friedman, i.e., θ is lower when $i = +\infty$ relative to $i = 0$. This guarantees that the relation between θ and i is nonmonotone: it is first increasing and it eventually decreases for sufficiently large values of i. Since wages increase with the firm profits (we show in the proof of Proposition 4 that w and θ comove as i changes), the relationship between w and i is also nonmonotone. A small, anticipated inflation pushes up wages while a large inflation rate depresses real wages. Hence, the unemployment minimizing level of i maximizes wages.

3.7 Long-run Phillips curve when market frictions vanish

We now characterize the equilibrium at two frictionless limits: (i) when search frictions in the goods amrket vanish and, (ii), when search frictions in labor market vanish. We interpret vanishing frictions in terms of matching technologies that become infinitly efficient at matching buyers and sellers.

Proposition 5 ***(Frictionless limits)*** *Suppose $\alpha(q) = A\bar{\alpha}(q)$ and $f(\theta) \equiv B\overline{f}(\theta)$.*

1. ***Limit as the goods market becomes frictionless.*** *Consider the limit as $A \to +\infty$. Then, $Z \to y^* - \varphi(y^*)$. If $x > b$, then $q \to +\infty$, and $\theta \to \theta^g_\infty$ where $\theta^g_\infty > 0$ is the unique solution of*

$$(\rho+\delta)\,\frac{k\theta^g_\infty}{f(\theta^g_\infty)} + \beta k\theta^g_\infty = (1-\beta)\,(x-b)\,. \tag{41}$$

   If $x \leq b$, then $\theta \to 0$.

2. ***Limit as the labor market becomes frictionless.*** *Consider the limit as $B \to +\infty$. Then, $(\theta, a, Z) \to \left(\theta^\ell_\infty, a^\ell_\infty, Z^\ell_\infty\right)$ where $\left(\theta^\ell_\infty, a^\ell_\infty, Z^\ell_\infty\right)$ is the solution of*

$$\begin{aligned} \beta k\theta^\ell_\infty &= (1-\beta)\left\{\alpha^s(\theta^\ell_\infty)\mu\left[\chi^m S^m(a^\ell_\infty, Z^\ell_\infty) + \chi^d S^d(Z^\ell_\infty)\right] + x - b\right\} && (42)\\ a^\ell_\infty &\in \arg\max_{a\geq 0}\left\{-ia + \alpha^b(\theta^\ell_\infty)(1-\mu)\chi^m S^m(a, Z^\ell_\infty)\right\} && (43)\\ (\rho+\lambda+\gamma)\,Z^\ell_\infty &= \max_{a\geq 0}\left\{-ia + \alpha^b(\theta^\ell_\infty)(1-\mu)\left[\chi^m S^m(a, Z^\ell_\infty) + \chi^d S^d(Z^\ell_\infty)\right]\right\}. && (44) \end{aligned}$$

   Moreover, $w \to \hat{x}^\ell_\infty$ where

$$\hat{x}^\ell_\infty = \alpha^s(\theta^\ell_\infty)\mu\left[\chi^m S^m(a^\ell_\infty, Z^\ell_\infty) + \chi^d S^d(Z^\ell_\infty)\right] + x. \tag{45}$$

3. ***Limit as both markets become frictionless***. *Consider the limit as $A \to +\infty$ and $B \to +\infty$. Then, $Z \to y^* - \varphi(y^*)$ and $w \to x$. If $x > b$, then $\theta \to \theta^g_\infty$ where*

$$\theta^g_\infty = \frac{(1-\beta)(x-b)}{\beta k}. \tag{46}$$

If $x \le b$, then $\theta \to 0$.

As the frictions in the goods market vanish, the value of consumers' outside options exhausts the gains from trade and allocations converge to a perfect competition outcome where the firm's surplus is driven to zero. The quantities are efficient, $y \to y^*$, and the payment just covers the worker's cost, $p \to \varphi(y^*)$, i.e., the average markup goes to zero. Provided $x > b$, market tightness remains positive at the limit but it is independent of monetary policy, i.e., the long-run Phillips curve becomes vertical.

As the frictions in the labor market vanish, market tightness approches a finite and positive limit defined in (42). The job finding rate, $B\overline{f}(\theta^\ell_\infty)$, and the vacancy filling rate, $B\overline{f}(\theta^\ell_\infty)/\theta^\ell_\infty$, go to infinity. Hence, unemployment vanishes asymptotically and the Phillips curve becomes vertical at $u = 0$. The real wage approaches the average productivity of a worker so that the markdown tends to 0.

4 Calibrated Example

We now carry out a quantitative analysis by calibrating our model to the US between 1955-2005. A unit of time corresponds to one month. We assume the labor market matching function is Cobb-Douglas, with job finding rate given by $f(\theta) = A\theta^\eta$, and the goods market matching function is $\alpha(q) = q/(1+q)$, following Kiyotaki and Wright (1993). The cost of production in bilateral meetings is $\varphi(y) = Gy^g$. We set $\lambda = 1$ and $\gamma = 0$, so that the desire to consume arrives, on average, once a month and never disappears. This captures a reasonable mid-point between goods that are purchased relatively frequently, such as groceries, and goods that are purchased relatively infrequently, such as durables, but we will also illustrate how our results depend on λ.

The parameters ρ, k, δ, A, η, β, and b are fixed using an approach close to that in BMW. We set $\rho = 0.001$ so that the real interest rate in the model matches the difference between the rate on Aaa bonds and realized inflation, on average. We use k and δ to match the average unemployment rate and unemployment-to-employment (UE) transition rate. The parameter A is normalized so that the vacancy rate is 1. The elasticity of the matching function η targets the regression coefficient of labor market tightness θ on the UE rate of 0.6. Firms' bargaining power in the labor market, $1-\beta$, is set to match an average wage markdown relative to the firm's marginal revenue product of labor, $1 - w/\hat{x}$, of 0.35 following evidence in Yeh et al. (2022). Unemployment benefits b are equal to 40% of w, following Shimer (2005) and we set the utility value of non-employment ℓ such that $(b+\ell) = 0.5\hat{x}$.[10]

[10] The effects of inflation on unemployment are channeled through labor productivity. Hence, we aim to capture the extent to which movements in labor productivity affect unemployment, as is studied in the large literature following Shimer (2005). Our target for ℓ represents a conservative estimate between the calibrations of Shimer (2005) with $\ell = 0$, Hall and Milgrom (2008) with $(b+\ell) = 0.71\hat{x}$, and Hagedorn and Manovskii (2008) with $(b+\ell) = 0.95\hat{x}$.

We follow Hall and Milgrom (2008), and set ℓ such that $(b+\ell) = 0.71\hat{x}$, where $\hat{x} = \alpha^s \mu \left[\chi^m S^m + \chi^d S^d\right] + x$ is average output per worker. We calibrate G, g, and μ as in the New Monetarist literature. We set G and g to match the relationship between money demand M/pY and i in the data, using the adjusted M1 series in Lucas and Nicolini (2015) as our measure of money. We set μ so the markup in the goods market is 30%, as discussed by Faig and Jerez (2005). We set $\chi_m = 0.8$ because in the Atlanta Fed data discussed by Foster et al. (2013), credit cards account for 23% of purchases in volume. In the Bank of Canada data discussed by Arango and Welte (2012), this number is 19%.

The targets and parameter values discussed above are summarized in Table 1.

Parameter	Description	Targets	Value
ρ	Rate of time preference	Average real interest rate	0.001
k	Vacancy cost	Average unemployment rate	0.39
δ	Job destruction rate	Unemployment-to-employment rate	0.03
A	Level of labor market matching	Average vacancies (normalization)	0.25
η	Elasticity of labor market matching	Elasticity of UE rate	0.60
β	Bargaining power of worker in labor market	Wage markdown	0.03
b	Unemployment benefits	$b = 0.4w$	0.31
ℓ	Value of leisure	$(b+\ell) = 0.71\hat{x}$	0.29
λ	Arrival of desire to consume	—	1
γ	Desire to consume disappears	—	0
χ_m	Fraction of monetary meetings	Fraction of credit card transactions	0.8
G	Level of production cost	Level of money demand	0.58
g	Elasticity of production cost	Elasticity of money demand	1.28
μ	Bargaining power of firms in goods market	Retail sector markup	0.88

Table 1: Calibrated parameters

In Figure 5, we illustrate how long-run inflation affects the unemployment rate (left panel), labor productivity (middle panel), and consumers' outside option (right panel) under the baseline calibration. As predicted by Proposition 4, the Philips curve and labor productivity are non-monotone in π. The reason is that consumers' outside option, Z, decreases with inflation since money holdings become more costly (right panel). Even though lower money holdings constrain the payment that consumers can make to firms, their lower outside option improves firms' market power in goods markets, increases their markup as illustrated in the left panel of Figure 6, and increases their expected revenue from a filled vacancy. The outside option effect is quantitatively dominant for annual inflation rates between the Friedman rule and 0.4%, the latter representing the unemployment-minimizing inflation rate. As inflation rises beyond 0.4%, the effect on liquidity constraints dominates and the Phillips curve turns upward-sloping.

While inflation unambiguously increases firms' market power in the goods market, there is a non-monotone effect on their market power in labor markets. The right and middle panels of Figure 6 illustrate the effect of inflation on wages and the wage markdown, respectively. For low inflation rates, higher inflation increases wages and reduces the wage markdown. The increase in firms' market power in goods markets passes through to higher wages and dominants the effect of inflation on decreasing consumers' payment ca-

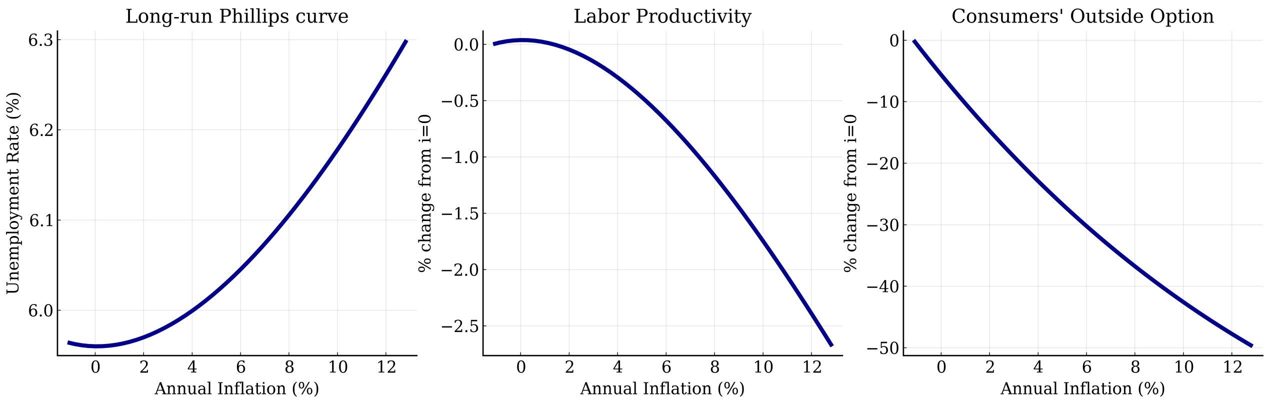

Figure 5: The effects of inflation on unemployment (left), labor productivity (middle), and consumers' outside options (right)

pacity. For high inflation rates, the second channel tends to dominate and wages fall, or the wage markdown rises, with inflation.

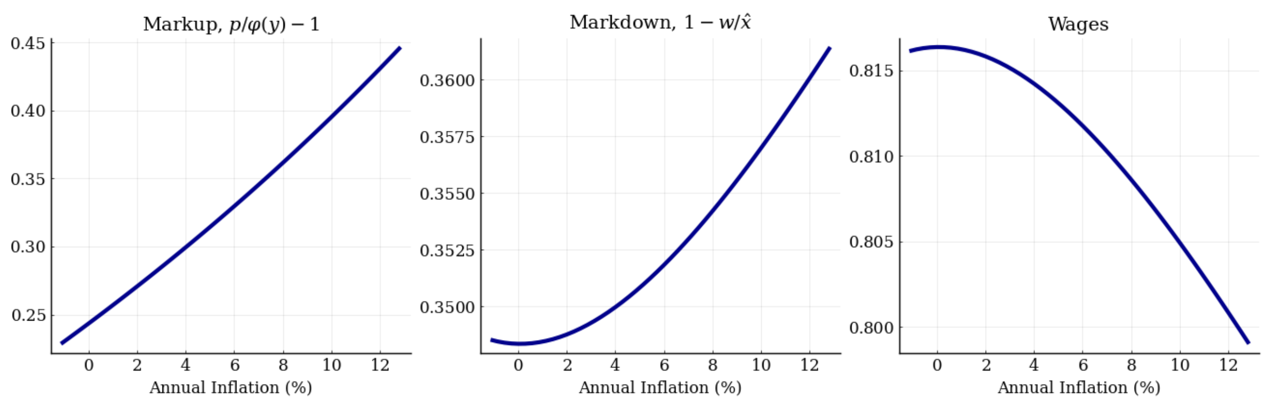

Figure 6: The effects of inflation on price markups (left), wage markdowns (middle), and wages (right)

The strength of the consumer search channel depends on the rate at which the desire to consume arrives, λ. If the desire to consume occurs relatively infrequently, then consumers' opportunity cost of consumption is relatively large since the value of returning to being idle is relatively low. On the other hand, if consumers spend little time remaining idle, the opportunity cost of consumption is low. In Figure 7, we illustrate the effects of λ on the shape of the long-run Phillips curve and the unemployment-minimizing level of inflation. For each value of λ, we recalibrate all other model parameters as outlined above.

The left panel of Figure 7 shows that for low values of λ the consumer search channel is quantitatively strong. For instance, if the desire to consume occurs on average once per quarter, illustrated in the solid-blue curve, the long-run Phillips curve is downward sloping for annual inflation rates up to 4%. The right panel illustrates how the unemployment-minimizing inflation rate changes as λ varies from close to zero to 6. For low values of λ, inflation as high as 9% can decrease unemployment. As the speed of the desire to consume

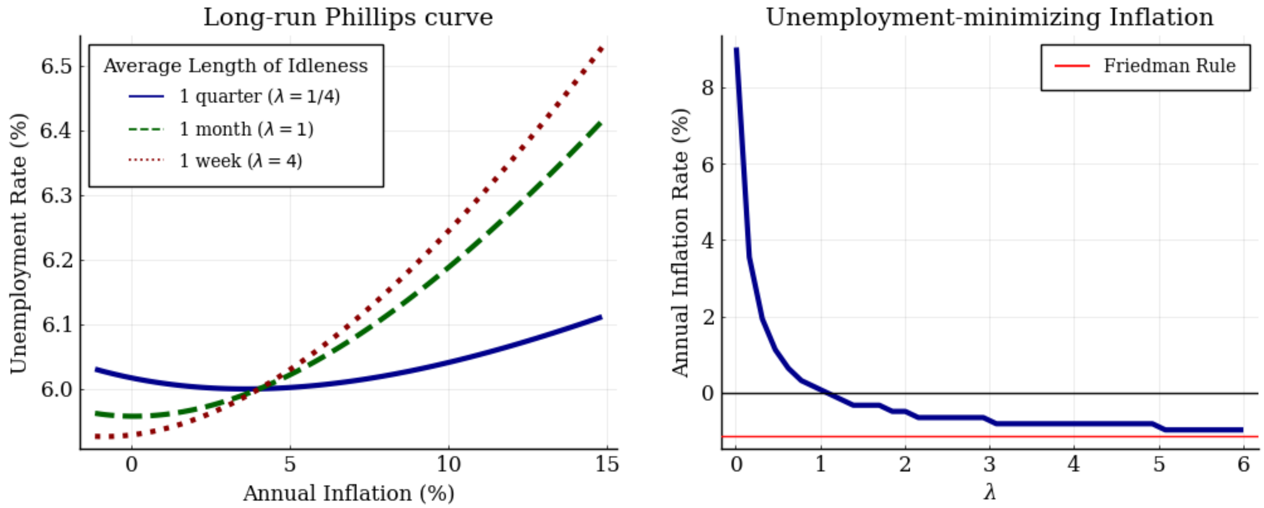

Figure 7: The Long-run Phillips curve and λ (left); Unemployment-minimizing inflation rate and λ (right)

increases, the unemployment-minimizing inflation rate decreases towards the Friedman Rule, without hitting it.

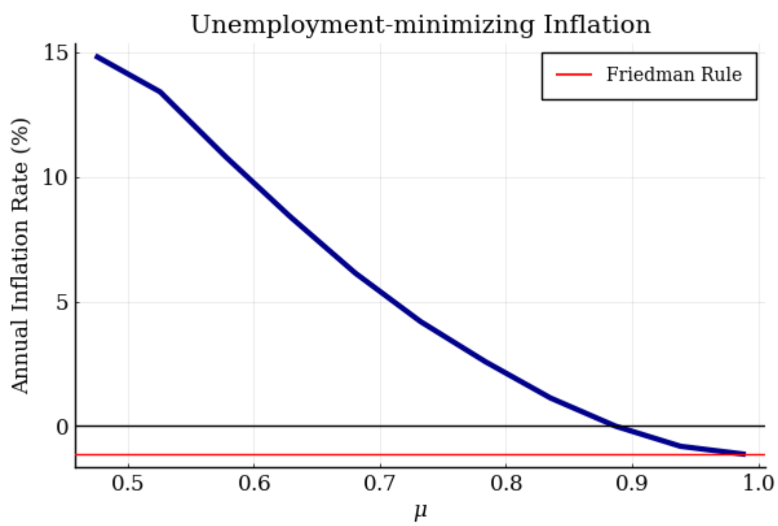

Figure 8: Unemployment-minimizing inflation rate and μ.

The role of consumer search also depends on the weight that consumers' outside option is given in bargaining in the goods market, $1-\mu$. Figure 8 illustrates how the unemployment minimizing rate of inflation depends on μ. As firms' bargaining power increases, consumers' outside option falls and the consumer search channel is diminished. This leads to a more positively-sloped long-run Phillips curve and a lower unemployment-minimizing inflation rate.

5 Equilibrium consumer search

In the model of Section 3, consumers never exert their option to search once matched with a firm on the equilibrium path. The option of searching for alternative producers is a threat that affects the surplus of the match and the terms of trade, and determines the extent of firms' market power. We now extend our model with horizontally differentiated products to introduce search on the equilibrium path.

The preference of a consumer for good y is now εy where ε is a random variable capturing the idiosyncratic taste of the consumer. When a consumer meets a firm, $\varepsilon \in [0, \bar{\varepsilon}]$ is drawn from a cumulative distribution $F(\varepsilon)$ and is common-knowledge in the match.[11] The consumer can then decide to engage a negotiation with the firm or to keep searching for an alternative producer.

Consider a match between a consumer with a^* real balances and a firm when the realization of the preference shock is ε. There are gains from trade if

$$\max_{y \geq 0} \{\varepsilon y - \varphi(y) : \varphi(y) \leq a^*\} > Z, \tag{47}$$

which has a similar interpretation as (4). The next lemma characterizes the threshold for ε above which the condition (47) holds.

Lemma 1 ***(Optimal search)*** *The threshold for ε above which gains from trade are positive obeys:*

$$\begin{aligned} \varepsilon_R(a^*, Z) &= \hat{\varepsilon}(Z) \quad \textit{if}\ \hat{\varepsilon}(Z) \leq \tilde{\varepsilon}(a^*) \\ &= \frac{a^* + Z}{\varphi^{-1}(a^*)} \quad \textit{otherwise,} \end{aligned} \tag{48}$$

where $\hat{\varepsilon}(Z)$ is the solution to $\varepsilon y_\varepsilon^* - \varphi(y_\varepsilon^*) = Z$ and $\tilde{\varepsilon}(a^*) \equiv \varphi'[\varphi^{-1}(a^*)]$.

The threshold ε_R can take two values. If a^* is large, it is equal to $\hat{\varepsilon}(Z)$ which only depends on the consumer's outside options. It is the value of ε such that the match surplus is zero when the quantity traded, y, is efficient. If a^* is small, the liquidity constraint binds and ε_R depends on both a^* and Z. It increases with Z but decreases with a^*. If the consumers'outside options improve, they become pickier. If they hold more real balances, they are willing to buy goods that they value less and compensate by buying larger quantities.

For all $\varepsilon \geq \varepsilon_R$, the determination of the terms of trade is given by the proportional solution,

$$\max_{p,y} \{p - \varphi(y)\} \quad \text{s.t. } p - \varphi(y) = \mu\left[\varepsilon y - \varphi(y) - Z\right] \quad \text{and} \quad p \leq a. \tag{49}$$

We denote $y_\varepsilon(a, Z)$ the solution to (49). Whenever there are gains from trade, the surpluses in monetary and credit matches are

$$S_\varepsilon^m(a, Z) \equiv \varepsilon y_\varepsilon(a, Z) - \varphi\left[y_\varepsilon(a, Z)\right] - Z \quad \text{if} \quad \varepsilon \geq \varepsilon_R(a, Z) \tag{50}$$

$$S_\varepsilon^d(Z) \equiv \varepsilon y_\varepsilon^* - \varphi(y_\varepsilon^*) - Z \qquad \text{if} \quad \varepsilon \geq \hat{\varepsilon}(Z). \tag{51}$$

[11] One can give several interpretations for ε. For instance, firms produce different varieties of good y and consumers value these different varieties differently. Alternatively, the intensity for the desire to consume could be varying over time.

If there are no gains from trade, then $S_\varepsilon^\chi = 0$ for $\chi \in \{m, d\}$. The difference between (50) and (51) is the fact that the determination of the surplus in monetary matches is subject to a liquidity constraint. Following the same reasoning as above, the outside option of the consumer, Z, solves:

$$(\rho + \lambda + \gamma) Z = \max_{a^* \geq 0} \left\{ -ia^* + \alpha(1-\mu) \int \left[\chi^m S_\varepsilon^m(a^*, Z) + \chi^d S_\varepsilon^d(Z) \right] dF(\varepsilon) \right\}. \tag{52}$$

This equation has a similar interpretation as (14). There is a unique solution $Z \in [0, \bar{\varepsilon} y_{\bar{\varepsilon}}^* - \varphi(y_{\bar{\varepsilon}}^*)]$. It is an increasing function of α and a decreasing function of μ. If the optimal choice of real balances is interior, it solves

$$\int_{\varepsilon_R(a^*,Z)}^{\bar{\varepsilon}} \frac{\alpha \chi^m (1-\mu) \left[\varepsilon - \varphi'(y_\varepsilon) \right]}{\mu \varepsilon + (1-\mu)\varphi'(y_\varepsilon)} dF(\varepsilon) = i, \tag{53}$$

where $y_\varepsilon = y_\varepsilon(a^*, Z)$. The left side, which represents the marginal benefits from holding real balances, is decreasing in a^* so, if it exists, there is a unique solution to (53).

The free-entry condition (22) in the labor market is generalized to give:

$$\begin{aligned} (\rho + \delta) \frac{k\theta}{f(\theta)} + \beta k \theta \quad = \quad & (1-\beta) \times \\ & \left\{ \alpha^s(\theta)\mu \left[\chi^m \int_{\varepsilon_{R(a^*,Z)}}^{\bar{\varepsilon}} S_\varepsilon^m(a^*, Z) dF(\varepsilon) + \chi^d \int_{\hat{\varepsilon}(Z)}^{\bar{\varepsilon}} S_\varepsilon^d(Z) dF(\varepsilon) \right] + x - b \right\}. \end{aligned} \tag{54}$$

The two integrals on the right side represent the firm surpluses in all monetary and credit matches where the gains from trade are positive.

The steady-state measure of active consumers solves

$$\left\{ \gamma + \alpha \chi^m \left[1 - F(\varepsilon_R) \right] + \alpha \chi^d \left[1 - F(\hat{\varepsilon}) \right] \right\} \omega_1 = \lambda(\omega - \omega_1). \tag{55}$$

The left side is the flow of consumers who become inactive, either because their desire for consumption vanishes at rate γ or because they meet a firm that produces a variety of the good that they want to consume. The right side represents the flow of inactive consumers that become active at rate λ. Using that $q\omega_1 = n$, (55) can be rewritten to obtain the following relationship between q and θ:

$$\frac{\lambda \omega q}{\gamma + \alpha(q)\chi^m \left[1 - F(\varepsilon_R) \right] + \alpha(q) \chi^d \left[1 - F(\hat{\varepsilon}) \right] + \lambda} = \frac{f(\theta)}{\delta + f(\theta)}. \tag{56}$$

So the tightness of the goods market increases with θ but decreases with ε_R and $\hat{\varepsilon}$.

An equilibrium is defined as a list, $\langle \varepsilon_R, Z, a^*, \theta, q \rangle$, solution to (48), (52), (53), (54), and (56). In the following we consider equilibria in the neighborhood of the Friedman rule when $i = 0^+$. After some substitutions, such equilibria can be reduced to pairs, (θ, Z), that are solution to (52) and (54).

Proposition 6 ***(Long-run Phillips curve and consumer search)*** *Assume $x > b$, $\rho + \lambda + \gamma$ is small, and $i = 0^+$.*

1. *$y_\varepsilon = y_\varepsilon^*$ for all $\varepsilon \geq \varepsilon_R(a^*, Z) = \hat{\varepsilon}(Z)$. An increase in λ or γ reduces Z and ε_R, and raises θ.*

2. *A small increase in i from $i = 0^+$ generates an increase in θ, and a decrease in u, Z, and ε_R.*

The assumption that the effective discount rate, $\rho + \lambda + \gamma$, is small allows us to focus on equilibria where the curve representing (54) cuts the curve representing (52) from above in the (θ, Z) space, as shown in Figure 9.

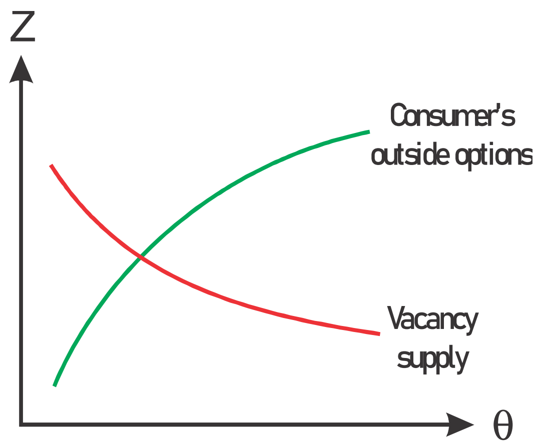

Figure 9: Equilibrium with consumer search when $i = 0^+$

The first part of Proposition 6 establishes the links between market power, velocity of money (as captured by $\alpha(q)\chi^m [1 - F(\varepsilon_R)] + \alpha(q)\chi^d [1 - F(\hat{\varepsilon})]$), and unemployment. As λ or γ rises, the value of searching for other producers falls and hence firms' market power raises. Consumers spend their real balances on goods that they value less, which raises the velocity of money. Firms' expected revenue rises, which incites them to open more vacancies and reduces unemployment.

The second part of Proposition 6 shows that the result according to which the long-run Phillips curve is downward sloping at low inflation rates is robust when one introduces differentiated goods and ex post match heterogeneity. As the inflation rate increases, consumer search becomes more costly. As a result, consumers become less choosy, ε_R decreases, and the value of their outside options decreases. As firms' market power increases, they open more vacancies, θ increases, and the unemployment rate decreases.

In Figure 10 we present a numerical example with horizontally differentiated products. The parameter values are the same as that in Table 1. We represent the baseline model ($\varepsilon = 1$) in blue and consider two examples of mean-preserving spread of ε: $\varepsilon \sim U[0.25, 1.75]$ in yellow and $\varepsilon \sim U[0, 2]$ in green. As the variance of ε increases, the option value of consumer search rises and thus Z rises. Consumers carry more real balances and more firms participate in the goods and labor market. Consequently, workers get a higher wage rate and the unemployment rate falls. The value of π^* falls as ε grows more dispersed and the Philips curve is more likely to be upward sloping.

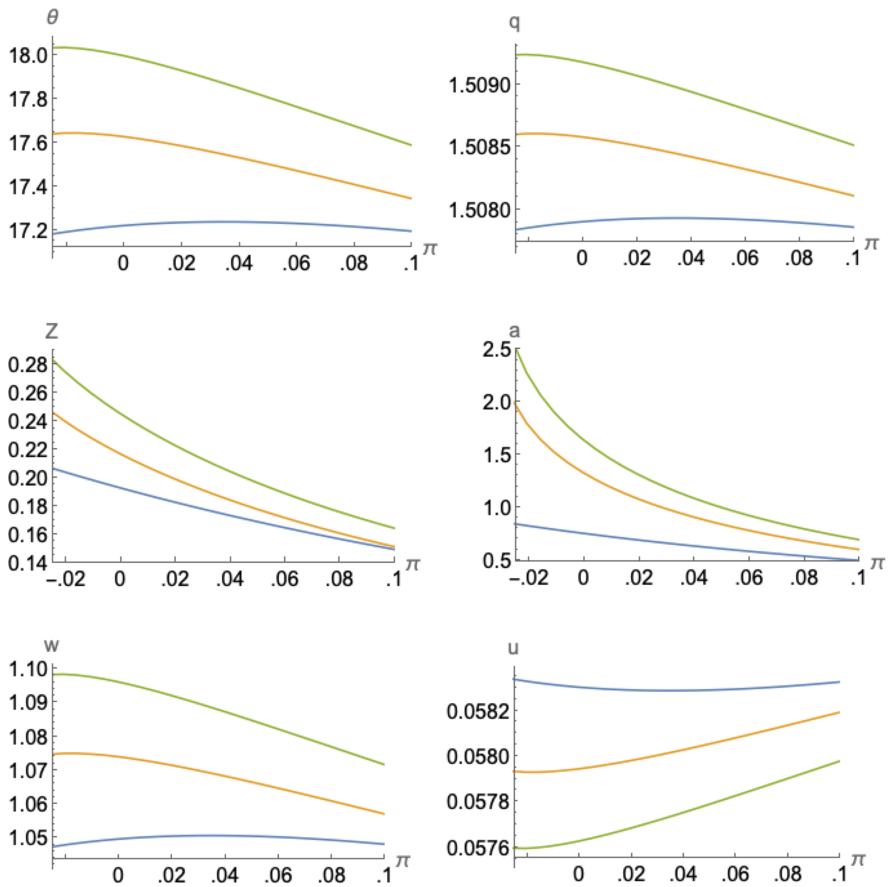

Figure 10: Mean-preserving spread of ε. Blue: $\varepsilon = 1$, yellow: $\varepsilon \sim U[0.25, 1.75]$ and green: $\varepsilon \sim U[0, 2]$.

6 Conclusion

The objective of this paper was to make a simple but robust observation regarding the long-run trade-off between unemployment and inflation. In the class of models pioneered by BMW, where goods and labor markets are frictional, and money plays an essential role to facilitate the exchange of goods and services, the relation between unemployment and inflation is $\cup$-shaped. As a result, the inflation rate that minimizes unemployment is above the one prescribed by the Friedman rule. This result is robust – it does not require parametric conditions to hold – once one introduces consumer search in order to endogenize consumer outside options and firms' market power.

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A Proofs of propositions and lemmas

Proof of Proposition 1.

Throughout we assume $\chi^d > 0$. Define $Z(\theta)$ the solution to (30) as a function of θ. To see that $Z(\theta)$ exists and is unique, note that the left side of (30) is linear, increasing in Z while the right side (RHS) is decreasing for all $Z \in (0, y^* - \varphi(y^*))$. If $Z = 0$ then $RHS \geq \alpha^b(\theta)(1-\mu)\chi^d [y^* - \varphi(y^*)] > 0$ if $\theta > 0$. If $Z = y^* - \varphi(y^*)$ then $RHS = 0$. Hence, for all $\theta > 0$, there is a unique $Z \in (0, y^* - \varphi(y^*))$ solving (30). Since $\alpha^b(\theta)$ is increasing in θ, it follows that RHS is increasing in θ, and hence $Z'(\theta) > 0$. Moreover, $\alpha^b(0) = 0$ implies $Z(0) = 0$.

We have seen that the set of solutions to (11) is either the corner solution 0, the interior solution given by (12), which we denote by $a_I^*(\theta)$, or both. Note that $a_I^*(\theta)$ is increasing continuously in θ. We define the correspondence $\text{N}^*(\theta)$ as the set of measures of consumers that choose to hold a_I^* solution to (12). If the interior solution, $a_I^*(\theta)$, is the unique solution to (11), then $\text{N}^*(\theta) = \{1\}$. If 0 is the unique solution, $\text{N}^*(\theta) = \{0\}$. When a_I^* and 0 are both solutions, then $\text{N}^*(\theta) = [0,1]$. By the Theorem of the Maximum, the set of maximizers, $a^*(\theta)$, is nonempty and thus $\text{N}^*(\theta)$ is nonempty and upper hemi-continuous.

We now define the following correspondence:

$$\begin{aligned}\Gamma(\theta) \equiv \Big\{ &(1-\beta)\left\{\alpha^s(\theta)\mu\left\{\chi^m \nu S^m\left[a^*(\theta), Z(\theta)\right] + \chi^d S^d\left[Z(\theta)\right]\right\} + x - b\right\} \\ &-\beta k\theta - (\rho+\delta)\frac{k\theta}{f(\theta)} : \nu \in \text{N}^*(\theta)\Big\}.\end{aligned} \tag{57}$$

Generalizing (28) to allow for asymmetric equilibria where buyers can hold different real balances, an equilibrium can be reduced to a θ solution to $0 \in \Gamma(\theta)$. As $\theta \to 0$, $a^*(\theta) \to 0$, so that any selection from $\Gamma(\theta)$ converges to the value

$$(1-\beta)\left\{\alpha^s(0)\mu\chi^d\left[y^* - \varphi(y^*)\right] + x - b\right\}.$$

Since $\alpha^s(0) = +\infty$, $\Gamma(0) = \{+\infty\}$. As $\theta \to +\infty$, $\alpha^s(\theta) \to 0$, $\theta/f(\theta) \to 1/f'(+\infty) = +\infty$ and hence any selection from $\Gamma(\theta)$ converges to $-\infty$. By the upper hemi-continuity of Γ, there exists a $\theta > 0$ such that $0 \in \Gamma(\theta)$. ■

Proof of Proposition 2. Equation (36) determines θ. The left side is increasing in θ from 0 to $+\infty$ while the right side is decreasing in θ from $+\infty$ to $z-b$. Hence, θ is unique. The right side of (36) is increasing in $\lambda+\gamma$. Hence, θ increases with λ and γ. It follows that the unemployment rate, $u = \delta/\left[\delta + f(\theta)\right]$, decreases with λ and γ. From (32), Z increases if and only if θ decreases. So Z is a decreasing function of λ and γ. Finally, by (34), w increases in λ and in γ. ■

Proof of Proposition 4. Part 1: Labor market tightness is determined by $\Gamma(\theta; i) = 0$ where from (57)

$$\Gamma(\theta;i) \equiv (1-\beta)\left\{\alpha^s(\theta)\mu\left\{\chi^m S^m\left[a^*(\theta;i), Z(\theta;i)\right] + \chi^d S^d\left[Z(\theta;i)\right]\right\} + x - b\right\} - \beta k\theta - (\rho+\delta)\frac{k\theta}{f(\theta)},$$

and we make the relationships between a^* and i and Z and i explicit. When $i = 0$,

$$\Gamma(\theta; 0) \equiv (1-\beta)\left\{\frac{\alpha^s(\theta)\mu\left(\rho+\lambda+\gamma\right)}{\rho+\lambda+\gamma+\alpha^b(\theta)(1-\mu)}\left[y^* - \varphi(y^*)\right] + x - b\right\} - \beta k\theta - (\rho+\delta)\frac{k\theta}{f(\theta)}. \tag{58}$$

It is monotone decreasing in θ, so equilibrium at the Friedman rule is unique.

We now show that $\Gamma(\theta; i)$ is increasing in i in the neighborhood of the Friedman rule. From (8)

$$\frac{\partial S^m(a, Z)}{\partial a} \equiv \left[1 - \varphi'(y)\right]\frac{\partial y}{\partial a}.$$

In the neighborhood of $i = 0^+$, $y = y^*$ and $1 - \varphi'(y) = 0$. Hence, $\partial S^m(a, Z)/\partial a = 0$. The effect of a change in a^* induced by an increase in i on the match surplus is second order when i is close to 0 because the match surplus is maximum. However, from (8)-(9), when y is in the neighborhood of y^*,

$$\frac{\partial S^m(a, Z)}{\partial Z} = \frac{\partial S^d(Z)}{\partial Z} = -1.$$

From (30),

$$\frac{\partial Z}{\partial i} = \frac{-a^*}{\rho+\lambda+\gamma+\alpha^b(\theta)(1-\mu)},$$

where, at the Friedman rule, by (12) and (35)

$$a^* = \varphi(y^*) + \frac{\left(\rho+\lambda+\gamma\right)\mu}{\rho+\lambda+\gamma+\alpha^b(\theta)(1-\mu)}\left[y^* - \varphi(y^*)\right] > 0.$$

Combining these results, $\chi^m S^m\left[a^*(\theta; i), Z(\theta; i)\right] + \chi^d S^d\left[Z(\theta; i)\right]$ is increasing in i, and hence $\Gamma(\theta; i)$ is also increasing in i. It follows that θ such that $\Gamma(\theta; i) = 0$ is increasing in i. The unemployment rate, $u = \delta/\left[\delta + f(\theta)\right]$, is decreasing in θ and hence decreasing in i. By the definition of $\Gamma(\theta; i)$ and (31), we can reexpress $\Gamma(\theta; i)$ as

$$\Gamma(\theta; i) = \frac{(1-\beta)}{\beta}(w - b) - k\theta - (\rho+\delta)\frac{k\theta}{f(\theta)}.$$

Since $\Gamma(\theta; i) = 0$ in equilibrium, w and θ must comove as i changes. Therefore, w rises in i.

Part 2: We now consider the limiting case $i = +\infty$. Since agents do not carry money, the outcome is similar to a pure credit economy but with $\chi^d < 1$. From (29) $a^* = 0$. By the steps leading to (35)

$$Z = \frac{\alpha^b(\theta)\chi^d(1-\mu)}{\rho+\lambda+\gamma+\alpha^b(\theta)\chi^d(1-\mu)}\left[y^* - \varphi(y^*)\right]$$

and hence

$$\Gamma(\theta; +\infty) \equiv (1-\beta)\left\{\frac{\alpha^s(\theta)\mu\chi_d\left(\rho+\lambda+\gamma\right)}{\rho+\lambda+\gamma+\alpha^b(\theta)\chi^d(1-\mu)}\left[y^* - \varphi(y^*)\right] + x - b\right\} - \beta k\theta - (\rho+\delta)\frac{k\theta}{f(\theta)}. \tag{59}$$

From (58) and (59), $\Gamma(\theta; +\infty) < \Gamma(\theta; 0)$ for all $\theta > 0$. So θ_0 solution to $\Gamma(\theta; 0) = 0$ is larger than $\theta_{+\infty}$ solution to $\Gamma(\theta; +\infty) = 0$. Hence, the unemployment rate when $i = 0$ is lower than the unemployment rate when $i = +\infty$. Since w and θ comove, w is lower when $i = +\infty$ than when $i = 0$. Finally, note that there is a finite upper bound for i above which a monetary equilibrium does not exist. This upper bound is

$i = \chi^m(1-\mu)\alpha^b(\theta)/\mu$ where θ is a bounded above by the market tightness of a pure credit economy with $Z=0$. This upper bounded of θ is finite and it solves

$$(1-\beta)\left\{\alpha^s(\theta)\mu\left[y^* - \varphi(y^*)\right] + x - b\right\} - \beta k\theta - (\rho+\delta)\frac{k\theta}{f(\theta)} = 0.$$

■

Proof of Proposition 5. From (27),

$$\frac{q}{\bar{\alpha}(q)} \to \frac{Af(\theta)}{\lambda\omega\left[\delta + f(\theta)\right]} = +\infty \text{ for all } \theta > 0.$$

So, if $\theta > 0$, $q \to +\infty$ and the firm matching rate with a consumer tends to $A\bar{\alpha}(q)/q \to \lambda\omega\left[\delta + f(\theta)\right]/f(\theta)$. Since $A\bar{\alpha}(q) \to +\infty$, from (36), $Z \to y^* - \varphi(y^*)$. Using that $S^m(a^*, Z) \to 0$ and $S^d(Z) \to 0$, from (28), θ solves

$$(\rho+\delta)\frac{k\theta}{f(\theta)} = (1-\beta)(x-b) - \beta k\theta.$$

It is consistent with $\theta > 0$ iff $x > b$. Hence, if $x \leq b$ then $\theta \to 0$ as $A \to +\infty$. Suppose $q \to q_\infty > 0$. Then, $\alpha^b = A\bar{\alpha}(q) \to +\infty$ and, from (30), $Z \to y^* - \varphi(y^*)$. If $q_\infty = 0$, then $A\bar{\alpha}(q)/q \to +\infty$ and, from (28), $\theta = 0$ implies $Z \to y^* - \varphi(y^*)$. ■

Proof of Lemma 1. The inequality (47) holds iff $\varepsilon > \varepsilon_R$ where the reservation value for the preference shock, $\varepsilon_R(a^*, Z)$, solves

$$\max_{y \geq 0}\left\{\varepsilon_R y - \varphi(y)\right\} = Z \quad \text{s.t.} \quad \varphi(y) \leq a^*. \tag{60}$$

We distinguish two cases depending on whether or not the constraint, $\varphi(y) \leq a^*$, binds.

Case #1. If the constraint $\varphi(y) \leq a^*$ is not binding, then $\varepsilon_R = \hat{\varepsilon}(Z)$ is the solution to $\varepsilon y_\varepsilon^* - \varphi(y_\varepsilon^*) = Z$ where $y_\varepsilon^* = \arg\max\left\{\varepsilon y - \varphi(y)\right\}$. We now check the condition under which $\varphi(y_{\hat{\varepsilon}}^*) \leq a^*$ is slack. Using that φ is an increasing bijection, we have:

$$\varphi(y_{\hat{\varepsilon}}^*) \leq a^* \Longleftrightarrow y_{\hat{\varepsilon}}^* \leq \varphi^{-1}(a^*).$$

Apply the increasing function φ' on both sides to obtain:

$$\varphi(y_{\hat{\varepsilon}}^*) \leq a^* \Longleftrightarrow \varphi'(y_{\hat{\varepsilon}}^*) \leq \varphi' \circ \varphi^{-1}(a^*).$$

Use the definition of $y_{\hat{\varepsilon}}^*$, i.e., $\varphi'(y_{\hat{\varepsilon}}^*) = \hat{\varepsilon}$, to rewrite the inequality above as:

$$\varphi(y_{\hat{\varepsilon}}^*) \leq a^* \Longleftrightarrow \hat{\varepsilon} \leq \tilde{\varepsilon}(a^*) \equiv \varphi' \circ \varphi^{-1}(a^*).$$

Case #2. If the constraint, $\varphi(y) \leq a^*$, is binding then $y = \varphi^{-1}(a^*)$ so that ε_R solves $\varepsilon_R\varphi^{-1}(a^*) - a^* = Z$. Solving for ε_R we obtain $\varepsilon_R = (a^* + Z)/\varphi^{-1}(a^*)$. ■

Proof of Proposition 6. Part 1. From (53), $y_\varepsilon = y_\varepsilon^*$ for all $\varepsilon \geq \varepsilon_R$, i.e., agents trade the efficient quantities in all matches where there are gains from trade. This requires $a^* = \varphi(y_{\bar{\varepsilon}}^*) + \mu\left[\bar{\varepsilon}y_{\bar{\varepsilon}}^* - \varphi(y_{\bar{\varepsilon}}^*) - Z\right]$.

We now prove that $\varepsilon_R(a^*, Z) = \hat{\varepsilon}(Z)$. From (52), $Z < \bar{\varepsilon} y^*_{\bar{\varepsilon}} - \varphi(y^*_{\bar{\varepsilon}})$ since otherwise $S^m_{\varepsilon}(a^*, Z) = S^d_{\varepsilon}(Z) = 0$ and

$$(\rho + \lambda + \gamma) Z = \alpha(1 - \mu) \int \left[\chi^m S^m_{\varepsilon}(a^*, Z) + \chi^d S^d_{\varepsilon}(Z)\right] dF(\varepsilon) = 0,$$

which is a contradiction. Using that $Z < \bar{\varepsilon} y^*_{\bar{\varepsilon}} - \varphi(y^*_{\bar{\varepsilon}})$, $\varphi^{-1}(a^*) > y^*_{\bar{\varepsilon}}$ and $\tilde{\varepsilon}(a^*) \equiv \varphi'[\varphi^{-1}(a^*)] > \varphi'(y^*_{\bar{\varepsilon}}) = \bar{\varepsilon}$. Moreover, $\hat{\varepsilon}(Z) < \bar{\varepsilon}$. Hence, by Lemma 1, $\varepsilon_R(a^*, Z) = \hat{\varepsilon}(Z)$.

We now show how to reduce an equilibrium to a pair (θ, Z) solution to two equations. From (56),

$$\frac{\lambda \omega q}{\gamma + \alpha(q)\left[1 - F(\hat{\varepsilon})\right] + \lambda} = \frac{f(\theta)}{\delta + f(\theta)}. \tag{61}$$

From (61), $q = Q(\theta, \hat{\varepsilon})$ where Q is increasing in θ and decreasing in $\hat{\varepsilon}$. From (52), Z solves

$$(\rho + \lambda + \gamma) Z = \alpha\left[Q(\theta, \hat{\varepsilon}(Z))\right](1 - \mu) \int_{\hat{\varepsilon}(Z)}^{\bar{\varepsilon}} S^d_{\varepsilon}(Z) dF(\varepsilon). \tag{62}$$

From (62), Z is an increasing function of θ. From (54), θ solves

$$(\rho + \delta) \frac{k\theta}{f(\theta)} + \beta k \theta = (1 - \beta) \times \left[\alpha^s\left[Q(\theta, \hat{\varepsilon}(Z))\right] \mu \int_{\hat{\varepsilon}(Z)}^{\bar{\varepsilon}} S^d_{\varepsilon}(Z) dF(\varepsilon) + x - b\right]. \tag{63}$$

When Z is close to $\bar{\varepsilon} y^*_{\bar{\varepsilon}} - \varphi(y^*_{\bar{\varepsilon}})$, the effect of a change in Q on the right side is negligeable. In that case, θ is a decreasing function of Z. From (62), as $\rho + \lambda + \gamma$ approaches 0, for all $\theta > 0$, Z approaches $\bar{\varepsilon} y^*_{\bar{\varepsilon}} - \varphi(y^*_{\bar{\varepsilon}})$. From (63), θ approaches the positive solution to

$$(\rho + \delta) \frac{k\theta}{f(\theta)} + \beta k \theta = (1 - \beta) \times (x - b).$$

An increase in λ or γ shifts the curve representing (62) downward in the space (θ, Z). See Figure 9. As a result Z decreases while θ increases. It follows that $\varepsilon_R = \hat{\varepsilon}(Z)$ decreases.

Part 2. Consider now a small increase in i from $i = 0^+$. We still have $\varepsilon_R(a^*, Z) = \hat{\varepsilon}(Z)$ so that $q = Q(\theta, \hat{\varepsilon})$ defined by (61). Any change in a^* only has a second-order effect on $S^m_{\varepsilon}(a, Z)$, hence (52) is approximated by

$$(\rho + \lambda + \gamma) Z = -ia^* + \alpha\left[Q(\theta, \hat{\varepsilon})\right](1 - \mu) \int S^d_{\varepsilon}(Z) dF(\varepsilon).$$

As i increases, the curve representing (52) shifts downward in the (θ, Z) space. The equilibrium condition (54) is still approximated by (63). Since we start from an equilibrium where the curve representing (63) intersects the curve representing (62) by above, θ increases, Z decreases, and $\varepsilon_R = \hat{\varepsilon}$ decreases. ■

B Alternative Interpretation of Production Cost

Previously we assume the production cost $\varphi(y)$ is a disutility paid by the entrepreneur or manager. An alternative interpretation is that $\varphi(y)$ is a production cost paid by the worker and it is compensated by the wage w. The interpretation of $\varphi(y)$ does not matter for allocations, but it affects the definition of wage w in (18) and thus the calculations of wage markdown. In this section we provide the formulas for wage and wage markdown under this alternative interpretation.

Given the new interpretation of w and $\varphi(y)$, the worker now pays the variable cost of productions, hence equation (18) now becomes

$$\rho E = w - \alpha^s \left[\chi^m \varphi(y^m) + \chi^d \varphi(y^*)\right] - \delta\beta J. \tag{64}$$

By the logic leading to (23), the wage w can be reexpressed as

$$w = \beta \left\{\alpha^s \mu \left[\chi^m S^m(a^*, Z) + \chi^d S^d(Z)\right] + x\right\} + \alpha^s \left[\chi^m \varphi(y^m) + \chi^d \varphi(y^*)\right] + (1-\beta)b + \beta k\theta. \tag{65}$$

When compared with (23), the key novelty is the presence of the second term on the right side, which represents the compensation to the worker for the variable cost of production.

We can compute the wage markdown as in (25). But since the variable cost of production is paid by the worker and compensated by wages, we do not need to subtract the variable cost when calculating the net expected revenue of a firm. Hence $\hat{x} = \mathbb{E}[p] + x$ or equivalently

$$\hat{x} = \alpha^s \left\{\chi^m \varphi(y^m) + \chi^d \varphi(y^*) + \mu \left[\chi^m S^m(a^*, Z) + \chi^d S^d(Z)\right]\right\} + x. \tag{66}$$

The markdown is

$$\begin{aligned} MKDOWN &\equiv \frac{\hat{x} - w}{\hat{x}} \\ &= \frac{(1-\beta)\left\{\alpha^s \mu \left[\chi^m S^m(a^*, Z) + \chi^d S^d(Z)\right] + x - b\right\} - \beta k\theta}{\hat{x}}. \end{aligned} \tag{67}$$

The numerator is the same as that in (25), which represents the net profit of the firm-work match. But now the net expected revenue of the firm in the denominator includes the variable cost of production, as captured by the difference between (24) and (66).