Monetary Policy in a Dollarized Economy*

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Abstract

We study different aspects of dollarization in an economy where both the local and a foreign currency can be used for transactions. In our benchmark model, agents face heterogeneous liquidity needs in a frictional goods market where fiat money is essential. In contrast to local currency, foreign currency offers a better rate of return, but sellers charge a fee to accept it as a payment. Different equilibria arise where one or both currencies circulate. We show that allowing foreign currency bank accounts (FCBAs) can have different welfare implications depending on parameter values. In addition, monetary policy have different welfare implications depending on whether FCBAs are allowed or not. Finally, we discuss the welfare implications of various policies that have been implemented in dollarized economies such as differential reserve requirements and a tax on FCBAs.

Keywords: money, dollarization, currency substitution, bank deposits, monetary policy, reserve requirements.

JEL classification: E41, E50.

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1 Introduction

Over the last thirty years, the holding by residents of a share of their liquid and illiquid assets as well as liabilities in the form of foreign currency-denominated instruments has been widespread in many developing and transition economies. Since the dollar is generally the main foreign currency chosen by domestic residents, this phenomenon is typically referred to as dollarization.[1] It is useful to distinguish three aspects of *dollarization* that broadly match the three functions of money. One is currency substitution, which reflects a situation where domestic agents switch part of their currency portfolio to foreign currency for transaction purposes.[2] Another aspect of dollarization is asset and/or deposit substitution, which consists when agents switch their financial assets or liabilities towards those that are denominated in foreign currency. Finally, there is the role of unit of account that foreign currencies can play when denominating prices of domestic goods and services.[3] Among the various aspects of dollarization, understanding currency substitution and the consequences of foreign borrowing has been extensively studied in the literature, while deposit substitution (also referred as deposit dollarization) has received much less attention.[4] This is despite evidence suggesting its quantitative importance in many emerging markets.[5] In this paper we contribute to this latter literature by considering market frictions in domestic financial markets and explore the consequences for domestic monetary policy and welfare of allowing foreign denominated bank accounts.

In spite of a major reduction in inflation, a shift towards fiscal consolidation, having more central bank independence and improvements in financial markets deposit; deposit dollarization, defined as the share of foreign currency deposits/credit in total deposits/credit, remains a common and persistent phenomenon around the world. Bannister et al. (2018), for instance, find that

[1]We refer to Gelos et al. (2008) for more on dollarization.

[2]Much of the literature has focused on this aspect of *dollarization*. These papers were driven by the hyperinflationary episodes of Latin American countries that induced agents to switch to dollars. More recently, authors have studied *euroization* as a response of Central, Eastern and South-Eastern European countries to turbulent episodes in the late 1990s and early 2000s.

[3]We refer to Drenik and Perez (2021) for more on this aspect.

[4]Prominent examples that studied foreign borrowing are Diaz-Alejandro (1985), Burnside et al. (2001b), Caballero and Krishnamurthy (2003), and Céspedes et al. (2004) among others. For currency substitution we refer to Savastano (1992), Calvo and Végh (1996) and Uribe (1997), among others.

[5]For the importance of deposit dollarization we refer to Bennett et al. (1999) and Gomis-Porqueras et al. (2005), Capasso and Neanidis (2019) and the seminal paper of Broda and Yeyati (2006).

average deposit dollarization in 2015 was approximately 30 percent. Thus, having a framework that allows us to explore how currency substitution and deposit dollarization affect consumption, savings, payment and settlement choices by domestic agents and how the presence of foreign currency impacts domestic policies is critically relevant for many small open economies. Following Broda and Yeyati (2006), we consider an environment where deposit dollarization is an equilibrium outcome and a direct consequence of agents optimally choosing their currency portfolios. They do so in response to frictions in the economic environment. Agents face stochastic trading opportunities in frictional goods markets, where buyers can not credibly commit to repay sellers. As a result, trade credit in these markets is not feasible, thus a medium of exchange is required for trade to take place in these frictional markets. Moreover, sellers find it costly to recognize foreign currency relative to the domestic one.

Other than the frictions previously described, accounting for tax evasion when studying dollarization is important, as Holman and Neanidis (2006) emphasize.[6] This is the case as these phenomena is a response to fiscal imbalances and high inflation rates.[7] Within Holman and Neanidis (2006) spirit, we allow agents to evade paying taxes by transacting with domestic and foreign money in decentralized and frictional markets, as in Gomis-Porqueras et al. (2014).[8] Once agents open bank accounts, then the holdings of fiat money are recorded and governments can use this information to levy taxes. In the benchmark environment we first assume that the taxing capacity is quite limited and the enforcement of such taxes is not possible. We then consider a situation where tax on foreign denominated bank deposits can be levied. As a result, allowing foreign denominated bank accounts can increase tax revenues, reducing the reliance on seignorage income to raise government revenues as reported by Reinhart et al. (2014).[9]

Given the economic environment that we previously described, some natural questions then

[6] The authors find that seigniorage finance has stronger negative implications for growth over income-tax finance, in countries with less-developed financial markets.

[7] In the 1980s, Latin America struggled with recession, inflation and unemployment. The repeated failure of stabilization policies resulted in higher inflation rates, larger fiscal deficits, deeper external imbalances and continuous capital flight. Under these circumstances, individuals used the U.S. dollar as hard currency to protect their income from the detrimental effects of inflation.

[8] We refer to Schneider and Enste (2000) for a survey on the informal sector and how the use of fiat money helps evade taxes.

[9] Reinhart et al. (2014) explore whether dollarization leads to significant differences in the ability to raise revenue from seigniorage.

arise. Does deposit dollarization allow for greater financial development and/or efficiency? What are the implications for monetary and fiscal policy when foreign denominated bank deposits are allowed? Does deposit dollarization always increase welfare? How reserve requirements in foreign denominated bank accounts differ from a tax on deposits?

2 Related Literature

This paper contributes to two different strands of the literature. On that studies and proposes a theoretical framework to generate currency substitution as an equilibrium outcome. The other one mainly focuses on the financial dollarization and considers various market imperfections that result in increases in welfare when foreign denominated assets and instruments are allowed.

Currency substitution is a topic that has a long tradition in monetary economics. Within the theoretical literature, Guidotti and Rodriguez (1992) present a model with perfect means of payment substitutability, in which agents face costs of adjusting their holdings of foreign currency. These costs result in an inflation band within which agents choose not to switch between currencies. Perfect means of payment substitutability is also assumed by Uribe (1997) and Valev (2010), who stress the role of network externalities as the source of non-linearities in the relationship between money demand and inflation.

Relative to these papers we consider a general equilibrium model where sellers find it costlier to settle transactions using foreign currency. In addition, agents have access to domestic and foreign denominated bank accounts that are able to provide some liquidity insurance. As a result of such financial services, they shape agents' portfolio choice between domestic and foreign currency.

Within the financial dollarization literature, Barajas and Morales (2003) point to the relative market power of borrowers and central bank intervention in foreign exchange markets in being important factors for economies to be financially dollarized. The role of incomplete credit markets is also highlighted by the fact that the presence of foreign banks tends to be associated with higher dollarization.[10] Along the same lines, Schneider and Tornell (2004) and Ranciere et al. (2010) show that implicit bail-out guarantees give incentives for debtors and creditors to write debt contracts

[10] Basso et al. (2011) suggest that in European transition economies there is a strong link between financial deepening, cross border banking activities and dollarization.

in foreign currency.[11] Such implicit guarantees are likely to be stronger, the larger the share of domestic borrowers which hold unhedged foreign currency debt. In addition, Broda and Yeyati (2006). These authors show that an equal treatment of domestic currency and dollar deposits in the event of a bank liquidation creates an incentive for banks to dollarize.[12] Moral hazard, related to government guarantees or other forms of regulation in the presence of asymmetric payoffs, can also lead to dollarization, to the extent that they insure dollar creditors and borrowers from large losses in the event of a large depreciation as in Burnside et al. (2001a).

Our paper is closely related to Ize and Yeyati (2003) propose a static CAPM model with risk-averse borrowers and lenders to deliver dollarization as an equilibrium outcome. They argue that domestic residents prefer to denominate contracts in foreign currency when its purchasing power in terms of domestic consumption is stable relative to that of domestic currency. Relative to these authors we consider a general equilibrium model where domestic and foreign denominated bank accounts provide some liquidity insurance.

3 Environment

The economic environment is based on Rocheteau and Wright (2005) and the open economy framework of Gomis-Porqueras et al. (2013). Time is discrete and continues forever. The small open economy is populated by two types of agents both of unit measure, labeled buyers and sellers. Agents discount periods at rate $\beta = \frac{1}{1+\rho} \in (0, 1)$. In each period, two markets open sequentially. These markets are characterized by different frictions and trading protocols.

At the beginning of a period, buyers and sellers enter a frictional decentralized goods market, which from now on we refer as DM. Here a subset of agents trade a perishable good in pairwise meetings.[13] In the second sub-period, all agents participate in a frictionless competitive centralized market, which from now on we denote as CM. In this market, all agents can produce and consume a perishable good as well as re-balance their portfolio. The CM good is taken to be the numéraire.

[11] The authors interpret the implicit bailout as a situation where there is a guarantee that the exchange rate of the local currency will not be allowed to depreciate.

[12] A bank liquidation is understood as the case in which the residual value of the bank is distributed on a pro rata basis among all depositors according to the value of their claims at the time of liquidation.

[13] As in Gomis-Porqueras et al. (2013), the DM good is only traded locally, capturing the non-tradable sector.

Meeting Technologies. Buyers and sellers face stochastic trading opportunities in DM. With probability σ, agents can find a counterparty to trade with, while with complementary probability $1-\sigma$ they do not. In contrast, in CM all agents can always trade with each other in competitive markets as well as have access to the foreign exchange market.

Preferences. Buyers obtain utility when consuming q units of the DM good and disutility when working h hours in CM. Thus, the buyer's lifetime expected utility is given by

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\varepsilon_t u(q_t)-h_t\right], \tag{1}$$

where ε_t represents a preference shock that buyers experience in period t in DM, but that they are aware of in CM of the previous period $t-1$. ε_t follows an *i.i.d.* process, and is drawn by agents at the beginning of the second sub-period of $t-1$. We assume ε_t follows a continuous distribution $F(\cdot)$ with support $[0,\bar{\varepsilon}]$.

Sellers, on the other hand, incur disutility when producing q units of the DM good and obtain utility when consuming x units of CM goods. Thus, the seller's lifetime expected utility is given by

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[-c(q_t)+x_t\right]. \tag{2}$$

Assets. The two durable assets in this economy are intrinsically useless fiat objects: a domestic currency (money 1), issued by the local government, and a foreign currency (money 2), issued by a foreign government. The supply of domestic currency $\{M_{1,t}\}$ is controlled by the local government through lump-sum CM taxes/transfers and grows at a constant rate $\gamma_1 = M_{1,t+1}/M_{1,t}$. Since we are dealing with a small open economy, domestic agents take the gross rate of return of the foreign currency as given. The exogenous return is equal to R_2. Agents can re-balance their portfolio of currencies in CM as they have access to a foreign exchange market where $\epsilon = \frac{\phi_{2,t}}{\phi_{1,t}}$ is the corresponding nominal exchange rate and $\phi_{1,t}$ ($\phi_{2,t}$) denotes the real value of domestic (foreign) currency in units of CM goods.[14]

[14] Since currencies are imperfect substitutes the nominal exchange rate is determinate. This is different from Gomis-Porqueras et al. (2017), where the threat of counterfeiting breaks the indeterminacy even when currencies are perfect substitutes.

Frictions. In DM buyers and sellers are anonymous. As a result, their trading histories are private, which implies that a medium of exchange is essential to trade in DM. Accepting domestic currency is costless. This is the case as buyers and sellers can easily identify domestic counterfeited currency. In contrast, the cost of accepting foreign currency is positive, and equal to κ. This reflects various sellers' costs such as the ones incurred when recognizing foreign notes in trade, verifying its authenticity, managing a second currency during every-days' transactions, transferring to the bank a foreign currency in addition to the domestic one, avoiding government foreign currency controls, among other reasons.

4 Benchmark model

As a benchmark we first consider and environment where domestic and foreign fiat currency can circulate. However, financial intermediation is not possible. Within this set up, we focus on stationary equilibria where allocations, the gross rates of return of domestic money $R_1 = \frac{\phi_{1,t+1}}{\phi_{1,t}} = \frac{1}{\gamma_1}$ and foreign currency $R_2 = \frac{\phi_{2,t+1}}{\phi_{2,t}}$ are constant over time. To economize on notation, we drop the subscript t unless necessary.

4.1 CM problem

At the beginning of the second sub-period, a buyer enters a frictionless competitive market with wealth ω, denominated in terms of the CM good. The buyer experiences a shock, ε, that determines the payoff from consuming in the next period. In this market, agents choose their portfolio of domestic and foreign money holdings to bring forward to the next period. Formally, buyers solve the following problem

$$W^b(\omega, \varepsilon) = \max_{h,z_1,z_2} \left\{-h + \beta V^b(z_1, z_2, \varepsilon)\right\} \text{ subject to } \frac{z_1}{R_1} + \frac{z_2}{R_2} = h + \omega + \tau \tag{3}$$

where W^b and V^b denote the buyer's CM and DM value functions, respectively. τ represents taxes/lump-sum transfers from the government and (z_1, z_2) are next-period real balances of domestic and foreign currency in units of the CM good, respectively. In particular, $z_j = \phi_j m_j$ and m_j are the nominal holdings of currency $j = \{1, 2\}$. Substituting h from the budget constraint into the objective function, we have the following

$$W^b(\omega,\varepsilon) = \omega + \tau + \max_{z_1,z_2} \left\{ -\frac{z_1}{R_1} - \frac{z_2}{R_2} + \beta V^b(z_1, z_2, \varepsilon) \right\}. \tag{4}$$

The expected value function $Z^b(\omega)$, before entering the CM, that is before ε is realized, is linear in wealth. In particular, we have that

$$Z^b(\omega) = \int_0^{\bar{\varepsilon}} W^b(\omega,\varepsilon) dF(\varepsilon) = \omega + Z^b(0). \tag{5}$$

Finally, the seller's CM value function is similar, but if carrying currencies across periods is costly (agents face positive nominal interest rates), sellers will choose not to hold any of the fiat currencies. This is the maintained assumption throughout the rest of the paper.

4.2 DM problem

Once the shock has been realized, some buyers and sellers are bilaterally matched and trade according to a buyer's take-it-or-leave-it bargaining protocol. The buyer's surplus is defined as $\mathcal{U}_\varepsilon^b = \varepsilon u(q) + Z^b(z_1 - p_1, z_2 - p_2) - Z^b(z_1, z_2)$, and he solves the following problem

$$\max_{p_1,p_2,q} \ \mathcal{U}_\varepsilon^b \quad \text{s.t.} \quad p_1 \leq z_1, \ \ p_2 \leq z_2, \ \ \text{and} \quad -c(q) + p_1 + p_2 - \kappa \, \mathbb{I}_{(p_2>0)} \geq 0 \tag{6}$$

where, because of linearity of Z^b, we have that $\mathcal{U}_\varepsilon^b = \varepsilon u(q) - p_1 - p_2$. Note that the first inequality is a feasibility constraint, which limits the payment (p_1, p_2) to the amount of domestic and foreign currency the buyer has brought into the match (z_1, z_2), respectively. The second constraint ensures the participation of the seller, where κ is the cost supported by sellers to accept foreign currency, and $\mathbb{I}_{(p_2>0)}$ is an indicator function, which takes value 1 if sellers pay the cost to accept foreign currency, and 0 if they only accept domestic currency. Next, when characterizing the optimal terms of trade, we analyze two cases.

Case A

Consider a match with a buyer that has experienced a preference shock ε and that the seller does not incur the cost κ to accept foreign currency. In that situation, foreign currency is not exchanged, resulting in the following problem

$$\widetilde{\mathcal{U}}_{\varepsilon}^{b}=\max_{q,p_1\geqslant 0}\left[\varepsilon u(q)-p_1\right] \text{ s.t } p_1\leq z_1 \text{ and } c(q)\leq p_1 \tag{7}$$

yielding the following optimal terms of trade

$$\widetilde{q}_{\varepsilon} = \begin{cases} q_{\varepsilon}^{*} \\ c^{-1}(z_1) \end{cases} \text{if } z_1 \begin{cases} \geqslant \\ < \end{cases} c\left(q_{\varepsilon}^{*}\right) \tag{8}$$

$$\widetilde{p}_{1,\varepsilon} = c(\widetilde{q}_{\varepsilon}) \tag{9}$$

where $\widetilde{q}_{\varepsilon}$ and $\widetilde{p}_{1,\varepsilon}$ are functions of ε and q_{ε}^{*} corresponds to the first best allocation in DM that satisfies $\varepsilon u'(q_{\varepsilon}^{*})=c'(q_{\varepsilon}^{*})$.

Lemma 1 $\widetilde{q}_{\varepsilon}\in[0,q_{\varepsilon}^{*})$ *is an increasing function of both the shock* ε *and real domestic currency* z_1.

<u>**Case B**</u>

Let us now consider a match between a buyer and seller where both domestic and foreign currencies are accepted as a means of payment. Then the buyer's problem is given by

$$\widehat{\mathcal{U}}_{\varepsilon}^{b}=\max_{q,p_1,p_2\geqslant 0}\left[\varepsilon u(q)-p_1-p_2\right] \text{ s.t } p_1\leq z_1,\;\; p_2\leq z_2,\;\; \text{and}\;\; c(q)+\kappa\leq p_1+p_2 \tag{10}$$

which results in the following optimal terms of trade

$$\widehat{q}_{\varepsilon} = \begin{cases} q_{\varepsilon}^{*} \\ c^{-1}(z_1+z_2-\kappa) \end{cases} \text{if } z_1+z_2-\kappa \begin{cases} \geqslant \\ < \end{cases} c\left(q_{\varepsilon}^{*}\right) \tag{11}$$

$$\widehat{p}_{1,\varepsilon}+\widehat{p}_{2,\varepsilon}-\kappa = c(\widehat{q}_{\varepsilon}). \tag{12}$$

where $\widehat{q}_{\varepsilon}$, $\widehat{p}_{1,\varepsilon}$ and $\widehat{p}_{2,\varepsilon}$ are functions of ε.

Lemma 2 $\widehat{q}_{\varepsilon}\in[0,q_{\varepsilon}^{*})$ *is an increasing function of both* ε *and total real money balances* z_1+z_2, *and a decreasing function of* κ.

Combining these two cases, we can obtain the general solution to the DM's optimal terms of trade. Since in a match the seller's participation constraint is satisfied in both cases, the decision of whether or not to use the foreign currency rests with the buyer. Given their preference shock

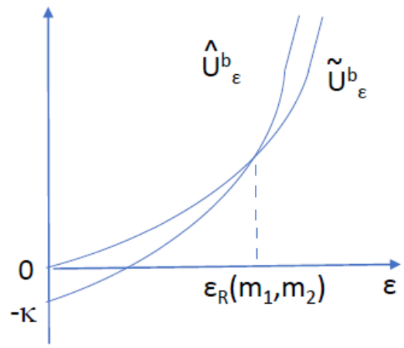

Figure 1: Determination of the preference threshold ε_R in the baseline economy.

ε and money holdings z_1 and z_2, the buyer is then confronted with the following choice

$$\max\left\{\widetilde{\mathcal{U}}^b_\varepsilon, \widehat{\mathcal{U}}^b_\varepsilon\right\} \tag{13}$$

where both $\widetilde{\mathcal{U}}^b_\varepsilon$ and $\widehat{\mathcal{U}}^b_\varepsilon$ are increasing functions of ε as illustrated in Figure 1.

Proposition 3 *There exists a threshold $\varepsilon_R \in [0, \bar{\varepsilon}]$ that satisfies $\widetilde{\mathcal{U}}^b_{\varepsilon_R} = \widehat{\mathcal{U}}^b_{\varepsilon_R}$.*[15] *This cutoff value ε_R is such that buyers with preference shock ε will trade in the DM with the domestic currency whenever $\varepsilon < \varepsilon_R$, whereas for $\varepsilon \geq \varepsilon_R$ they will be willing to trade with both the domestic and foreign currencies. Moreover, when κ increases, the threshold ε_R increases, making it more difficult for agents to accept and use foreign currency.*

4.3 Optimal currency portfolio

The expected value function of a buyer at the beginning of the period, with initial real money holdings z_1 and z_2 and preference shocks ε, is given by

$$\begin{aligned} V^b(z_1, z_2, \varepsilon) &= \sigma\left[\varepsilon u(q) + Z^b(z_1 - p_1, z_2 - p_2)\right] + (1-\sigma)Z^b(z_1, z_2); \\ &= \sigma \max\left\{\varepsilon u(\widetilde{q}_\varepsilon) + Z^b(z_1 - \widetilde{p}_{1,\varepsilon}, z_2), \varepsilon u(\widehat{q}_\varepsilon) + Z^b(z_1 - \widehat{p}_{1,\varepsilon}, z_2 - \widehat{p}_{2,\varepsilon})\right\} \\ &\quad + (1-\sigma)Z^b(z_1, z_2) \end{aligned} \tag{14}$$

where σ denotes the probability of trading in DM, and p_1 (p_2) the real value of domestic (foreign) currency exchanged in the match. Substituting V^b into W^b and exploiting the linearity in wealth,

[15]This can be formulated as the complementarity problem $f(\varepsilon_R) \perp 0 \leq \varepsilon_R \leq \bar{\varepsilon}$ where $f(\varepsilon_R) = \widetilde{\mathcal{U}}^b_{\varepsilon_R} - \widehat{\mathcal{U}}^b_{\varepsilon_R}$ which can be solved numerically as the rootfinding problem $\min\{\max\{f(\varepsilon_R), 0 - \varepsilon_R\}, \bar{\varepsilon} - \varepsilon_R\} = 0$.

a buyer's optimal choice for real balances in CM solves the following problem

$$\max_{z_1, z_2} \left\{ -\frac{z_1}{R_1} - \frac{z_2}{R_2} + \beta\sigma \max\left[\widetilde{S}_\varepsilon\left(\widetilde{q}_\varepsilon\right), \widehat{S}_\varepsilon\left(\widehat{q}_\varepsilon\right)\right] + \beta\left(z_1 + z_2\right) \right\} \tag{15}$$

where $\widetilde{S}_\varepsilon(\widetilde{q}_\varepsilon) = \varepsilon u(\widetilde{q}_\varepsilon) - c(\widetilde{q}_\varepsilon)$ is the surplus of the buyer if he holds the domestic currency only ($\widetilde{z}_1 > 0, z_2 = 0$), and $\widehat{S}_\varepsilon(\widehat{q}_\varepsilon) = \varepsilon u(\widehat{q}_\varepsilon) - c(\widehat{q}_\varepsilon) - \kappa$ is his surplus if he uses both the domestic and foreign currencies ($\widehat{z}_1 > 0, \widehat{z}_2 > 0$). As $\widetilde{q}_\varepsilon = c^{-1}\left(\widetilde{z}_1\right)$, and $\widehat{q}_\varepsilon = c^{-1}\left(\widehat{z}_1 + \widehat{z}_2 - \kappa\right)$, this maximization problem can be rewritten as follows

$$\max\left\{\widetilde{X}^b_\varepsilon, \widehat{X}^b_\varepsilon\right\} \tag{16}$$

where

$$\widetilde{X}^b_\varepsilon \equiv \max_{\widetilde{z}_1}\left[-s_1\widetilde{z}_1 + \sigma\widetilde{S}_\varepsilon(\widetilde{z}_1)\right] \tag{17}$$

with $s_1 = \frac{1+\rho-R_1}{R_1}$, and

$$\widehat{X}^b_\varepsilon \equiv \max_{\widehat{z}_1, \widehat{z}_2}\left[-s_1\widehat{z}_1 - s_2\widehat{z}_2 + \sigma\widehat{S}_\varepsilon(\widehat{z}_1 + \widehat{z}_2 - \kappa)\right] \tag{18}$$

with $s_2 = \frac{1+\rho-R_2}{R_2}$. Note that the optimal currency portfolio choice depends on the currencies relative returns. We now explore the various cases that can emerge.

Case 1 : $R_2 = R_1$

When the rates of return of the two currencies are identical, for any ε, there is no reason to incur the cost κ to use foreign currency as the two currencies are perfect substitutes.[16] As a consequence, it is rational for buyers not to hold foreign currency ($z_2 = 0$) as the quantity of goods they would exchange in a match ($\widehat{q}_\varepsilon$), and their surplus ($\widehat{S}_\varepsilon$), would be lower than the quantity of goods ($\widetilde{q}_\varepsilon$), and surplus ($\widetilde{S}_\varepsilon$), they could get when trading with just the domestic currency. Consequently, when $R_2 = R_1$, buyers choose to use only the domestic currency $\widetilde{z}_1$ that maximizes $\widetilde{X}^b_\varepsilon$. Their problem is then given by

$$\max_{\widetilde{z}_1}\left[-s_1\widetilde{z}_1 + \sigma\widetilde{S}_\varepsilon(\widetilde{z}_1)\right] \tag{19}$$

[16]Note that the use of domestic currency generates no additional cost.

which results in the following first order condition

$$\widetilde{S}'_{\varepsilon}(\widetilde{z}_1) = \frac{s_1}{\sigma}. \tag{20}$$

Using the definition of the surplus $\widetilde{S}_{\varepsilon}$ and the optimal terms of trade (8) and (9), we solve for $\widetilde{S}'_{\varepsilon}$ and obtain the optimality condition for DM consumption

$$\varepsilon \frac{u'(\widetilde{q}_{\varepsilon})}{c'(\widetilde{q}_{\varepsilon})} - 1 = \frac{s_1}{\sigma} \tag{21}$$

with real balances given by

$$\widetilde{z}_{1,\varepsilon} = \begin{cases} c(\widetilde{q}_{\varepsilon}) & \text{for } \widetilde{q}_{\varepsilon} < q^*_{\varepsilon} \\ [c(q^*_{\varepsilon}), +\infty) & \text{for } \widetilde{q}_{\varepsilon} = q^*_{\varepsilon} \end{cases} \tag{22}$$

where the first best consumption level q^*_{ε} is the solution to $\varepsilon u'(q^*_{\varepsilon}) = c'(q^*_{\varepsilon})$. Clearly, in the case where carrying real balances is costly, the amount of currency 1 carried by each individual $\widetilde{z}_{1,\varepsilon}$ depends on the idiosyncratic preference shock ε. This results in a non-degenerate distribution of real balances in equilibrium.

Case 2 : $R_2 < R_1$

When the rate of return of foreign currency is smaller than the rate of return of domestic currency, for any ε, buyers have no incentive to hold foreign currency (as its holding cost is higher than the domestic one). As a result, the foreign currency does not circulate in the economy. The buyer's problem is the same as in the previous case.

Case 3 : $R_2 > R_1$

When the rate of return of the foreign currency is higher than the rate of return of the domestic currency, the foreign currency is a more attractive savings instrument. However, sellers have to pay the cost κ to accept this currency. This in turn reduces the benefits of acquiring it as it reduces the surplus of a DM match where foreign currency is accepted.

According to Proposition 3, buyers with preference shock $\varepsilon < \varepsilon_R$ prefer to use only the domestic currency in their DM purchases. Since holding money from one CM to the next CM is costly for both currencies, these buyers have no incentive to carry foreign currency. These agents will hold just enough domestic currency to spend in the DM in case they're matched with a seller. This means they will choose their real balances of domestic currency according to the following problem

$$\max_{\widetilde{z}_1}\left[-s_1\widetilde{z}_1+\sigma\widetilde{S}_{\varepsilon}(\widetilde{z}_1)\right]. \tag{23}$$

As in Case 1, the solution to the above problem is given by (21) and (22). Buyers with preference shock $\varepsilon \geq \varepsilon_R$, are willing to use either currencies in DM. Both currencies are costly to hold. However, the domestic currency is dominated in rate of return by the foreign currency. As a consequence, these buyers will hold just enough of the foreign currency to settle their DM trades and will not acquire any domestic currency. The amount of foreign currency they carry is the solution to the following problem

$$\max_{\widehat{z}_2}\left[-s_2\widehat{z}_2+\sigma\widehat{S}_{\varepsilon}(\widehat{z}_2-\kappa)\right] \tag{24}$$

which yields the first order condition

$$\widehat{S}'_{\varepsilon}(\widehat{z}_2-\kappa)=\frac{s_2}{\sigma}. \tag{25}$$

Using the definition of the surplus $\widehat{S}_{\varepsilon}$ and the optimal DM terms of trade (11) and (12), we obtain the optimality condition for DM consumption:

$$\varepsilon\frac{u'(\widehat{q}_{\varepsilon})}{c'(\widehat{q}_{\varepsilon})}-1=\frac{s_2}{\sigma} \tag{26}$$

with real balances given by

$$\widehat{z}_{2,\varepsilon}=\begin{cases}c(\widehat{q}_{\varepsilon})+\kappa & \text{for } \widehat{q}_{\varepsilon}<q^*_{\varepsilon}\\ [c(q^*_{\varepsilon})+\kappa,+\infty) & \text{for } \widehat{q}_{\varepsilon}=q^*_{\varepsilon}\end{cases}. \tag{27}$$

As is the case with currency 1, when carrying currency 2 is costly, the amount $\widehat{z}_{2,\varepsilon}$ is a function of ε. This results in a non-degenerate distribution of real balances of currency 2 in equilibrium.

Having characterized all possible cases we can establish the following result.

Proposition 4 *For $\varepsilon \in (0,\varepsilon_R]$, if sellers' cost κ of accepting the foreign currency is sufficiently high, buyers will not use the foreign currency although its rate of return is higher than the domestic one ($\widetilde{z}_{2,\varepsilon}=0$). For $\varepsilon \in [\varepsilon_R,\bar{\varepsilon}]$, buyers will prefer to carry the foreign currency only as its higher return compensates the cost of accepting it in transactions ($\widehat{z}_{1,\varepsilon}=0$).*

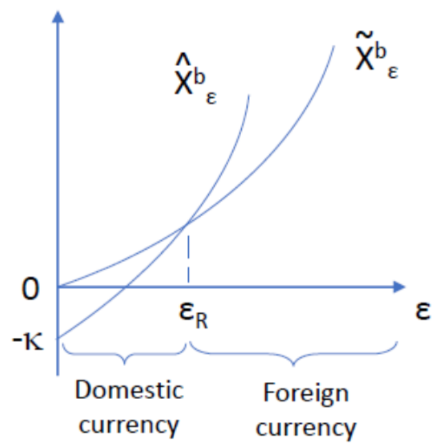

Figure 2: Optimal currency holdings.

Figure 2 illustrates the buyers'optimal currency portfolio decision.

As we can see, the domestic economy is partially dollarized, i.e. the foreign currency is circulating and being accepted as a means of payment in the domestic economy, as long as it yields a rate of return that is high enough relative to the domestic currency. In particular, only buyers that are making larger DM purchases use the foreign currency to make such purchases.

4.4 Stationary monetary equilibrium

We focus on equilibria where money is valued and quantities are constant over time. In particular, we have the following definition.

Definition 5 *A stationary monetary equilibrium consists of allocations, real balances ($\tilde{q}_\varepsilon$, $\hat{q}_\varepsilon$, $\tilde{z}_{1,\varepsilon}$, $\hat{z}_{2,\varepsilon}$) and a preference threshold (ε_R) that satisfy optimal DM consumption equations* (21) *and* (26)*, real balances equations* (22) *and* (27) *and the indifference equation*

$$\tilde{X}^b(\varepsilon_R) = \hat{X}^b(\varepsilon_R). \tag{28}$$

4.5 Comparative statics

Next, we analyze how the degree of dollarization in the domestic economy responds to changes in the exogenous parameters describing the economic environment.

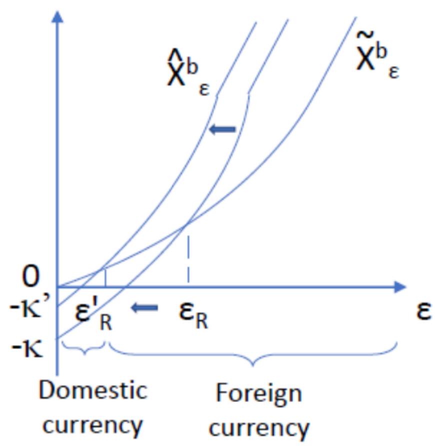

Figure 3: Decrease in the acceptability cost ($\kappa' < \kappa$).

Lemma 6 *When the cost of accepting foreign currency κ decreases, the value of the threshold ε_R decreases.*

Lemma 6 is depicted by Figure 3. Note that currency substitution between domestic and foreign currencies takes place. In particular, more buyers hold foreign currency, more foreign currency circulates, and more sellers accept it. This results in a higher degree of dollarization of the domestic economy.

Lemma 7 *When the rate of return of foreign currency R_2 increases, relative to the domestic currecncy rate of return R_1, the slope of $\widehat{X}^b_\varepsilon$ increases, reducing the value of the threshold ε_R.*

When conditions for Lemma 7 hold we have that more buyers hold foreign currency, more foreign currency circulates in the domestic economy, and more sellers accept it. This results in more dollarization of the local economy, which is illustrated in Figure 4.

In order to understand further the equilibrium properties of the benchmark model, we resort to numerical analysis. For that we assume the following functional forms and parameter values. The DM utility is assumed to take the CRRA form $u(q) = \frac{Aq^{1-a}}{1-a}$ where we set $A = 1$ and $a = 0.3$. The DM cost function is assumed linear i.e. $c(q) = q$. We set the discount factor β at 0.99, the probability of finding a DM seller σ at 0.8 and the transaction cost of foreign currency κ at 0.01. The preference shock ε is assumed to be uniformly distributed over the interval $[0, 1]$. Finally, we set the domestic inflation rate at 10% which yields a value for R_1 of about 0.91. R_2 is set at 0.98.

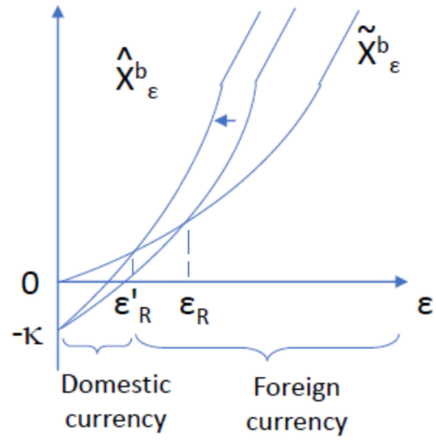

Figure 4: Increase in the rate of return of foreign currency.

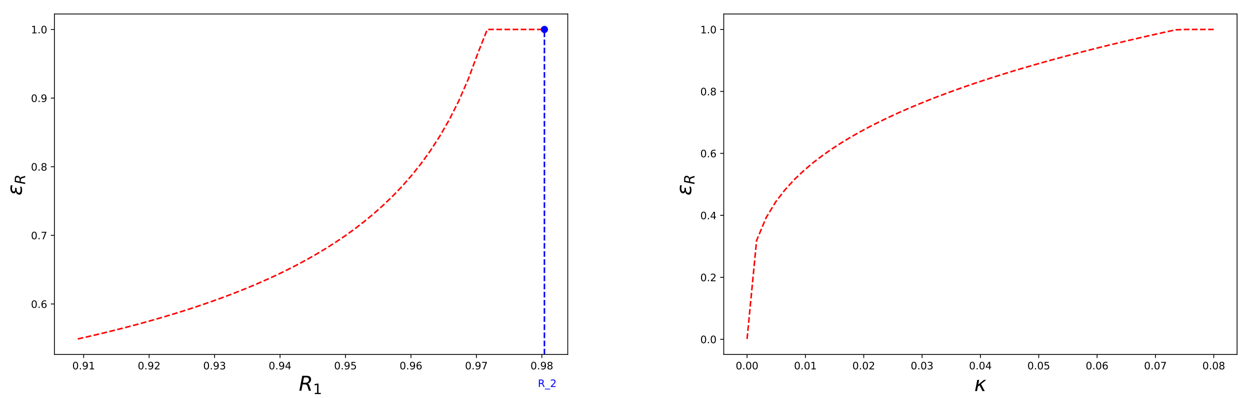

Figure 5: Foreign currency adoption as a function of parameters, benchmark model.

In our first numerical experiment, we focus on the effect of monetary policy on dollarization in the baseline economy. In particular, the left panel of Figure 5 depicts the effect of increasing the return on the domestic currency, through lower inflation, while keeping the return on the foreign currency constant. As the return on domestic currency improves, the share of buyers using the foreign currency slowly decreases. As we further improve the return, dollarization starts decreasing at an accelerating rate. It's worth noting that complete de-dollarization (i.e. $\varepsilon_R = 1$) occurs well before the two currencies have the same return. This is because even though the domestic currency is still costlier to hold, buyers find it attractive enough as to avoid paying the cost κ on the use of the foreign currency.

This finding suggests that, in order to avoid dollarization in equilibrium, the central bank does not have to reduce inflation all the way down to the level that makes the return on holding the domestic currency the same as the foreign currency. Rather, it needs to ensure that inflation is low enough to make it sub-optimal for consumers to incur the transaction costs associated with holding foreign currency. The higher the transaction costs, the higher the inflation rate the economy can tolerate without transitioning to a dollarization equilibrium.

Next, we look at how the cost of using the foreign currency affects dollarization. The right panel depicts ε_R as an increasing function of κ. In particular, when $\kappa = 0$, the foreign currency is used in all transactions as it has a higher return compared to the domestic one. However, as κ increases the rate of adoption of the foreign currency drops until it reaches 0, i.e. $\varepsilon_R = \bar{\varepsilon}$.

5 Model with banks

A key feature of most dollarized economies is that not only the foreign currency circulates in the domestic economy, but also that foreign denominated bank accounts are available to domestic agents. To capture this important feature, we now allow banks to play a key role in terms of money reallocation between agents with excess demand of domestic or foreign currency, and agents with excess supply of these currencies. Whether or not bankers can accept foreign currency deposits depends on the type of banking regulations that are in place.

Relative to the benchmark environment, we now introduce a third set of agents: bankers. In CM, bankers can consume and produce the perishable good using the same linear production technology as buyers and sellers. In contrast to the benchmark, now at the beginning of each period, after a trading opportunity occurs (or not), buyers and sellers have access to banking services. As in Berentsen et al. (2007), bankers are endowed with a record-keeping technology that allows them to write deposits and loan contracts. Agents, including banks, cannot default on their financial commitments as the government enforces loan and deposit contracts in CM.

Before the exchange of money for goods takes place in DM, buyers and sellers can trade with bankers who accept one-period nominal deposits and offer one-period nominal loans. Banks pay a nominal interest rate i_j^d for currency j deposits and charge nominal rate i_j^l for currency j loans, with $j \in \{1, 2\}$.

In what follows we consider a banking sector that is perfectly competitive, so bankers take interest rates as given. Thus, there is no strategic interaction among bankers, or between bankers and private agents. In particular, there is no bargaining over the terms of deposit and loan contracts.

5.1 CM problem

At the beginning of the final sub-period, a buyer enters CM with real domestic and foreign fiat money holdings, z_1 and z_2 respectively, as well as loans l_j and deposits d_j in currency j, all expressed in units of the numéraire CM good. In CM, agents repay their outstanding loans and redeem their deposits, paying and receiving their corresponding interest payments as well as having access to the foreign exchange market. The value function of a buyer in the CM is now given by

$$\begin{aligned} W^b(\omega, d_1, d_2, l_1, l_2, \varepsilon) &= \max_{h, z_1, z_2} \left\{ -h + \beta V^b(z_1, z_2, \varepsilon) \right\} \\ \text{s.t. } \frac{z_1}{R_1} + \frac{z_2}{R_2} &= h + \omega + \sum_{j=1}^{2} \left(1 + i_j^d\right) d_j - \sum_{j=1}^{2} \left(1 + i_j^l\right) l_j + \tau \end{aligned} \tag{29}$$

where ω is sum of real balances held by the buyer in both currencies at the start of the CM, $\left(1 + i_j^d\right) d_j$ corresponds to the value of deposits of currency j, and $\left(1 + i_j^l\right) l_j$ is the repayment, with interests, of the loan contracted in currency j. Substituting h from the budget constraint into the objective function, we can rewrite the previous problem as follows

$$W^b(\omega, d_1, d_2, l_1, l_2, \varepsilon) = \omega + \sum_{j=1}^{2} \left(1 + i_j^d\right) d_j - \sum_{j=1}^{2} \left(1 + i_j^l\right) l_j + \tau + \max_{z_1, z_2} \left\{ -\frac{z_1}{R_1} - \frac{z_2}{R_2} + \beta V^b(z_1, z_2, \varepsilon) \right\}. \tag{30}$$

5.2 DM problem

At the beginning of the period, the buyer's expected DM value function with initial real balance holdings (z_j), loans (l_j) and deposits (d_j), for $j \in \{1, 2\}$, is given by

$$V^b(z_1, z_2, \varepsilon) = \sigma \left[\varepsilon u(q) + Z^b\left(z_1 + l_1 - p_1 + z_2 + l_2 - p_2, 0, 0, l_1, l_2\right)\right] + (1-\sigma) Z^b(z_1 - d_1 + z_2 - d_2, d_1, d_2, 0, 0) \tag{31}$$

where

$$
\begin{aligned}
Z^b(\omega, d_1, d_2, l_1, l_2) &= \int_0^{\bar{\varepsilon}} W^b(\omega, d_1, d_2, l_1, l_2, \varepsilon) dF(\varepsilon) \\
&= \omega + \sum_{j=1}^{2} \left(1 + i_j^d\right) d_j - \sum_{j=1}^{2} \left(1 + i_j^l\right) l_j + Z^b(0).
\end{aligned} \tag{32}
$$

Note that when buyers do not have a trading opportunity in DM, they may find it profitable to deposit their money holdings in order to earn some interest income. Since carrying currency across periods is costly, non-trading buyers will deposit all of their domestic and foreign money holdings with the bank. Additionally, depending on buyers' preferences, the loans and deposits in domestic and foreign currencies might be different across buyers.

When banks are active, the terms of trade are such that buyers solve the following problem

$$
\begin{aligned}
\max_{p_1, p_2, q} \mathcal{U}_{\varepsilon}^b &= \max_{p_1, p_2, q} \left[\varepsilon u(q) - p_1 - p_2\right] \\
\text{s.t. } p_1 &\leq z_1 + l_1, p_2 \leq z_2 + l_2, \text{ and } -c(q) + p_1 + p_2 - \kappa \mathbb{I}_{(p_2 > 0)} \geq 0
\end{aligned} \tag{33}
$$

where the first constraint is again a liquidity constraint, which limits the payment (p_1, p_2) to the amount of real balances the buyer has brought into the DM (z_1, z_2) plus any loans he contracted with the bank (l_1, l_2).

As in the benchmark model, agents in DM trade according to a buyer's take it or leave offer and we distinguish two cases.

Case A

First, suppose the seller in a given match with a buyer, with preference shock ε, is not willing to incur the cost κ. Then foreign currency is not exchanged in that match. In this case the DM optimal terms of trade solve the following problem

$$
\widetilde{\mathcal{U}}_{\varepsilon}^b = \max_{q, p_1 \geqslant 0} \left[\varepsilon u(q) - p_1\right] \text{ s.t } p_1 \leq z_1 + l_1 \text{ and } c(q) \leq p_1 \tag{34}
$$

which yields the solution

$$\widetilde{q}_{\varepsilon} = \begin{cases} q_{\varepsilon}^{*} \\ c^{-1}(z_1+l_1) \end{cases} \text{if } z_1+l_1 \begin{cases} \geqslant \\ < \end{cases} c(q_{\varepsilon}^{*}) \tag{35}$$

$$\widetilde{p}_{1,\varepsilon} = c(\widetilde{q}_{\varepsilon}) \tag{36}$$

where q_{ε}^{*} satisfies $\varepsilon u'(q_{\varepsilon}^{*}) = c'(q_{\varepsilon}^{*})$. We can now establish the following result.

Lemma 8 *When sellers do not incur the cost κ, then $\widetilde{q}_{\varepsilon} \in [0, q_{\varepsilon}^{*})$ is an increasing function of ε and real domestic currency, $z_1 + l_1$.*

Case B

Suppose now that both domestic and foreign currencies are accepted in a match. In that case, the DM optimal terms of trade solve the following problem

$$\widehat{\mathcal{U}}_{\varepsilon}^{b} = \max_{q,p_1,p_2 \geqslant 0} [\varepsilon u(q) - p_1 - p_2] \text{ s.t. } c(q) + \kappa \leq p_1 + p_2 \text{ and } p_1 + p_2 \leq z_1 + l_1 + z_2 + l_2 \tag{37}$$

which delivers the following optimal solution

$$\widehat{q}_{\varepsilon} = \begin{cases} q_{\varepsilon}^{*} \\ c^{-1}(z_1+l_1+z_2+l_2-\kappa) \end{cases} \text{if } z_1+l_1+z_2+l_2-\kappa \begin{cases} \geqslant \\ < \end{cases} c(q_{\varepsilon}^{*}) \tag{38}$$

$$\widehat{p}_{1,\varepsilon} + \widehat{p}_{2,\varepsilon} = c(\widehat{q}_{\varepsilon}) + \kappa \,. \tag{39}$$

We can now establish the following result.

Lemma 9 *When sellers incur the cost κ, then $\widehat{q}_{\varepsilon} \in [0, q_{\varepsilon}^{*})$ is an increasing function of ε and of total real money balances, $z_1 + l_1 + z_2 + l_2$, and a decreasing function of the cost κ.*

Combining these two cases, we can obtain the general solution to the DM's optimal terms of trade. As in benchmark model, sellers' participation constraint is always satisfied. As such, buyers can choose which currency to use by solving the following problem

$$\max\left\{\widetilde{\mathcal{U}}_{\varepsilon}^{b}, \widehat{\mathcal{U}}_{\varepsilon}^{b}\right\} \tag{40}$$

with both $\widetilde{\mathcal{U}}_{\varepsilon}^{b}$ and $\widehat{\mathcal{U}}_{\varepsilon}^{b}$ increasing functions of ε. Having characterized all possible cases we can establish the following result.

Proposition 10 *There exists a threshold $\varepsilon_R \in [0, \bar{\varepsilon}]$ that satisfies $\widetilde{\mathcal{U}}^b_{\varepsilon_R} = \widehat{\mathcal{U}}^b_{\varepsilon_R}$.*[17] *This threshold ε_R is such that for all $\varepsilon < \varepsilon_R$ agents will only trade with domestic currency, whereas for all $\varepsilon > \varepsilon_R$ agents will exchange both domestic and foreign currency for goods.*

Lemma 11 *When κ increases, the threshold ε_R increases also, meaning it becomes more difficult for agents to accept and use foreign currency.*

5.3 Banking decisions

We now study the deposit and loan decisions that buyers and sellers face once all shocks have been realized. Agents who know they will be able to purchase goods, or not, have to decide what type of banking services they want. The loan and deposit decisions of a buyer solve the following problem

$$\begin{aligned} &\sigma \max_{l_1,l_2} \left\{ \max \left[\begin{array}{c} \varepsilon u(\widetilde{q}_\varepsilon) + Z^b(z_1 + l_1 - \widetilde{p}_1, 0, 0, l_1, 0) \ ; \\ \varepsilon u(\widehat{q}_\varepsilon) + Z^b(z_1 + l_1 - \widehat{p}_1 + z_2 + l_2 - \widehat{p}_2, 0, 0, l_1, l_2) \end{array} \right] \right\} \\ &\qquad + (1-\sigma) \max_{d_1,d_2} Z^b(z_1 - d_1 + z_2 - d_2, d_1, d_2, 0, 0) \\ &\qquad \text{s.t. } d_1 \leq z_1, d_2 \leq z_2 \end{aligned} \tag{41}$$

where the optimal terms of trade are given by (35), (36), (38) and (39). This problem can be rewritten as

$$\sigma \max_{l_1,l_2} \left\{ \max \left[\varepsilon u(\widetilde{q}_\varepsilon) - \left(1 + i^l_1\right) l_1 \ ; \varepsilon u(\widehat{q}_\varepsilon) - \sum_{j=1}^{2} \left(1 + i^l_j\right) l_j \right] \right\} + (1-\sigma) \max_{d_1,d_2} \left[\sum_{j=1}^{2} i^d_j d_j \right]. \tag{42}$$

subject to the same constraints. As in the benchmark model, we distinguish two cases, first when only the domestic currency is accepted, and then when both can be used to settle transactions.

<u>Case A</u>

If sellers do not incur the cost κ, foreign currency is not accepted, the resulting buyer's banking decision is then given by

$$\sigma \max_{l_1} \left[\varepsilon u(\widetilde{q}_\varepsilon) - \left(1 + i^l_1\right) l_1 \right] + (1-\sigma) \max_{d_1} \left[i^d_1 d_1 \right] \tag{43}$$

[17] This is equivalent to the complementarity problem $f(\varepsilon_R) \perp 0 \leq \varepsilon_R \leq \bar{\varepsilon}$ where $f(\varepsilon_R) = \widetilde{\mathcal{U}}^b_{\varepsilon_R} - \widehat{\mathcal{U}}^b_{\varepsilon_R}$.

subject to the terms of trade (35) and (36). This yields the following optimal conditions

$$\frac{\varepsilon u'(\widetilde{q}_\varepsilon)}{c'(\widetilde{q}_\varepsilon)} - 1 = i_1^l \text{ if } l_1 > 0 \tag{44}$$

$$d_1 = z_1 \text{ if } i_1^d > 0, \text{ and } d_1 \geq 0 \text{ if } i_1^d = 0. \tag{45}$$

Equation (44) determines the amount of DM consumption $\widetilde{q}_\varepsilon$ when borrowing is available. Notice that $\widetilde{q}_\varepsilon$ does not depend on the amount of real balances the buyer has brought with them. It only depends on the marginal rate of borrowing i_1^l. Given $\widetilde{q}_\varepsilon$ and z_1, the amount borrowed by the buyer can be obtained from equation (35), which is given by

$$l_{1,\varepsilon} = c(\widetilde{q}_\varepsilon) - z_1. \tag{46}$$

This amount is a function of the idiosyncratic preference shock ε which, in equilibrium, would result in a non-degenerate distribution of loans.

From equation (45), buyers, who are not matched with a seller, will deposit all their money holdings as long as $i_1^d > 0$. For $i_1^d = 0$, they are indifferent between depositing or not.

Case B

If foreign currency is accepted, the buyer's banking decision is then given by

$$\sigma \max_{l_1,l_2} \left[\varepsilon u(\widehat{q}_\varepsilon) - \sum_{j=1}^{2} \left(1 + i_j^l\right) l_j \right] + (1-\sigma) \max_{d_1,d_2} \left[\sum_{j=1}^{2} i_j^d d_j \right] \tag{47}$$

which yields the following first order conditions

$$\frac{\varepsilon u'(\widehat{q}_\varepsilon)}{c'(\widehat{q}_\varepsilon)} - 1 = i_1^l \text{ if } l_1 > 0 \tag{48}$$

$$\frac{\varepsilon u'(\widehat{q}_\varepsilon)}{c'(\widehat{q}_\varepsilon)} - 1 = i_2^l \text{ if } l_2 > 0 \tag{49}$$

$$d_1 = z_1 \text{ if } i_1^d > 0, \text{ and } d_1 \geq 0 \text{ if } i_1^d = 0 \tag{50}$$

$$d_2 = z_2 \text{ if } i_2^d > 0, \text{ and } d_2 \geq 0 \text{ if } i_2^d = 0. \tag{51}$$

Equations (44) and (49) determine the amount of DM consumption $\widehat{q}_\varepsilon$ when borrowing is available. Notice that $\widetilde{q}_\varepsilon$ does not depend on the amount of real balances the buyer has brought with them and depends only on the rates of borrowing i_1^l and i_2^l. In particular, for buyers to borrow in both currencies it must be that their borrowing rates are equal. If not, buyers will borrow only in the currency with the lowest rate. Given $\widehat{q}_\varepsilon$, z_1 and z_2, the amount borrowed by the buyer in each currency can be obtained from equation (38). This implies that

$$l_{1,\varepsilon} + l_{2,\varepsilon} = c(\widehat{q}_\varepsilon) + \kappa - z_1 - z_2. \tag{52}$$

These amounts are a function of the idiosyncratic preference shock ε which, in equilibrium, would result in a non-degenerate distribution of loans.

From equations (50) and (51), buyers who are not matched with a seller will deposit all their money holdings in both currencies as long as the deposit rates are positive. If the deposit rates are zero, buyers are indifferent between depositing or not.

Lemma 12 *Loan demand is an increasing function of ε, and a decreasing function of the loan rate i_j^l. The loan demand of the foreign currency is an increasing function of the sellers' cost of accepting such currency κ.*

Finally, sellers have no strict incentive to borrow or lend domestic and foreign currency as they do not need any type of money to trade in DM.

5.4 Optimal currency portfolio

Once we have characterized the buyer's optimal loan demand and supply decisions, we can determine the corresponding marginal values of holding domestic and foreign currencies. The expected value function of a buyer at the beginning of the period, with initial real money holdings z_1 and z_2, is given by

$$\begin{aligned} V^b(z_1, z_2, \varepsilon) =& \sigma \max \begin{bmatrix} \varepsilon u(\widetilde{q}) + Z^b(z_1 + l_1 - \widetilde{p}_1 + z_2, 0, 0, l_1, 0)\ ; \\ \varepsilon u(\widehat{q}) + Z^b(z_1 + l_1 - \widehat{p}_1 + z_2 + l_2 - \widehat{p}_2, 0, 0, l_1, l_2) \end{bmatrix} \\ &+ (1-\sigma) Z^b(z_1 - d_1 + z_2 - d_2, d_1, d_2, 0, 0). \end{aligned}$$

Substituting V^b into W^b and exploiting the linearity in wealth, the buyer's choice for real balances can be written as

$$\max_{z_1,z_2}\left\{-\sum_{j=1}^{2}\frac{z_j}{R_j}+\beta\sigma\max\left[\widetilde{S}_\varepsilon\left(\widetilde{q}_\varepsilon\right),\widehat{S}_\varepsilon\left(\widehat{q}_\varepsilon\right)\right]+\beta(1-\sigma)\sum_{j=1}^{2}\left(1+i_j^d\right)z_j+\beta\sigma\sum_{j=1}^{2}z_j\right\} \tag{53}$$

where $\widetilde{S}_\varepsilon(\widetilde{q}_\varepsilon)=\varepsilon u(\widetilde{q}_\varepsilon)-c(\widetilde{q}_\varepsilon)$ is the DM surplus of the buyer if he uses domestic currency only to trade in DM, and $\widehat{S}_\varepsilon(\widehat{q}_\varepsilon)=\varepsilon u(\widehat{q}_\varepsilon)-c(\widehat{q}_\varepsilon)-\kappa$ is his DM surplus if he uses both domestic and foreign currencies. As $\widetilde{q}_\varepsilon=c^{-1}\left(\widetilde{z}_1+l_1\right)$, and $\widehat{q}_\varepsilon=c^{-1}\left(\widehat{z}_1+l_1+\widehat{z}_2+l_2-\kappa\right)$. Since both currencies are costly to hold, a buyer will not carry a currency unless he plans to use it in DM trades. Because of that, the choice of which currency to carry out of CM depends on which currency the buyer will trade in DM. Hence the maximization problem can be rewritten as follows

$$\max\left\{\widetilde{\Omega}_\varepsilon^b,\widehat{\Omega}_\varepsilon^b\right\} \tag{54}$$

where $\widetilde{\Omega}_\varepsilon^b$ is given by $\max_{\widetilde{z}_1}\left[-s_1\widetilde{z}_1+\sigma\widetilde{S}_\varepsilon(\widetilde{z}_1+l_1)+(1-\sigma)i_j^d\widetilde{z}_1\right]$ with $s_1=\frac{1+\rho-R_1}{R_1}$, and $\widehat{\Omega}_\varepsilon^b$ is $\max_{\widehat{z}_1,\widehat{z}_2}\left[-s_1\widehat{z}_1-s_2\widehat{z}_2+\sigma\widehat{S}_\varepsilon(\widehat{z}_1+l_1+\widehat{z}_2+l_2-\kappa)+(1-\sigma)\left(i_1^d\widehat{z}_1+i_2^d\widehat{z}_2\right)\right]$ with $s_2=\frac{1+\rho-R_2}{R_2}$.

As in the benchmark model, the optimal currency portfolio choice depends on the relative returns. We now explore the various scenarios that can emerge.

Case 1: $R_2=R_1$

For any ε, sellers have no reason to incur the cost κ to accept foreign currency as the two currencies yield the same return. Additionally, it is rational for buyers not to hold foreign currency ($z_2=0$) as the quantity of goods they would exchange in a match ($\widehat{q}_\varepsilon$), and their surplus ($\widehat{S}_\varepsilon$), would be lower than the quantity of goods ($\widetilde{q}_\varepsilon$), and surplus ($\widetilde{S}_\varepsilon$), they could get with the domestic currency. Consequently, when $R_2=R_1$, $\widetilde{\Omega}_\varepsilon^b$ maximizes buyers' utility. Their currency portfolio problem then becomes

$$\max_{\widetilde{z}_1}\left[-s_1\widetilde{z}_1+\sigma\widetilde{S}_\varepsilon(\widetilde{z}_1+l_1)+(1-\sigma)i_1^d\widetilde{z}_1\right] \tag{55}$$

which delivers the following optimal decision

$$\widetilde{S}'_\varepsilon(\widetilde{z}_1+l_1)=\frac{s_1-(1-\sigma)i_1^d}{\sigma} \tag{56}$$

where the cost of accumulating an additional unit of money ($\frac{s_1-(1-\sigma)i_1^d}{\sigma}$) is lower when agents have access to the banking system, and can deposit their idle cash, than when there are no banks to reallocate their idle liquidity. Consequently, in equilibrium, agents accumulate more domestic currency when access to loans and deposits is available in DM. This in turn increases the quantities exchanged in a DM meeting relative to the benchmark model.

Using the definition of the surplus $\widetilde{S}_\varepsilon$ and the terms of trade (35) and (36), we obtain the optimality condition

$$\varepsilon\frac{u'(\widetilde{q}_\varepsilon)}{c'(\widetilde{q}_\varepsilon)} - 1 = \frac{s_1 - (1-\sigma)i_1^d}{\sigma} \tag{57}$$

with real balances given by

$$\widetilde{z}_{1,\varepsilon} = \begin{cases} c(\widetilde{q}_\varepsilon) - l_{1,\varepsilon} & \text{for } \widetilde{q}_\varepsilon < q_\varepsilon^* \\ [c(q_\varepsilon^*) - l_{1,\varepsilon}, +\infty) & \text{for } \widetilde{q}_\varepsilon = q_\varepsilon^* \end{cases}. \tag{58}$$

As in the benchmark model, when carrying currency 1 is costly, the amount $\widetilde{z}_{1,\varepsilon}$ is a function of ε. This results in a non-degenerate distribution of real balances of currency 1 in equilibrium.

Case 2: $R_2 < R_1$

When the rate of return of foreign currency is smaller than the rate of return of domestic currency, buyers have no incentive to hold foreign currency, and foreign currency does not circulate in the economy. In this scenario the buyer's currency portfolio problem is the same as the previous case.

Case 3: $R_2 > R_1$

As in the benchmark model, if sellers' cost κ of accepting the foreign currency is sufficiently high, buyers with $\varepsilon < \varepsilon_R$ will not use the foreign currency even though its rate of return is higher than the domestic one. For $\varepsilon > \varepsilon_R$, buyers will prefer to use the foreign currency only as its benefits compensate buyers for the cost of accepting it. Consequently, in equilibrium, there will be coexistence of domestic and foreign currency in the economy.

Buyers with $\varepsilon < \varepsilon_R$ will choose their real balances of domestic currency by solving the following problem

$$\max_{\widetilde{z}_1} \left[-s_1\widetilde{z}_1 + \sigma\widetilde{S}_\varepsilon(\widetilde{z}_1 + l_1) + (1-\sigma)i_1^d\widetilde{z}_1\right]. \tag{59}$$

As in Case 1, the solution is given by (57) and (58).

When $\varepsilon > \varepsilon_R$, buyers will choose the amount of real balances of foreign currency that solve the following problem

$$\max_{\widehat{z}_2} \left[-s_2 \widehat{z}_2 + \sigma \widehat{S}_\varepsilon(\widehat{z}_2 + l_2 - \kappa) + (1-\sigma) i_2^d \widehat{z}_2 \right] \tag{60}$$

which yields the following

$$\widehat{S}'_\varepsilon(\widehat{z}_2 + l_2 - \kappa) = \frac{s_2 - (1-\sigma) i_2^d}{\sigma}. \tag{61}$$

Using the definition of $\widehat{S}_\varepsilon$ and the terms of trade (38) and (39), we obtain the optimality condition for DM consumption

$$\varepsilon \frac{u'(\widehat{q}_\varepsilon)}{c'(\widehat{q}_\varepsilon)} - 1 = \frac{s_2 - (1-\sigma) i_2^d}{\sigma} \tag{62}$$

with real balances given by

$$\widehat{z}_{2,\varepsilon} = \begin{cases} c(\widehat{q}_\varepsilon) + \kappa - l_2 & \text{for } \widehat{q}_\varepsilon < q_\varepsilon^* \\ [c(q_\varepsilon^*) + \kappa - l_2, +\infty) & \text{for } \widehat{q}_\varepsilon = q_\varepsilon^* \end{cases}. \tag{63}$$

where the left side of equation (62) corresponds to the marginal benefit from accumulating foreign currency, whereas the right side corresponds to its marginal cost. Compared to the benchmark model this cost is reduced as idle currency balances can be reallocated to buyers thanks to the banking system. In particular, idle cash can indeed be remunerated in this case. Note that the higher payoff to consume in DM (higher ε), the higher the money and loan demand.

5.5 Equilibrium with domestic and foreign currency bank accounts

In what follows and without loss of generality, we focus on Case 3.[18] Given the optimal terms of trade, the demand for money and loans as well as the supply of deposits, we can solve for equilibrium in the loans and deposits markets. The assumption of perfect competition and free entry in the banking sector implies that $i_1^l = i_1^d$ and $i_2^l = i_2^d$. Combining (44) (or (48)) and (57) we obtain the following equilibrium condition

$$i_1^l = i_1^d = s_1. \tag{64}$$

[18]Notice that Cases 1 and 2 are the same as Case 3 with $\varepsilon_R = \bar{\varepsilon}$.

Similarly for currency 2, we combine (49) and (62) to get

$$i_2^l = i_2^d = s_2. \tag{65}$$

Equations (64) and (89) imply that the equilibrium interest rates prevailing in the markets for banking services in currencies 1 and 2 are equal to the opportunity costs of holding currencies 1 and 2, respectively. Using the above in (57) and (62), we get

$$\varepsilon \frac{u'(\widetilde{q}_\varepsilon)}{c'(\widetilde{q}_\varepsilon)} - 1 = s_1 \tag{66}$$

for the domestic currency users (with $\varepsilon < \varepsilon_R$) and

$$\varepsilon \frac{u'(\widehat{q}_\varepsilon)}{c'(\widehat{q}_\varepsilon)} - 1 = s_2 \tag{67}$$

for foreign currency users (with $\varepsilon \geq \varepsilon_R$). It is not too surprising to see that the quantities exchanged with the two active loan markets are higher than the quantities exchanged when these loan markets are inactive (*i.e.* in the benchmark case). The banking system permits indeed the reallocation of domestic and foreign currencies between buyers with and without liquidity needs.

The aggregate supply of deposits in currency 1 and currency 2 are given by

$$D_1 = (1-\sigma) \int_0^{\varepsilon_R} d_{1,\varepsilon} dF(\varepsilon) \tag{68}$$

and

$$D_2 = (1-\sigma) \int_{\varepsilon_R}^{\overline{\varepsilon}} d_{2,\varepsilon} dF(\varepsilon) \tag{69}$$

respectively. The aggregate demand for loans in currency 1 is given by

$$L_1 = \sigma \int_0^{\varepsilon_R} l_{1,\varepsilon} dF(\varepsilon) \tag{70}$$

and in currency 2 by

$$L_2 = \sigma \int_{\varepsilon_R}^{\overline{\varepsilon}} l_{2,\varepsilon} dF(\varepsilon). \tag{71}$$

Clearing of the banks' loans and deposits market in the domestic currency requires that

$$(1-\sigma)\int\limits_{0}^{\varepsilon_R} d_{1,\varepsilon} dF(\varepsilon) = \sigma \int\limits_{0}^{\varepsilon_R} l_{1,\varepsilon} dF(\varepsilon) \tag{72}$$

and in the foreign currency that

$$(1-\sigma)\int\limits_{\varepsilon_R}^{\bar{\varepsilon}} d_{2,\varepsilon} dF(\varepsilon) = \sigma \int\limits_{\varepsilon_R}^{\bar{\varepsilon}} l_{2,\varepsilon} dF(\varepsilon). \tag{73}$$

Using the individual loan demand (46) and deposit supply (45), we get

$$(1-\sigma)\int\limits_{0}^{\varepsilon_R} \widetilde{z}_{1,\varepsilon} dF(\varepsilon) = \sigma \int\limits_{0}^{\varepsilon_R} c(\widetilde{q}_{\varepsilon}) - \widetilde{z}_{1,\varepsilon} dF(\varepsilon) \tag{74}$$

which when simplified yields the aggregate demand for real money balances

$$\int\limits_{0}^{\varepsilon_R} \widetilde{z}_{1,\varepsilon} dF(\varepsilon) = \sigma \int\limits_{0}^{\varepsilon_R} c(\widetilde{q}_{\varepsilon}) dF(\varepsilon). \tag{75}$$

for which clearly the function

$$\widetilde{z}_{1,\varepsilon} = \sigma c(\widetilde{q}_{\varepsilon}) \tag{76}$$

is a solution, with $\widetilde{q}_{\varepsilon}$ given by (66). The amount borrowed by a given buyer is then

$$l_{1,\varepsilon} = (1-\sigma) c(\widetilde{q}_{\varepsilon}). \tag{77}$$

In the same way, the aggregate demand for currency 2 is given by

$$\int\limits_{\varepsilon_R}^{\bar{\varepsilon}} \widehat{z}_{2,\varepsilon} dF(\varepsilon) = \sigma \int\limits_{\varepsilon_R}^{\bar{\varepsilon}} \left(c(\widehat{q}_{\varepsilon}) + \kappa \right) dF(\varepsilon) \tag{78}$$

which yields the individual demand for currency 2

$$\widehat{z}_{2,\varepsilon} = \sigma \left(c(\widehat{q}_{\varepsilon}) + \kappa \right) \tag{79}$$

and loan demand

$$l_{2,\varepsilon} = (1-\sigma) \left(c(\widehat{q}_{\varepsilon}) + \kappa \right). \tag{80}$$

Clearly, money demand as a fraction of consumption is lower in an economy with both domestic and foreign bank accounts. However, consumption is higher.

Definition 13 *A stationary equilibrium with domestic and foreign currency bank accounts consists of allocations, real balances, loans* $(\widetilde{q}_{\varepsilon},\ \widehat{q}_{\varepsilon},\ \widetilde{z}_{1,\varepsilon},\ \widehat{z}_{2,\varepsilon},\ l_{1,\varepsilon},\ l_{2,\varepsilon})$ *and the preference threshold* (ε_R) *that satisfy optimal DM consumption equations* (66) *and* (67), *real balances equations* (76) *and* (79), *loans demand equations* (77) *and* (80) *and the indifference condition* $\widetilde{\Omega}^b_{\varepsilon_R} = \widehat{\Omega}^b_{\varepsilon_R}$.

5.6 Equilibrium without foreign currency bank accounts

In practice, banks tend to be restricted when offering foreign currency denominated deposits and loans to domestic customers.[19] This type of regulation affects the use of foreign currency relative to the domestic one.

When foreign currency bank accounts are not available, buyers using the foreign currency are unable to deposit their money holdings or borrow additional funds. In general, this makes the use of foreign currency less attractive for buyers as we will see below. The amount consumed by foreign currency buyers is determined as in the no-banking equilibrium, given by equation (26), and the amount of real foreign currency balances is given by (27). The optimality conditions for domestic currency buyers are the same as in the equilibrium with both domestic and foreign currency accounts described in the previous section. This leaves us with the following equilibrium definition.

Definition 14 *A stationary monetary equilibrium with only domestic currency bank accounts consists of allocations, real balances, loans* $(\widetilde{q}_{\varepsilon},\ \widehat{q}_{\varepsilon},\ \widetilde{z}_{1,\varepsilon},\ \widehat{z}_{2,\varepsilon},\ l_{1,\varepsilon})$ *and the preference threshold* (ε_R) *that satisfy the optimal DM consumption equations* (66) *and* (26), *real balances equations* (76) *and* (27), *loans demand equations* (77) *and the indifference equation* $\widetilde{\Omega}^b_{\varepsilon_R} = \widehat{\Omega}^b_{\varepsilon_R}$.

As we discussed previously, restricting access to foreign currency denominated bank accounts implies that there will be no active foreign currency loan market in the domnestic economy i.e.

[19]For example, foreign denominated bank accounts are outlawed in Brazil and Venezuela but not in Argentina, Uruguay, Mexico among other Latin American countries. We refer to Gomis-Porqueras et al. (2005) for more on foreign denominated bank accounts and its impact on the banking system.

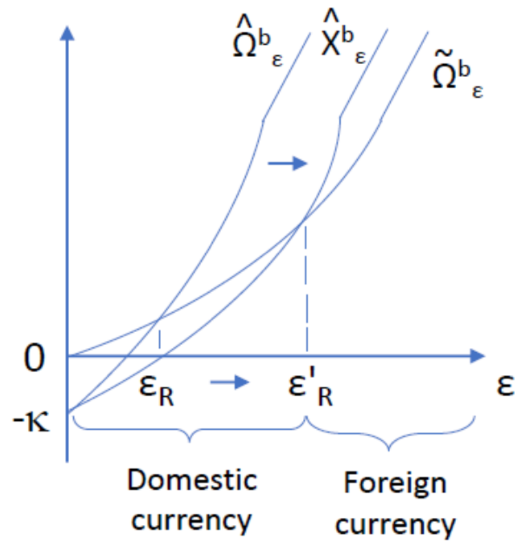

Figure 6: Shut-down of the foreign currency loan market.

$l_2 = d_2 = 0$. As a consequence, the holding cost of foreign money will increase relative to the domestic currency. This is the case as it would not possible for foreign currency buyers to earn an interest on their idle real balances in case they don't meet a seller. Thus, fewer buyers will hold foreign currency, and the quantity of goods exchanged in DM will decrease. Figure 6 illustrates this effect. From equations (67) and (26), the cost of holding foreign currency will increase from s_2 to $\frac{s_2}{\sigma}$. Consequently, as we can see from Figure 6, we have that$\widehat{\Omega}^b_\varepsilon$ curve will shift down to $\widehat{X}^b_\varepsilon$, and that ε_R will increase to ε'_R as a result. In conclusion, less people will use and accept the foreign currency in the economy, implying a reduction in dollarization.

5.7 Comparative statics

We now explore how different changes in the underlying economic environment affect currency substitution and the demand for foreign denominated bank accounts.

Figure 7 depicts how the dollarization preference threshold changes as a function of model parameters in the three different environments we previously presented and analyzed. The left panel of Figure 7 depicts dollarization as a function of monetary policy across all three environments. In all three models, increasing the relative return on the domestic currency by decreasing the inflation rate leads to a decrease in dollarization. It does so at an increasing rate. However, the return at which complete de-dollarization occurs varies depending which of the three models we consider. More precisely, the first economy to de-dollarize is the one where only domestic currency deposits are available. This is because the access to domestic deposit accounts offers the domestic currency an advantage over the foreign currency in terms of liquidity insurance. This is provided through interest rate payment on their deposits. Furthermore, increasing the return on domestic currency, R_1, results in the de-dollarization of the economy where both types of deposit accounts are available. Not surprisingly, the last economy to de-dollarize is the economy without deposit accounts. Interestingly, de-dollarization happens in the economy with both foreign and domestic bank accounts faster than in the economy without banks. The intuition behind this result is that introducing bank accounts mostly benefits holders of the currency with the lowest return, namely the domestic currency.

As can be seen in the right panel of Figure 7, an increase in the cost of accepting foreign currency reduces dollarization. However, this increase has differential effects depending on the type of deposit accounts that are available to domestic agents. The effect is strongest when only domestic currency deposit accounts are available. In this case complete de-dollarization happens the fastest. This is the case as the availability of domestic currency deposit accounts increase the demand for the domestic currency relative to the foreign currency.

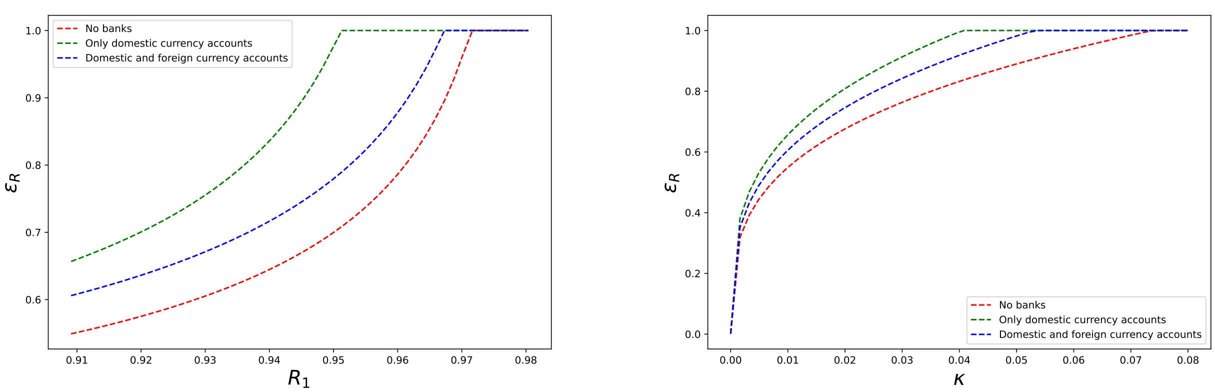

Figure 7: Foreign currency adoption as a function of parameters, model comparison.

6 Welfare implications of deposit dollarization

In this section we compare welfare under three different environments: the baseline economy without banks, an economy with only domestic currency accounts and finally an economy with domestic and foreign currency accounts. To do so, we first define our welfare measure as

$$\mathcal{W} = \sigma \int_0^{\varepsilon_R} (u(\tilde{q}_\varepsilon) - c(\tilde{q}_\varepsilon))\, dF(\varepsilon) + \sigma \int_{\varepsilon_R}^{\bar{\varepsilon}} (u(\hat{q}_\varepsilon) - c(\hat{q}_\varepsilon) - \kappa)\, dF(\varepsilon) + U(x^*) - x^* \tag{81}$$

which sums the DM net surplus across both matches using domestic currency and matches using foreign currency in addition to the CM surplus.

The implications for welfare of changing the rate of return of the domestic currency in the three different economies are depicted in Figure 8. First, welfare is increasing in the return on the domestic fiat currency, R_1, in all three models. When R_1 is sufficiently low, welfare is highest in the economy where both types of bank accounts is available (blue line). Welfare levels in economies with no banks (red line) and with only domestic currency accounts (green line) are close. As R_1 increases relative to the return on the foreign currency, R_2, welfare in the economy with only domestic bank accounts increases the most until we reach complete de-dollarization. Surprisingly, this is exactly the point where it surpasses welfare in the economy with both foreign and domestic bank accounts.

To better understand this result, let us start from the right side of the Figure where both the blue and green lines are identical. At this level of R_1, only the domestic currency circulates and domestic bank accounts are used in both economies. Hence the two economies are identical. As we reduce R_1, welfare in both economies is reduced at the same rate. This is the case until we reach a level where the two lines seperate. This is when the economy with both types of bank accounts start to dollarize. At this point the blue line falls at a faster rate than the red line. This is a direct result of a pecuniary externality resulting from some buyers decision to switch to the foreign currency. Pecuniary externalities are common in models with heterogeneous agents where different sets of agents face different constraints. The externality happens when the decisions of one set of agents affect prices which tightens the constraint on another set of agents as pointed by (Loong and Zeckhauser (1982); Greenwald and Stiglitz (1986); Dávila and Korinek (2018), among others).

In our model, buyers using the domestic currency face a different liquidity constraint compared to buyers using the foreign currency. When buyers start holding the foreign currency this has the effect of reducing the demand for the domestic currency and hence it's value. Buyers who still hold the domestic currency see the value of their currency further reduced. This in turn leads to lower welfare. This finding suggest then that it might be welfare improving to restrict access to foreign currency bank accounts for intermediate values of R_1, i.e. when domestic inflation is not too high. This results also indicates that a Pigouvian tax might improve welfare for intermediate values of R_1.

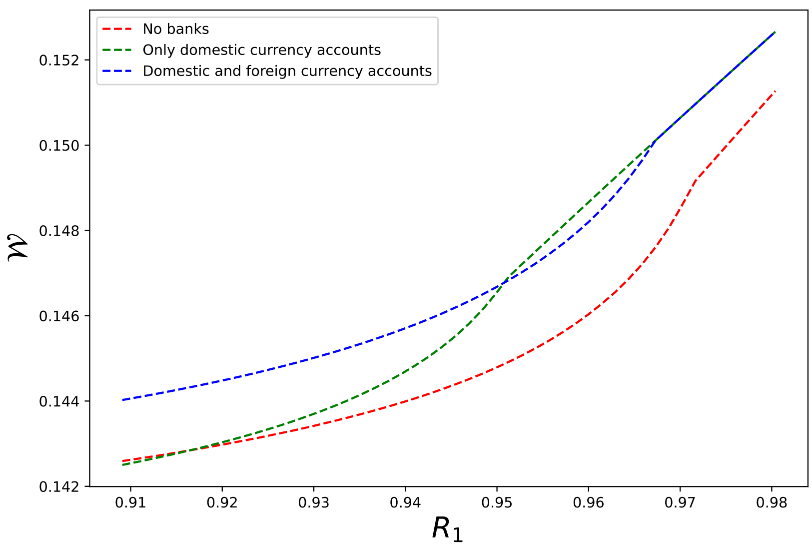

Figure 8: Welfare as a function of R_1, model comparison.

6.1 A tax on foreign currency deposits

We now relax our assumption that the domestic taxing authority can not tax any DM activity. In particular, we assume bank deposits in currency j are taxed at rate $\tau_j \in [0, 1]$.[20] The tax base includes both the deposited amount as well as the interests earned. In contrast real balances (domestic or foreign) held by DM buyers can not be taxed by the domestic fiscal authority. Then

[20]This taxing practice or differential reserve requirements have been used in dollarized economies. We refer to Reinhart and Reinhart (1999) and Coelho et al. (2001) for more on this matter.

the value function of a buyer in the CM is then given by

$$W^b(\omega, d_1, d_2, l_1, l_2, \varepsilon) = \max_{h,z_1,z_2} \left\{-h + \beta V^b(z_1, z_2, \varepsilon)\right\}$$
$$\text{s.t. } \frac{z_1}{R_1} + \frac{z_2}{R_2} = h + \omega + \sum_{j=1}^{2} (1-\tau_j)\left(1+i_j^d\right) d_j - \sum_{j=1}^{2}\left(1+i_j^l\right) l_j + \tau. \tag{82}$$

The rest of the model derivation is exactly the same except for the choice of deposits which becomes

$$\max_{d_1,d_2} \sum_{j=1}^{2} \left[i_j^d - \tau_j(1+i_j^d)\right] d_j$$
$$\text{s.t. } d_1 \le z_1,\ d_2 \le z_2,\ d_1 \ge 0,\ d_2 \ge 0. \tag{83}$$

The optimal choice of deposits satisfies the following first order conditions

$$d_j = z_j \text{ if } i_j^d > \frac{\tau_j}{1-\tau_j} \quad , \quad d_j \in [0, z_j] \text{ if } i_j^d = \frac{\tau_j}{1-\tau_j} \quad , \quad d_j = 0 \text{ if } i_j^d < \frac{\tau_j}{1-\tau_j} \tag{84}$$

for $j \in \{1, 2\}$. Notice that the positivity constraint on deposits matters now since it's possible that the after-tax return on deposits becomes negative if $i_j^d < \frac{\tau_j}{1-\tau_j}$. In this case, buyers holding a foreign currency would be better off keeping their liquidity in the form of cash to avoid the tax on deposits.[21] Given the above choice of deposits and assuming $i_j^d > \frac{\tau_j}{1-\tau_j}$, the problem of determining the optimal currency portfolio becomes

$$\max_{z_1,z_2} \left\{-\sum_{j=1}^{2} \frac{z_j}{R_j} + \beta\sigma \max\left[\widetilde{S}_\varepsilon\left(\widetilde{q}_\varepsilon\right), \widehat{S}_\varepsilon\left(\widehat{q}_\varepsilon\right)\right] + \beta(1-\sigma)\sum_{j=1}^{2}(1-\tau_j)\left(1+i_j^d\right) z_j + \beta\sigma\sum_{j=1}^{2} z_j\right\}. \tag{85}$$

In what follows we focus on Case 3, where both currencies are used in equilibrium. We also assume $\tau_1 = 0$ and let $\tau_2 \in (0, 1)$. As in the banking model without taxes, there exist a threshold ε_R such that buyers with preferences shock $\varepsilon > \varepsilon_R$ will use foreign currency while buyers with $\varepsilon < \varepsilon_R$ will use the local currency. This threshold satisfies the indifference equation $\widetilde{\Omega}^b_{\varepsilon_R} = \widehat{\Omega}^b_{\varepsilon_R}$ where $\widetilde{\Omega}^b_{\varepsilon_R}$ is the same as in the setting without a deposit tax and $\widehat{\Omega}^b_{\varepsilon_R}$ is given by

$$\max_{\widehat{z}_2} \left[-s_2\widehat{z}_2 + \sigma\widehat{S}_\varepsilon(\widehat{z}_2 + l_2 - \kappa) + (1-\sigma)\left(i_2^d - \tau_2(1+i_2^d)\right)\widehat{z}_2\right] \tag{86}$$

[21] This is similar to the mechanism highlighted in Gomis-Porqueras et al. (2014) in the context of a sales tax when characterizing the equilibrium size of the informal economy.

which assumes $i_2^d > \frac{\tau_2}{1-\tau_2}$.

Since we set $\tau_1 = 0$, the optimal amount of real balances for buyers using domestic currency is the same as we derived in the previous section. For buyers with $\varepsilon > \varepsilon_R$, the optimal amount of real balances of foreign currency solves the above optimization problem yielding the following optimality condition

$$\varepsilon \frac{u'(\widehat{q}_\varepsilon)}{c'(\widehat{q}_\varepsilon)} - 1 = \frac{s_2 - (1-\sigma)\left(i_2^d - \tau_2(1+i_2^d)\right)}{\sigma}. \tag{87}$$

Clearly, increasing taxes on foreign deposits, τ_2, increases the cost of holding the foreign currency by lowering the return on deposits. This in turn reduces $\widehat{q}_\varepsilon$. Since the tax is paid in CM, well after the banking market has closed, the market clearing conditions for that market will be the same as in the previous section.

Since banks do not pay any taxes, under perfect competition in the banking sector we have that $i_2^l = i_2^d$. Combining (49) and (87) we get that the interest rate on foreign deposits satisfies the following condition

$$i_2^d = \frac{s_2 - (1-\sigma)\left(i_2^d - \tau_2(1+i_2^d)\right)}{\sigma}. \tag{88}$$

Using the previous expression we can solve for the interest rate that clears the market for deposits and loans, which delivers the following equilibrium rate

$$i_2^l = i_2^d = \frac{s_2 + (1-\sigma)\tau_2}{1-(1-\sigma)\tau_2} \tag{89}$$

as long as $i_2^d > \frac{\tau_2}{1-\tau_2}$ which ensures foreign currency buyers deposit all their money balances. If this condition is not satisfied, i.e. if the tax is too high, buyers will choose to keep their real balances in the form of cash to avoid the deposit tax.

As we can see from equation (89), imposing a tax on foreign deposits increases the interest rate earned on these deposits since buyers require a higher interest payment to partially compensate for the deposit tax. Since the banking interest rate determines the marginal cost of liquidity in the DM, a tax on foreign currency deposits implies a lower DM consumption for buyers using foreign currency. This in turn should translate into a lower demand for the foreign currency and hence a decrease in dollarization, ie. a higher ε_R.

Next, we investigate whether this tax can (partially) correct the pecuniary externality described in Figure 8. For that, we proceed with a numerical example. Figure 9 depicts a contour plot the difference in welfare between the model with a foreign currency deposit tax and the model without as a function of both the return on the domestic currency R_1 and the deposit tax τ_2. What interests us the yellow and yellow-green region where the difference in welfare is positive. In our parametrization, this region occurs when inflation is moderate (roughly between 3% and 5%, i.e. R_1 between 0.95 and 0.97). This is the region where the pecuniary externality is operating and hence imposing a tax on foreign currency deposits is welfare-improving.

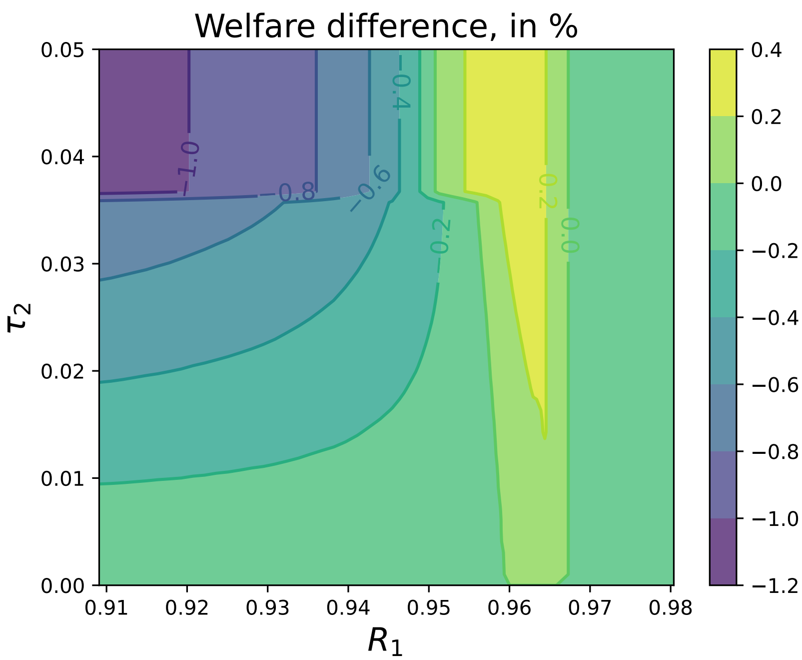

Figure 9: Welfare difference as a function of R_1 and τ_2, in %.

6.2 Differential reserve requirements

7 Seigniorage income and deposit dollarization

TBC

8 Conclusion

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Appendix

Proof of Proposition 3

Note that ε_R is such that $\widetilde{\mathcal{U}}_\varepsilon^b = \widehat{\mathcal{U}}_\varepsilon^b$, where $\widetilde{\mathcal{U}}_\varepsilon^b = \varepsilon u(\widetilde{q}^*) - c(\widetilde{q}^*)$ if $c(q_\varepsilon^*) \leq z_1$ and $\widetilde{\mathcal{U}}_\varepsilon^b = \varepsilon u\left[c^{-1}(z_1)\right] - z_1$ if $z_1 < c(q_\varepsilon^*)$. The derivatives relative to ε are given by $\frac{\partial \widetilde{\mathcal{U}}_\varepsilon^b}{\partial \varepsilon} = u(\widetilde{q}^*)$ if $c(q_\varepsilon^*) \leq z_1$, and $\frac{\partial \widetilde{\mathcal{U}}_\varepsilon^b}{\partial \varepsilon} = u(\widetilde{q}_\varepsilon)$ with $\widetilde{q}_\varepsilon = c^{-1}(z_1)$ if $z_1 < c(q_\varepsilon^*)$. When $\varepsilon = 0$, $\widetilde{\mathcal{U}}_\varepsilon^b = 0$, and $\widetilde{\mathcal{U}}_\varepsilon^b$ is an increasing function of ε, and becomes linear in ε at q_ε^*.

$\widehat{\mathcal{U}}_\varepsilon^b = \varepsilon u(\widehat{q}) - c(\widehat{q}) - \kappa$ if $c(q_\varepsilon^*) \leq z_1 + z_2 - \kappa$ and $\widehat{\mathcal{U}}_\varepsilon^b = \varepsilon u\left[c^{-1}(z_1 + z_2 - \kappa)\right] - (z_1 + z_2 - \kappa)$ if $z_1 + z_2 - \kappa < c(q_\varepsilon^*)$. The derivatives relative to ε are given by $\frac{\partial \widehat{\mathcal{U}}_\varepsilon^b}{\partial \varepsilon} = u(\widehat{q}^*)$ if $c(q_\varepsilon^*) \leq z_1 + z_2 - \kappa$, and $\frac{\partial \widehat{\mathcal{U}}_\varepsilon^b}{\partial \varepsilon} = u(\widehat{q}_\varepsilon)$ with $\widehat{q}_\varepsilon = c^{-1}(z_1 + z_2 - \kappa)$ if $z_1 + z_2 - \kappa < c(q_\varepsilon^*)$. When $\varepsilon = 0$, $\widehat{\mathcal{U}}_\varepsilon^b = -\kappa$, and $\widehat{\mathcal{U}}_\varepsilon^b$ is an increasing function of ε. If $z_2 - \kappa < 0$, the slopes of $\widetilde{\mathcal{U}}_\varepsilon^b$ and $\widehat{\mathcal{U}}_\varepsilon^b$ are equal. If $z_2 - \kappa > 0$ the slope of $\widehat{\mathcal{U}}_\varepsilon^b$ becomes steeper than the slope of $\widetilde{\mathcal{U}}_\varepsilon^b$ as $\widehat{q}_\varepsilon$ is an increasing function of total real balances. As the slope of $\widehat{\mathcal{U}}_\varepsilon^b$ is higher than the slope of $\widetilde{\mathcal{U}}_\varepsilon^b$, and increases with ε from below, the two curves intersect at ε_R, such that $\widehat{\mathcal{U}}_{\varepsilon_R}^b = \widetilde{\mathcal{U}}_{\varepsilon_R}^b \iff u(\widehat{q}_\varepsilon) - c(\widehat{q}_\varepsilon) - \kappa = u(\widetilde{q}_\varepsilon) - c(\widetilde{q}_\varepsilon)$, and become linear in ε at q_ε^*.

Proof of Proposition 4

Recall that $\widetilde{S}_\varepsilon(\widetilde{q}_\varepsilon) = \varepsilon u(\widetilde{q}^*) - c(\widetilde{q}^*)$ if $c(q_\varepsilon^*) \leq z_1$ and $\widetilde{S}_\varepsilon(\widetilde{q}_\varepsilon) = \varepsilon u\left[c^{-1}(z_1)\right] - z_1$ if $z_1 < c(q_\varepsilon^*)$. The derivatives of the surplus are $\frac{\partial \widetilde{S}_\varepsilon}{\partial \varepsilon} = u(\widetilde{q}^*)$ if $c(q_\varepsilon^*) \leq z_1$, and $\frac{\partial \widetilde{S}_\varepsilon}{\partial \varepsilon} = u(\widetilde{q}_\varepsilon)$ with $\widetilde{q}_\varepsilon = c^{-1}(z_1)$ if $z_1 < c(q_\varepsilon^*)$. Therefore, when $\varepsilon = 0$, $\widetilde{X}_\varepsilon^b = 0$, and $\widetilde{X}_\varepsilon^b$ is an increasing function of ε, and becomes linear in ε at q_ε^*.

When $z_1 = 0$, $\widehat{S}_\varepsilon(\widehat{q}_\varepsilon) = \varepsilon u(\widehat{q}_\varepsilon) - c(\widehat{q}_\varepsilon) - \kappa$ if $c(q_\varepsilon^*) \leq z_2 - \kappa$ and $\widehat{S}_\varepsilon(\widehat{q}_\varepsilon) = \varepsilon u\left[c^{-1}(z_2 - \kappa)\right] - (z_2 - \kappa)$ if $z_2 - \kappa < c(q_\varepsilon^*)$. The derivatives of the surplus are $\frac{\partial \widehat{S}_\varepsilon}{\partial \varepsilon} = u(\widehat{q}^*)$ if $c(q_\varepsilon^*) \leq z_2 - \kappa$, and $\frac{\partial \widehat{S}_\varepsilon}{\partial \varepsilon} = u(\widehat{q}_\varepsilon)$ with $\widehat{q}_\varepsilon = c^{-1}(z_2 - \kappa)$ if $z_2 - \kappa < c(q_\varepsilon^*)$. When $\varepsilon = 0$, $\widehat{X}_\varepsilon^b = -\kappa$, and $\widehat{X}_\varepsilon^b$ is an increasing function of ε, with a higher slope than $\widetilde{X}_\varepsilon^b$ as the rate of return of the foreign currency (and the quantity exchanged for a given ε) is higher than the domestic one, and becomes linear in ε at q_ε^*. As the slope of $\widehat{X}_\varepsilon^b$ is higher than the slope of $\widetilde{X}_\varepsilon^b$, the two curves intersect in ε_R, such that $\widehat{X}_{\varepsilon_R}^b = \widetilde{X}_{\varepsilon_R}^b$.