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Fairness in Voters' Ethical Calculus

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Abstract

The *ethical voting* solution to the paradox of voting has been based on a *rule-utilitarian* calculus within each group of supporters. This paper reassesses the utilitarian foundation by considering a setup in which the intensity of support differs among members of the same group. In this case, it is optimal for utilitarian agents to ignore the intensity dimension and vote or abstain based only on their voting cost. This contradicts the evidence that voter turnout correlates with voter preferences at the individual level. I argue that such a correlation may be captured by considerations of fairness, which require members with stronger preferences to vote on average for higher costs. I examine *fair* voting rules, based on either a proportionality or an egalitarian principle, and compare the equilibrium outcomes from such rules with the standard case of utilitarian rules. Postulating a role for fairness aligns the ethical framework with a broad range of voting models, according to which participation increases in the personal benefit from the election or in the ideological proximity between voters and candidates.

Keywords: voter turnout, ethical voting, fairness, rule-utilitarianism, egalitarianism.

JEL Classification: D72, D71, D63

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1 Introduction

Collective decision-making in democracies relies on citizens to vote in elections. Yet, rational choice theory has long struggled to explain electoral participation. If agents vote only to affect the outcome, the number of voters is large, and voting is even slightly costly, the negligible chance of being pivotal should bind turnout close to zero, in stark contrast to what we typically observe. A way out of this impasse has been to consider citizens' sense of duty as a driver of participation. In this more recent line of work, voting is "ethical" rather than instrumental, in that voters set a rule of behavior within their group and follow it irrespective of pivotality considerations.

This paper deals with what ethical calculus voters appear to follow. In the two seminal contributions, by Feddersen and Sandroni (2006a) and Coate and Conlin (2004), members of each group differ only in their voting cost, and a rule of behavior is given by a threshold cost below which members should vote. Their framework is inspired by Harsanyi's (1980) *rule-utilitarianism*, in that the threshold cost is set ex-ante in order to maximize an aggregate utility.[1] In what follows, I reexamine the extent to which turnout behavior is consistent with a purely utilitarian maximization. Specifically, I argue that, if members of a group differ also in the intensity of their preference, a rule that includes a concern for fairness is more in line with the empirical evidence.

Consider a group of supporters, who differ both in their voting costs and in the intensity of support for their candidate, relative to an opponent. An optimal utilitarian rule would command them to ignore the intensity dimension and to condition voting only on their cost. Indeed, from an aggregate (utilitarian) perspective, the total benefit depends on the probability that the group's candidate wins, and thus on the number of votes cast by the group, but not on which members provide those votes. The total cost, instead, is minimized by making sure that members with low costs vote, irrespective of the intensity of their preference. If the intensity of support and the voting cost are independently distributed, the implication is that strong and weak supporters should vote on average at the same rate.

Yet, the previous pattern is at odds with the theory and evidence from most voting models, according to which a higher benefit from the outcome of the election translates into a higher likelihood of voting. Empirically, most studies that look at the strength of voters' preference for a candidate or at the extent of their stakes in the election find a positive effect on the probability of voting (Holbrook et al. 2001, Harder and Krosnick 2008, Smets and Van Ham 2013). The *spatial voting* literature, pioneered by Downs (1957), represents voters' evaluation

[1] Specifically, members in each group maximize the aggregate group utility in Coate and Conlin (2004), and a welfare measure which includes all social costs of voting in Feddersen and Sandroni (2006a).

of candidates in terms of distance in an ideological or policy space. In this framework, the proximity of voters to candidates is a measure of the intensity of their preference, and scholars have indeed shown that such ideological distance affects not only the vote choice but also the likelihood of voting (Zipp 1985, Plane and Gershtenson 2004, Adams, Dow and Merrill 2006).

In light of the previous limitation, I look for an alternative calculus, within the ethical voting framework, that can account for the empirical correlation. The hypothesis considered in the paper is that a higher turnout rate by stronger supporters may reflect considerations of fairness within a group, by which members with "more at stake" should contribute more in the effort of providing votes. I develop this intuition by considering two rules based on principles associated to fairness. The first is a proportional rule, according to which the expected cost of voting for each member should be proportional to the intensity of the preference. The second is an egalitarian rule, which equalizes as much as possible members' expected utility. The qualifier *as much as possible* is important, because if some members have a very low intensity of preference - i.e. they are almost indifferent between the competing candidates - a fully egalitarian rule can only dictate abstention for all members. Instead, the proposed egalitarian rule conditions the equality of expected utility on voting for at least some cost realizations, allowing very weak supporters to always abstain and to obtain a lower expected utility.

I first show that both the proportional and the (conditionally) egalitarian rule require stronger supporters to vote for a wider range of voting costs than weaker supporters. Then, I evaluate whether the different calculus affects the existence and properties of an equilibrium configuration, in which the rules set by the two groups are consistent with each other. For the equilibrium analysis, I rely on simple uniform distributions for all random variables in the model, which, at a cost in generality, guarantee closed-form solutions. In all three cases, i.e. if groups use utilitarian, proportional or (conditionally) egalitarian rules, there exists a unique symmetric equilibrium, whose implications are similar in terms of aggregate turnout in the election. Only the two rules based on fairness, however, capture the positive correlation between turnout and the intensity of preferences at the individual level.

In the costly voting literature, the role of duty has been acknowledged since the seminal work by Riker and Ordeshook (1968). In their paper, however, duty is introduced as an exogenous term in voters' utility function. The important contribution of the ethical voting framework by Feddersen and Sandroni (2006a) and Coate and Conlin (2004) has been to endogenize the duty component. Their models highlight that the ethical motivation does not exclude strategic thinking but can rather interact with it. The equilibrium rules are thus endogenous to the strategic interaction between the groups, which makes participation responsive to the characteristics of the election. Feddersen and Sandroni address some technical aspects of their theory in a companion paper (2006b), while Feddersen (2004) offers a review of the rational-choice literature on voter turnout.[2]

The ethical voting model has been extended by several papers, which capture additional patterns of voting behavior. Ali and Lin (2013) examine the interaction between ethical motives and social rewards. Jorgenson and Saavedra (2018) adapt the model in the presence of an Electoral College with partisan and battleground districts, while Levine and Mattozzi (2020) augment it with an enforcement mechanism for the voting rules, through costly peer punishment. Bierbrauer, Tsyvinsky, and Werquin (2021) study how parties' platforms are determined endogenously with ethically-motivated turnout. The implications of the ethical calculus for the allocation of voting duties in heterogenous groups, however, have not received the deserved attention.[3]

Assuming different intensities of support among group members challenges the model's premises more than superficially. In the standard framework, the two groups have opposite views, and there is thus a degree of heterogeneity in citizens' preferences. Yet, all members of the same group obtain the same benefit from the outcome of the election. Harsanyi's rule-utilitarianism is then applied under the assumption that the rule is set before the realizations of the voting costs. Behind this "veil of ignorance", members of the same group are ex-ante identical and benefit equally from maximizing the aggregate utility. Instead, in my framework, different intensities of support may correspond to agents with idiosyncratic preferences over a policy (or ideological) space. This heterogeneity relates to the conflict of interest that is inherent to politics and is therefore less suitable to be put behind a veil of ignorance. As such, even ex-ante with respect to the realization of the voting costs, group members are not identical as in Harsanyi's perspective, and so the utilitarian criterion loses its theoretical appeal.

The general issue underlying the analysis concerns what norms of cooperation emerge among agents who have a common objective but differ in some important dimension. In this respect, much evidence shows that a concern for fairness is strong in humans' interactions (Kahneman, Knetsch, and Thaler 1986, Roth 1995, Fehr and Schmidt 1999, Faravelli 2007). Economists have recently started to delve into the pluralism of fairness, which embraces notions of merit, desert, or accountability in more complex situations (Konow 2003, Cappelen et al. 2007). Yet, in my framework, the problem of sharing voting costs within a group seems

[2]The ethical voting framework is formally very similar to models of elite-driven turnout (Shachar and Nalebuff 1999, Morton 1991), in which participation is determined by costly efforts of candidates or parties.

[3]Despite Coate and Conlin (2004) pointing to this direction of future research in the conclusion of their paper, where they wrote *"it would also be interesting to think through the implications of heterogeneity in supporters' and opposers' preferences. It seems likely that, within groups, those voters who care less intensely about an issue will have lower critical-cost levels."* (page 1497).

relatively free from these difficulties, and I look at rules which capture the essence of a fairness principle. Between the two proposed solutions, the proportional rule is more tractable than the egalitarian one and fits well within the model because voters' expected utility is linear in both the benefit from the election and the expected voting cost. The egalitarian rule is less tractable, although perhaps based on a stronger ethical principle, which would be appealing even outside a linear model.[4] The ability of both rules to account for the correlation between preferences and participation suggests that fairness may indeed enter people's assessment of their duty to vote.

The remainder of the paper is organized as follows. Section 2 presents the model and derives the implications of agents adopting a utilitarian, a proportional, or a (conditionally) egalitarian turnout rule within their group. Section 3 solves the three variants of the model for an equilibrium outcome and discusses the resulting properties. Section 4 offers a concluding discussion.

2 Model

Two candidates, A and B, run for election. An electorate of mass 1 is divided between groups A and B, according to which candidate citizens prefer. The size of the two groups is uncertain: μ is the size of group A and $1-\mu$ the size of group B, with μ being distributed according to a continuous distribution F_μ on $\mathcal{M} \subseteq [0,1]$.

Groups are heterogeneous in two dimensions. First, members have different intensities of preference for their candidate. I normalize to zero the benefit that each member receives if the candidate of the opposite group wins, and I denote wz_i the benefit of member i from the victory of the own candidate: in each group, z_i is drawn from a continuous distribution F_{z_i} on $\mathcal{Z} \subseteq \mathbb{R}_+$, while w is a common parameter measuring the importance of the election. Heterogeneous intensities of preference can account for differences in voters' degree of partisanship or concern about the election, and also allow for a spatial interpretation of the model, in which voters are distributed over a policy space and evaluate candidates based on their distance from them. In this case, groups are composed by the voters who are closer to one candidate rather than the other, and the intensity of preference corresponds, for each voter, to the utility difference between the closest and the farthest candidate.

Second, members have different costs of voting. In each group, χc_i denotes the cost of

[4]See Fowler, Johnson, and Smirnov (2005) and Dawes et al. (2007) for evidence that egalitarian motives are widespread in humans; Fehr, Bernhard, and Rockenbach (2008) and Basic, Falk, and Kosse (2019) for the emergence of such motives in children; and Binmore (2006) and references therein for a discussion on the occurrence of equal-sharing norms among hunter-gatherer societies that have survived to modern times.

voting for member i: c_i is drawn from a continuous distribution F_{c_i} on $\mathcal{C} \subseteq \mathbb{R}_+$, and χ is a common cost parameter. The cost of voting is interpreted as the opportunity cost of the time spent in the process of voting. The intensity of preference and the cost of voting, i.e. the realizations of z_i and c_i, are independent. The distribution of both variables within groups is common knowledge.

A turnout rule in each group is a function mapping the intensity of preference and the cost of voting into a choice between voting and abstaining:

$$t_A(z_i, c_i) : \mathcal{Z} \times \mathcal{C} \to \{0, 1\}$$
$$t_B(z_i, c_i) : \mathcal{Z} \times \mathcal{C} \to \{0, 1\}$$

where $t_A(z_i, c_i) = 1$ means that member i (in group A) votes and $t_A(z_i, c_i) = 0$ means that member i abstains. The turnout rate in each group is obtained by aggregating the turnout rules across the two dimensions of heterogeneity:

$$\tau_A = \int_{\mathcal{Z}} \int_{\mathcal{C}} t_A(z_i, c_i)\, dF_{z_i}\, dF_{c_i}$$
$$\tau_B = \int_{\mathcal{Z}} \int_{\mathcal{C}} t_B(z_i, c_i)\, dF_{z_i}\, dF_{c_i}$$

For given turnout rates τ_A and τ_B in the two groups, the candidate who wins the election depends on the realization of the groups' sizes, μ and $1 - \mu$. The probability that candidate A wins is

$$P\left(\mu \tau_A > (1 - \mu)\, \tau_B\right) = P\left(\mu > \frac{\tau_B}{\tau_A + \tau_B}\right) = 1 - F_\mu \left(\frac{\tau_B}{\tau_A + \tau_B}\right)$$

while candidate B wins with the complementary probability.

The perspective of the standard ethical voting model is that members agree on a rule before they learn their voting cost, since this may depend on the circumstances of the election day, e.g. the weather or any family or work obligations. To better compare the implications of the different rules, I will keep the same perspective throughout the paper. The expected utility of a member i in group A, with respect to both the groups' size μ and the cost of voting c_i, for a given intensity of preference z_i, is

$$u_i(z_i) = \left[1 - F_\mu \left(\frac{\tau_B}{\tau_A + \tau_B}\right)\right] w z_i - \chi\, E[c_i\, t_A(z_i, c_i)] \tag{1}$$

The first term gives the benefit from a victory discounted by the probability of winning, the second term gives the expected cost of voting given the turnout rule. An analogous expression for the expected utility holds for members of group B, for a turnout rule $t_B(z_i, c_i)$ and a probability of winning for candidate B equal to $F_\mu\left(\frac{\tau_B}{\tau_A+\tau_B}\right)$. An underlying assumption of the framework is that inter-personal comparisons of utilities across members are meaningful.

The ethical calculus in each group determines the two rules $t_A(z_i, c_i)$ and $t_B(z_i, c_i)$ and thus the turnout rates τ_A, τ_B. I assume that all citizens are ethical, i.e. they all follow the turnout rule that is set collectively in their group. This assumption and the fact that uncertainty concerns the size of the two groups makes the model closer to the one by Coate and Conlin (2004) than to the one by Feddersen and Sandroni (2006a), in which the uncertainty is on the share of ethical voters within groups of known size. As such, the rule-utilitarian calculus in the following section is formally a *group* rule-utilitarian calculus, identical to the one in Coate and Conlin: the optimal rule maximizes the aggregate utility of the group. Yet, all insights of the analysis also apply to the setting by Feddersen and Sandroni, in which the welfare measure maximized by the turnout rule includes all social costs of voting.

2.1 Utilitarian rule

I first analyze the behavior of utilitarian members, who follow the rule that maximizes the aggregate utility in their group. The aggregation of the individual utilities within group A is over the three variables μ, z_i, and c_i. Consider first the benefit term in equation (1), i.e. $\left[1 - F_\mu\left(\frac{\tau_B}{\tau_A+\tau_B}\right)\right] w z_i$. This is aggregated over z_i and over the values of μ corresponding to a victory of candidate A, i.e. $\mu > \frac{\tau_B}{\tau_A+\tau_B}$. Hence, the expected aggregate benefit is

$$\int_{\mu>\frac{\tau_B}{\tau_A+\tau_B}} \mu \left(w \int_{\mathcal{Z}} z_i \, dF_{z_i}\right) dF_\mu = w \; E[z_i] \int_{\mu>\frac{\tau_B}{\tau_A+\tau_B}} \mu \, dF_\mu \tag{2}$$

where the term $E[z_i]$ is the average intensity of preference in the group. Instead, the cost term $\chi c_i \, t_A(z_i, c_i)$ is aggregated over c_i, over z_i, and over all values of μ in its support $\mathcal{M}$. Hence, the expected aggregate cost is

$$\int_{\mathcal{M}} \mu \left(\int_{\mathcal{Z}} \int_{\mathcal{C}} \chi \, c_i \, t_A(z_i, c_i) \, dF_{z_i} \, dF_{c_i}\right) dF_\mu = \chi \; E[\mu] \int_{\mathcal{Z}} \int_{\mathcal{C}} c_i \, t_A(z_i, c_i) \, dF_{z_i} \, dF_{c_i} \tag{3}$$

where $E[\mu]$ is the expected size of the group. Putting (2) and (3) together, the expected aggregate utility of group A, denoted u_A, is given by

$$u_A = w \; E[z_i] \int_{\mu>\frac{\tau_B}{\tau_A+\tau_B}} \mu \, dF_\mu - \chi \; E[\mu] \int_{\mathcal{Z}} \int_{\mathcal{C}} c_i \, t_A(z_i, c_i) \, dF_{z_i} \, dF_{c_i} \tag{4}$$

The problem of utilitarian supporters in group A is to set the turnout rule $t_A(z_i, c_i)$ which maximizes the expected aggregate utility in (4). The following result holds.

Proposition 1. *If there exists a rule $t_A(z_i, c_i)$ such that the expected aggregate utility in group A is maximized, it is equal to*

$$t_A(z_i, c_i) = \begin{cases} 1 & \text{if} \quad c_i \leq \bar{c}_A \\ 0 & \text{if} \quad c_i > \bar{c}_A \end{cases}$$

for some threshold cost $\bar{c}_A$ which does not depend on z_i.

Proof. See Appendix A. □

The intuition for the presence of a threshold cost is that any level of turnout τ_A is less costly for the group if the required votes are provided by members with low costs rather than by those with high costs. For a utilitarian aggregation, it is actually optimal to fix the same threshold cost for all z_i. That is because the total benefit depends only on the turnout rate τ_A, while the total cost depends on the costs of those who are required to vote. Hence, starting from any non-constant threshold $\bar{c}_A(z_i)$, equalizing the threshold across different supporters z_i, while keeping the same aggregate turnout rate τ_A, does not change the total benefit but lowers the total cost, since members voting with high costs are substituted by members with low costs.

In light of the previous proposition, the turnout rate in group A is equal to

$$\tau_A = \int_{\mathcal{Z}} \int_{c_i < \bar{c}_A} 1 \, dF_{z_i} \, dF_{c_i} = F_{c_i}(\bar{c}_A) \tag{5}$$

and the expected aggregate utility u_A in (4) can be expressed as a function of the threshold cost $\bar{c}_A$, as follows

$$u_A = w \, E[z_i] \int_{\mu > \frac{\tau_B}{F_{c_i}(\bar{c}_A) + \tau_B}} \mu \, dF_\mu - \chi \, E[\mu] \int_{c_i < \bar{c}_A} c_i \, dF_{c_i} \tag{6}$$

Maximizing u_A with respect to $\bar{c}_A$ determines the optimal threshold cost as a function of the turnout rate τ_B in the opposite group. Given the symmetry between groups, an analogous maximization problem concerns group B, whose solution yields the threshold cost $\bar{c}_B$ as a function of τ_A. An equilibrium of the model is then given by a pair of threshold costs such that groups' behavior is consistent with each other. The problem of finding the equilibria is technical in its general formulation but, as I show in section 3, an appropriate choice of distributions F_μ, F_{c_i}, and F_{z_i} yields simple closed-form solutions.

The general result of this section is that, from the point of view of an observer who does not know the individual voting costs, the same turnout rate should be observed on average among supporters with different intensities of preference. Yet, as I have stressed in the introduction, this result is at odds with the empirical evidence that the intensity of preferences correlates positively with participation.

2.2 Rules based on fairness

I propose two different rules, related to standard concepts of fairness. The first is based on a principle of proportionality. Such a rule dictates that, for all members of a group, the effort in providing a vote should be proportional to the gain obtained from the victory of the group. The second rule is based on a principle of equalizing the utility of the voting members of the group.

2.2.1 Proportional rule

Consider a rule dictating that the expected cost of voting should be proportional to the intensity of preference z_i, for every member i. In group A, such a rule is characterized by the following expression

$$E[c_i\, t_A(c_i, z_i)] = \gamma_A\, z_i \tag{7}$$

where the left term is the expected cost of voting for each member i from the rule $t_A(c_i, z_i)$, and γ_A is the coefficient of proportionality. The proportional rule is well-suited to the model because, by assumption, voters' expected utility in (1) is linear in the benefit from the election and in the expected cost, and the benefit is linear in z_i. Hence, under such a rule, the expected utility of member i (with respect to both μ and c_i) is also proportional to z_i, and specifically equal to

$$u_i = \left[\left(1 - F_\mu\left(\frac{\tau_B}{\tau_A + \tau_B}\right)\right) w - \chi\gamma_A\right] z_i$$

It follows that, if members were to collectively decide about a turnout rule and a coefficient of proportionality γ_A, they could agree on choosing those which maximize the term $\left[\left(1 - F_\mu\left(\frac{\tau_B}{\tau_A+\tau_B}\right)\right) w - \chi\gamma_A\right]$. The optimal proportional rule thus solves

$$\max_{\{t_A(z_i,c_i)\,,\,\gamma_A\}} \left[1 - F_\mu\left(\frac{\tau_B}{\tau_A + \tau_B}\right)\right] w - \chi\gamma_A \quad \text{subject to (7)} \tag{8}$$

The following result holds.

Proposition 2. *If there exists an optimal turnout rule for the problem in (8), it is equal to*

$$t_A(z_i, c_i) = \begin{cases} 1 & \text{if} \quad c_i \leq \bar{c}_A(z_i) \\ 0 & \text{if} \quad c_i > \bar{c}_A(z_i) \end{cases}$$

for some threshold cost function $\bar{c}_A(z_i)$ which is increasing in z_i.

Proof. See Appendix A. □

Hence, the turnout rate τ_A solves

$$\tau_A = \int_{\mathcal{Z}} \int_{c_i < \bar{c}_A(z_i)} 1 \, dF_{c_i} \, dF_{z_i} = \int_{\mathcal{Z}} F_{c_i}(\bar{c}_A(z_i)) dF_{z_i} \tag{9}$$

The optimality of threshold costs comes again from their efficiency: for any level of turnout, the expected cost of voting for any z_i is minimized by ensuring that votes are preferentially provided by supporters with low costs. Now, however, the constraint in (7) requires that the threshold cost be a function of z_i. To see why such a function must be increasing, we can rewrite the expression in (7) as

$$\int_{c_i < \bar{c}_A(z_i)} c_i \, dF_{c_i} = \gamma_A \, z_i \tag{10}$$

The left-hand side is increasing in $\bar{c}_A(z_i)$. Because the right-hand side is increasing in z_i, also the threshold cost in the left-hand side must be. One can then solve the equation to obtain the threshold $\bar{c}_A(z_i)$ as a function of γ_A. Hence, solving the problem in (8) corresponds to choosing the optimal γ_A, given (9) and (10). The optimal γ_A determines the turnout behavior in group A, as a function of the turnout rate τ_B in the opposite group. Given the symmetry between groups, an analogous maximization problem concerns group B, and again, in equilibrium, the choices of the two groups must be consistent with each other.

2.2.2 Egalitarian rule

An alternative way to incorporate fairness in the voting rule is based on applying an egalitarian criterion. The justification for the egalitarian criterion may come from what would happen if all members of a group abstained because of pivotality considerations. In this case, for any positive turnout by the opposite group, the opposite candidate would win, and the utility of all members would indeed be equal to zero. One should note, however, that a strictly egalitarian rule, imposing the equality of expected utility for all members of the group, would be unable to solve the collective action problem. Indeed, it would dictate abstention for all members as the only feasible rule, given that the expected utility of supporters with z_i close to 0 is bounded from above at a level close to 0. This implies that any rule that can solve the collective action problem, even if inspired by a fairness principle, should allow for some difference in expected utility between voters with stronger and weaker preferences.

One way to relax the strict egalitarian requirement is to impose it only for the voting members of the group, while abstaining members - more precisely, those who abstain for any cost realization - are allowed to receive a lower expected utility. According to such a rule, the individual expected utilities in group A should satisfy

$$\begin{aligned} \left[1 - F_\mu\left(\frac{\tau_B}{\tau_A + \tau_B}\right)\right] wz_i - \chi\, E[c_i\, t_A(z_i, c_i)] &= k \qquad \forall i : E[t_A(z_i, c_i)] \in (0, 1) \\ \left[1 - F_\mu\left(\frac{\tau_B}{\tau_A + \tau_B}\right)\right] wz_i &\leq k \qquad \forall i : E[t_A(z_i, c_i)] = 0 \end{aligned} \tag{11}$$

The first line shows the constraint of equal expected utility (to the endogenous value k) for the members of the group who vote for some realizations of their cost, while the second line shows that members who always abstain may obtain a lower utility than k. The expectation is taken again with respect to both μ (the size of the group) and c_i (the voting cost), for a given value of z_i (the intensity of preference).

I call such a rule *conditionally egalitarian*, as the egalitarian principle is conditioned on the fact of voting for at least some cost realization. The rule naturally results in a monotonicity property, according to which members' expected utility is weakly increasing in their intensity of preference z_i, and strictly increasing in z_i within the set of abstaining members. The optimal rule maximizes the endogenous value of k and, in doing so, determines both the expected utility of the voting members and the two subgroups of the voting and abstaining members. It solves

$$\max_{t_A(z_i, c_i)} \left[1 - F_\mu\left(\frac{\tau_B}{\tau_A + \tau_B}\right)\right] wz_i - \chi\, E[c_i\, t_A(z_i, c_i)] \quad \text{subject to (11)} \tag{12}$$

The following result holds.

Proposition 3. *If there exists an optimal turnout rule for the problem in (12), it is equal to*

$$t_A(z_i, c_i) = \begin{cases} 1 & \text{if} \quad c_i \leq \bar{c}_A(z_i) \\ 0 & \text{if} \quad c_i > \bar{c}_A(z_i) \end{cases}$$

for some threshold cost function $\bar{c}_A(z_i)$ which is increasing in z_i.

Proof. See Appendix A. □

As before, since the optimal rule seeks to minimize the expected voting cost for a given turnout rate τ_A, it takes the form of a threshold cost: for any level of turnout, within each tranche of supporters z_i, voting duties are preferentially assigned to those with low costs. The fact that threshold costs be a function of z_i follows from the first line of the constraint in (11), which requires that the expected voting cost be a function of z_i. The property of the threshold cost being increasing in z_i follows again from rewriting the expected cost term $E[c_i\, t_A(z_i, c_i)]$ as $\int_{c_i<\bar{c}_A(z_i)} c_i\, dF_{c_i}$, which is increasing in $\bar{c}_A(z_i)$. Since the individual benefit is also increasing in z_i, the equality of expected utilities requires the threshold cost to be also increasing in z_i.

As with the proportional rule, the turnout rate τ_A solves

$$\tau_A = \int_{\mathcal{Z}} \int_{c_i<\bar{c}_A(z_i)} 1\, dF_{c_i}\, dF_{z_i} = \int_{\mathcal{Z}} F_{c_i}(\bar{c}_A(z_i)) dF_{z_i}$$

Yet, contrary to the analysis in sections 2.1 and 2.2.1, the problem in (12) does not simplify to a maximization problem with respect to a single variable, but remains a problem of calculus of variation: the objective is a functional which is maximized with respect to a function $\bar{c}_A(z_i)$. An analogous problem concerns group B. In the next section, under a specific choice of the model's distributions, I show that the problem can be simplified and solved in closed-form.

The general result of this and the previous sections is that both a proportional rule and a (conditionally) egalitarian rule require members with higher intensity of preferences to vote for a greater range of voting costs. Consider again an observer who does not know the voting costs but observes only the average turnout rate of each tranche of voters with intensity z_i. If members follow either of these two rules, the observer indeed observes that voters' turnout correlates positively with the intensity of the preference.

3 Solving for the equilibrium

Propositions 1, 2 and 3 characterize how the voting costs are shared in the group as a function of the intensity of members' preferences, for the utilitarian, the proportional, and the (conditionally) egalitarian rule, respectively. In its general formulation, however, the model is not easily solvable for an equilibrium, i.e. for turnout rules in the two groups that are consistent with each other. In this section, I show that specifying uniform distributions for the variables in the model yields a simple existence result for the equilibrium, in closed-form solutions.

Assumption 1. *The distribution function for the group size μ, the distribution function for the intensity of preferences z_i, and the distribution function for the voting cost c_i are all uniform on* $[0, 1]$.

The assumption of a uniformly distributed μ involves a simpler expression than in Coate and Conlin (2004), where the group sizes follow a Beta distribution, but it keeps the model tractable in my richer framework. For the individual variables z_i and c_i, given the two parameters w and χ in equation (1), restricting the support of the distributions to $[0, 1]$ is without loss of generality.[5] The fact that the distribution of the voting cost is uniform does have some implications for the application of the two fairness-based rules. In particular, with a finite support of c_i, depending on the parameters, the optimal turnout rule may require a positive mass of members to vote for any realization of their cost. In this case, within the set of those members, the rule cannot yield an expected voting cost that is proportional to z_i (for the proportional rule) or an expected utility that is the same for all voting members (for the egalitarian rule). Hence, in what follows, I will adjust the two rules such that the proportional rule will indeed be proportional and the egalitarian rule will indeed be egalitarian only for members who vote for some and only some realizations of their cost, i.e. ex-ante with a probability strictly between 0 and 1.

3.1 Equilibrium with utilitarian rules

Consider the utilitarian version of the model, under Assumption 1. We have $\tau_A = \bar{c}_A$ from (5) and thus the group utility in (6) becomes

$$u_A = \frac{w}{2} \int_{\frac{\tau_B}{\bar{c}_A + \tau_B}}^{1} \mu \, d\mu - \frac{\chi}{2} \int_0^{\bar{c}_A} c_i \, dc_i = \frac{w}{4} \left[1 - \left(\frac{\tau_B}{\bar{c}_A + \tau_B} \right)^2 \right] - \frac{\chi}{4} (\bar{c}_A)^2 \tag{13}$$

The first-order condition of the maximization problem with respect to $\bar{c}_A$ is

$$\frac{w}{2} \frac{(\tau_B)^2}{(\bar{c}_A + \tau_B)^3} - \frac{\chi}{2} \bar{c}_A = 0 \tag{14}$$

while the second-order condition holds because u_A is concave in $\bar{c}_A$. Given the symmetry between groups, an analogous condition holds in group B, whose solution yields the threshold cost $\bar{c}_B$ as a function of τ_A.

For an equilibrium, consistency requires that the turnout rate τ_B be equal to the one resulting from the solution of the equivalent problem in group B. This means that, in equilibrium, the first-order conditions of the groups' maximization problems hold jointly. Since we also have $\tau_B = \bar{c}_B$, the solution can be obtained by solving the corresponding system of two equations in the two variables $\bar{c}_A$ and $\bar{c}_B$. We obtain a unique solution $\bar{c}_A = \bar{c}_B = \sqrt{\frac{w}{8\chi}}$ and

[5] The models by Coate and Conlin and Feddersen and Sandroni also assume a uniform distribution of the voting cost on $[0, 1]$.

therefore the following result.

Proposition 4. *If the members of both groups are utilitarians, there exists a unique equilibrium, in which the turnout rules are*

$$t_A(z_i, c_i) = t_B(z_i, c_i) = \begin{cases} 1 & \textit{if} \quad c_i \leq \sqrt{\dfrac{w}{8\chi}} \\ 0 & \textit{otherwise} \end{cases}$$

The corresponding turnout rates of the groups are equal to

$$\tau_A = \tau_B = \min\{\sqrt{\frac{w}{8\chi}}\,, 1\}$$

Proof. See Appendix A. □

The existence of an equilibrium is guaranteed by the fact that, under Assumption 1, the group utility u_A in (13) is continuous and quasi-concave in the choice variable $\bar{c}_A$. Assuming more general distributions that preserve those properties for the group utility would also ensure that an equilibrium exists, although in practice it is difficult to find a reasonable combination of alternative distributions that preserves quasi-concavity. As discussed by Coate and Conlin (2004), for example, an equilibrium may not exist under a general Beta distribution for the group size variable μ.

3.2 Equilibrium with proportional rules

Consider the proportional version of the model, under Assumption 1. Since the expected voting cost is equal to $\int_0^{\bar{c}_A(z_i)} c_i \, dc_i = \frac{(\bar{c}_A(z_i))^2}{2}$, equation (10) yields

$$\bar{c}_A(z_i) = \sqrt{2\gamma_A z_i} \tag{15}$$

and thus the turnout rate in (9) is equal to

$$\tau_A = \int_0^1 \min\{\sqrt{2\gamma_A z_i}\,, 1\} \, dz_i \tag{16}$$

in which I have taken into account that, given $c_i \sim U[0, 1]$, depending on γ_A, the threshold cost in (15) may imply that some voters vote for any cost realization. Solving the integral in equation (16) yields a solution for τ_A as a function of γ_A that is

$$\tau_A^*(\gamma_A) = \begin{cases} \frac{2}{3}\sqrt{2\gamma_A} & \text{if} \quad \gamma_A \leq \frac{1}{2} \\ 1 - \frac{1}{6\gamma_A} & \text{if} \quad \gamma_A > \frac{1}{2} \end{cases} \tag{17}$$

Moreover, we have $1 - F_\mu\left(\frac{\tau_B}{\tau_A+\tau_B}\right) = \frac{\tau_A}{\tau_A+\tau_B}$ and thus the problem in (8) corresponds to

$$\max_{\gamma_A} \quad \frac{\tau_A^*(\gamma_A)}{\tau_A^*(\gamma_A)+\tau_B} w - \chi\gamma_A \tag{18}$$

in which $\tau_A^*(\gamma_A)$ is given by (17). The problem in (18) is concave in γ_A and thus the first order condition identifies the optimal solution as a function of τ_B. As before, group B solves an analogous problem. We can then solve for a symmetric equilibrium by replacing $\tau_B = \tau_A$ in the first order condtion.[6] We obtain

$$\gamma_A = \gamma_B = \begin{cases} \frac{w}{8\chi} & \text{if} \quad w \leq 4\chi \\ \frac{\chi + \sqrt{\chi^2 + 6\chi w}}{12\chi} & \text{if} \quad w > 4\chi \end{cases}$$

and therefore the following result.

Proposition 5. *If the members of both groups follow the proportional rule, there exists a unique symmetric equilibrium, in which the turnout rules are*

$$t_A(z_i, c_i) = t_B(z_i, c_i) = \begin{cases} 1 & \textit{if} \quad c_i \leq \sqrt{\frac{w}{4\chi}} z_i & \textit{and if} \quad w \leq 4\chi \\ 1 & \textit{if} \quad c_i \leq \sqrt{\frac{\chi + \sqrt{\chi^2+6\chi w}}{6\chi}} z_i & \textit{and if} \quad w > 4\chi \\ 0 & \textit{otherwise} \end{cases}$$

The corresponding turnout rates of the groups are equal to

$$\tau_A = \tau_B = \begin{cases} \sqrt{\frac{w}{9\chi}} & \textit{if} \quad w \leq 4\chi \\ 1 - \frac{2\chi}{\chi + \sqrt{\chi^2 + 6\chi w}} & \textit{if} \quad w > 4\chi \end{cases}$$

Proof. See Appendix A. □

[6]Because equation (17) and the analogous one for group B have a two-part structure, it is now difficult to rule out asymmetric equilibria without checking for the absence of global deviations. However, even if they exist, asymmetric equilibria do not seem of particular interest, given the strong symmetry between the two groups in the model.

Generalizing the previous result for different distributions than those in Assumption 1 is again difficult. In particular, now the domain of the choice variable γ_A is $\mathbb{R}_+$, hence not compact, which calls for caution with respect to stating existence of an equilibrium even if the continuity and quasi-concavity of the objective function are preserved under the more general assumptions.

3.3 Equilibrium with egalitarian rules

Consider the (conditionally) egalitarian version of the model, under Assumption 1. The problem of finding the optimal function $\bar{c}_A(z_i)$ can be transformed into a standard maximization problem with respect to a variable as follows. First, as in the previous section, the expected cost term is equal to $E[c_i\, t_A(z_i, c_i)] = \frac{(\bar{c}_A(z_i))^2}{2}$. Hence, the equality condition in the first line of (11) becomes

$$\frac{\tau_A}{\tau_A + \tau_B} w z_i - \chi\, \frac{(\bar{c}_A(z_i))^2}{2} = k \qquad \forall i : E[t_A(z_i, c_i)] \in (0, 1)$$

and thus it implies that

$$\bar{c}_A(z_i) = \sqrt{\frac{2}{\chi}\left(\frac{\tau_A}{\tau_A + \tau_B} w z_i - k\right)} \qquad \forall i : E[t_A(z_i, c_i)] \in (0, 1) \tag{19}$$

where both k and the coefficient $\frac{\tau_A}{\tau_A+\tau_B}$ are to be determined. Since k should be as high as possible and the threshold cost $\bar{c}_A(z_i)$ must be weakly positive for all z_i, it is indeed optimal to have an interval of members with low values of z_i who always abstain. Let us denote $\underline{z}$ the highest value of z_i for which a member always abstains, i.e. such that $\bar{c}_A(z_i) = 0 \;\; \forall z_i \leq \underline{z}$ and $\bar{c}_A(z_i) > 0 \;\; \forall z_i > \underline{z}$. By a continuity argument, this value must solve

$$\frac{\tau_A}{\tau_A + \tau_B} w \underline{z} = k \tag{20}$$

Substituting k from (20) into (19), we have

$$\bar{c}_A(z_i) = \sqrt{\frac{2}{\chi} \frac{\tau_A}{\tau_A + \tau_B} w (z_i - \underline{z})} \quad \text{if} \quad z_i > \underline{z} \tag{21}$$

The corresponding aggregate turnout rate τ_A then solves

$$\tau_A = \int_{\underline{z}}^{1} \int_{0}^{\bar{c}_A(z_i)} 1 \; dc_i \; dz_i = \int_{\underline{z}}^{1} \min\{\sqrt{\frac{2}{\chi} \frac{\tau_A}{\tau_A + \tau_B} w (z_i - \underline{z})}\, , 1\}\; dz_i \tag{22}$$

in which I have taken into account that, given $c_i \sim U[0,1]$, the threshold cost in (21) is bounded at 1. In the appendix, I show that equation (22) yields the following solution for $\underline{z}$ as a function of τ_A:

$$\underline{z}^*(\tau_A) = \begin{cases} 1 - \left(\dfrac{9\chi\tau_A(\tau_A+\tau_B)}{8w}\right)^{\frac{1}{3}} & \text{if} \quad \dfrac{{\tau_A}^2}{\tau_A+\tau_B} \leq \dfrac{\chi}{3w} \\ 1 - \tau_A - \dfrac{\chi}{6w}\dfrac{\tau_A+\tau_B}{\tau_A} & \text{if} \quad \dfrac{{\tau_A}^2}{\tau_A+\tau_B} > \dfrac{\chi}{3w} \end{cases} \tag{23}$$

Now, the maximization problem in (12) is equivalent to choosing τ_A in order to maximize the endogenous value of k given (20) and (22). This corresponds to solving

$$\max_{\tau_A} \frac{\tau_A}{\tau_A+\tau_B} w \underline{z}^*(\tau_A) \tag{24}$$

in which $\underline{z}^*(\tau_A)$ is given by (23). The problem in (24) is concave in τ_A, and thus the first order condition identifies the optimal τ_A as function of the turnout rate τ_B in the opposite group. Again, group B solves an analogous problem, and we can solve for a symmetric equilibrium by replacing $\tau_A = \tau_B$ in the first order condition. We obtain the following result.

Proposition 6. *If the members of both groups are (conditionally) egalitarians, there exists a unique symmetric equilibrium, in which the turnout rules are*

$$t_A(z_i,c_i) = t_B(z_i,c_i) = \begin{cases} 1 \quad \textit{if} \quad c_i \leq \sqrt{\dfrac{w}{\chi}\left(z_i - \dfrac{1}{2}\right)} & \textit{and if} \quad w \leq 2\chi \\ 1 \quad \textit{if} \quad c_i \leq \sqrt{\dfrac{1}{3} + \dfrac{w}{\chi}\left(z_i - \dfrac{2}{3}\right)} & \textit{and if} \quad w > 2\chi \\ 0 \quad \textit{otherwise} \end{cases}$$

The corresponding turnout rates of the groups are equal to

$$\tau_A = \tau_B = \begin{cases} \sqrt{\dfrac{w}{18\chi}} & \textit{if} \quad w \leq 2\chi \\ \dfrac{1}{3} & \textit{if} \quad w > 2\chi \end{cases}$$

Proof. See Appendix A. $\square$

Similar considerations as with the previous rules can be made with respect to the possibility of extending the existence result to more general distributions. As with the utilitarian rule, the domain of the choice variable τ_A in the group's optimization problem is compact, hence any more general distribution that preserves the continuity and quasi-concavity of the objective function would guarantee the existence of an equilibrium. The caveat is again that quasi-concavity of the objective function (as well as analytical tractability) is easily lost under more general distributions.

3.4 Comparison

Figure 1 shows graphically how the three rules assign voting duties among group members. Recall that, since $c_i \sim U[0,1]$, the threshold cost $\bar{c}_A(z_i)$ on the vertical axis corresponds to the ex-ante probability of voting.

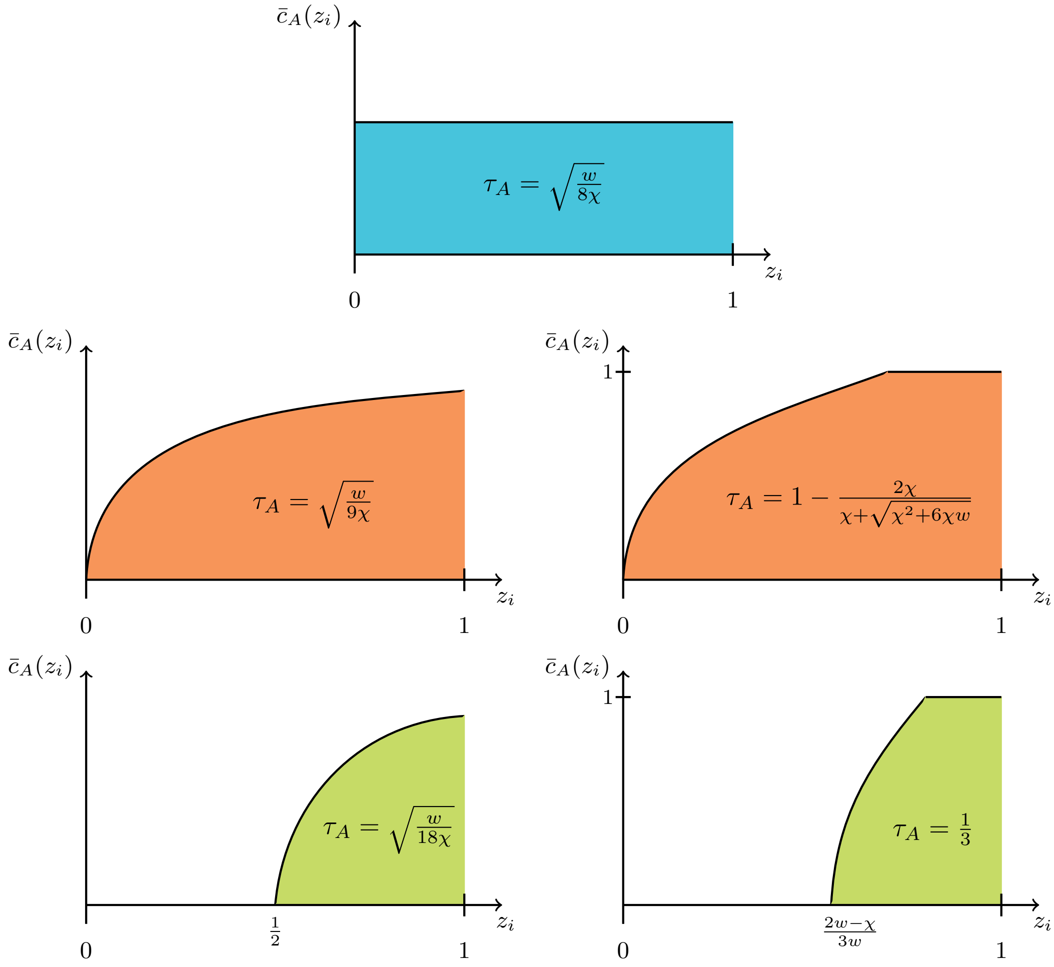

Figure 1: Threshold costs and turnout rates.
Top: Utilitarian rule.
Middle: Proportional rule, for $w \leq 4\chi$ (left) and $w > 4\chi$ (right).
Bottom: Egalitarian rule, for $w \leq 2\chi$ (left) and $w > 2\chi$ (right).

We can observe that both the proportional and the (conditionally) egalitarian rule require members with higher intensity of preference z_i to vote for a greater range of voting costs. The noticeable difference between the two rules is the presence of always abstainers with the (conditionally) egalitarian rule. In all graphs, the aggregate turnout rate τ_A corresponds to the integral of the curve and it is increasing in the ratio between the importance of the election w and the cost parameter χ. The expected total turnout rate is also equal to τ_A, since both groups are of size 0.5 in expectation.

Two additional comments are in order. First, in the parameter region in which the threshold cost is lower than 1 for all z_i and all rules, we observe that aggregate turnout is the highest if agents are utilitarians ($\tau_A = \sqrt{\frac{w}{8\chi}}$), it is lower if agents follow the proportional rule ($\tau_A = \sqrt{\frac{w}{9\chi}}$), and the lowest if they follow the (conditionally) egalitarian rule ($\tau_A = \sqrt{\frac{w}{18\chi}}$). Yet, I believe that it is difficult to translate this result into a testable prediction able to discriminate between the three models, since it seems more a mathematical artifact of the chosen distributions than a property based on well-grounded intuition. I would rather argue that the implications for aggregate turnout under the three rules are similar, in terms of being increasing in $\frac{w}{\chi}$.

Second, the expected total turnout in the (conditionally) egalitarian model is bound to be in any case weakly lower than $\frac{1}{3}$. This is because, as w increases above 2χ, the optimal rule asks more voters with low preferences to abstain in order for those who vote with probability between 0 and 1 to have a higher (equal) utility. This may be seen as an inconvenient property of the (conditionally) egalitarian rule, yet it depends on the assumption of a uniform distribution of voting costs more than on a structural limitation of the rule itself. Intuitively, one would be able to elicit a substantially higher turnout under the same rule if the assumption of a uniform distribution of c_i was replaced by a distribution with only a small fraction of agents having large costs. Yet, the model under such an assumption would be much less tractable.

4 Discussion

The paper has argued that an ethical calculus of voting which accounts for fairness in the distribution of duties among group members is consistent with the empirical correlation between participation and the intensity of preferences. Developing an appropriate setup to test the explanatory power of the different fairness principles, with respect to the choice of voting in elections, is a direction for future empirical research. I conclude by briefly discussing three points which help contextualize the previous results, in comparison to the standard rule-utilitarian framework.

First, a relevant assumption of the model is that the realization of the voting cost c_i and that of the intensity of preference z_i are independent. Assuming a negative correlation between these two variables could also drive, mechanically, a result of positive correlation between participation and the intensity of preferences. But the assumption of independence between the two variables is sound conceptually: assuming a lower cost of voting for voters with a strong preference would be a reduced-form way to embed the empirical result into the model rather than an attempt to explain it.

Second, the result of a constant threshold cost for all members that emerged in the utilitarian model may not be robust to changing the level of aggregation from the group to a subgroup. In this case, if voters are utilitarians only within subgroups, the strategic interaction between these subgroups could result in different threshold costs across different subgroups. But within each subgroup, the same result of irrelevance of the intensity of support would hold, which points to a general limitation of the utilitarian criterion in dealing with heterogeneous agents. Moreover, in my framework, subgroups of voters with the same intensity of preferences have mass zero, which implies that any turnout rule decided at the subgroup level would stumble back into the issue of pivotality.

Third, a more subtle assumption of the model is that there is no uncertainty concerning the term $\int_{\mathcal{Z}} z_i \, dF_{z_i}$, i.e. the aggregate (or average) intensity of preferences in a group. In an alternative (and more difficult) framework, voters may not know the intensity of preference of their fellow group members and may try to infer it from their own intensity. In this case, the perceived aggregate benefit would depend on the individual benefit, and it seems reasonable that even utilitarian members could follow a turnout rule as an increasing function of their intensity of preference.

Overall, however, a departure from a purely utilitarian framework has strong normative grounds in the presence of a relevant heterogeneity. In many models of voting, the heterogeneity in voters' preferences, typically interpreted in terms of distance from candidates on some ideological space, is indeed very relevant. It is thus important to assess and eventually revisit the theory of ethical voting in such richer settings.[7] This paper is an attempt in this direction.

[7] As a comparison, the theory of optimal income taxation is an example of a field whose original formulation was in utilitarian terms but has later been extended to account for agents' heterogeneity (Fleurbaey and Maniquet 2018).

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A Appendix

Proof of Proposition 1:

Assume that group A utility is maximized by a rule $t_A(z_i, c_i)$ such that, for a set of agents with positive measure, it holds $t_A(z_i, c_i) \neq \mathbb{1}(c_i \leq \bar{c}_A)$. This implies that there exist two subsets of agents (with positive measure), such that agents' costs of voting in the first are higher than in the second, agents in the first subset vote, and agents in the second abstain. Consider any rule that makes some agents in the first subset abstain and an equal positive mass of agents in the second subset vote, i.e. such that the mass of votes cast by the group remains the same. Then aggregate voting costs are reduced, while the aggregate benefit is the same, and therefore the group utility must increase, which contradicts the fact that the original rule is optimal.

Proof of Proposition 2:

Assume that the objective function in problem (8) is maximized by a rule $t_A(z_i, c_i)$ such that, for a set of agents with positive measure, it holds $t_A(z_i, c_i) \neq \mathbb{1}(c_i \leq \bar{c}_A(z_i))$. This implies that for each z_i within some set Z with positive measure, there exist at least two subsets of agents, such that agents' costs of voting in the first are higher than in the second, agents in the first subset vote, and agents in the second abstain. Consider a rule r that, for each of those z_i, makes some agents in the first subset abstain and some agents in the second subset vote, in a way such that the mass of votes cast by the group remains the same. Such a rule r reduces the expected cost term $E[c_i\, t_A(z_i, c_i)]$ for the concerned z_i. We could then construct another proportional rule that is preferred by all agents in the group as follows. If all agents in the group are concerned, i.e. if Z has mass 1, then by a continuity argument, the rule r can be taken in a way that reduces the expected cost for everyone while remaining proportional to z_i (and such that the mass of votes cast by the group is the same). This rule would be preferred by all agents, which contradicts the fact that the original rule is optimal. If the mass of Z is less than 1, and thus there exist a set Z' of agents z_i with positive mass for which the original rule takes the form of a threshold cost, then by a continuity argument, it is possible to construct a rule r' that (i) increases the expected cost for the agents z_i in Z with respect to the rule r and increases the mass of votes provided by those agents, (ii) reduces the expected cost for the agents z_i in Z' with respect to the original rule and reduces the mass of votes provided by those agents, in a way such that the expected voting cost of all agents under the rule r' is proportional to z_i and lower than under the original rule and such that the mass of votes cast by the group is the same as under the original rule. This rule is again preferred by all agents, which contradicts the fact that the original rule is optimal. Hence, the optimal rule takes the form of a threshold cost function $\bar{c}_A(z_i)$. Then, the expected cost $E[c_i\, \mathbb{1}(c_i \leq \bar{c}_A(z_i))]$ is increasing in $\bar{c}_A(z_i)$. Hence, for the constraint in (7) to hold, $\bar{c}_A(z_i)$ must be increasing in z_i.

Proof of Proposition 3:

Assume that the objective function in problem (12) is maximized by a rule $t_A(z_i, c_i)$ such that for a set of agents with positive measure, it holds $t_A(z_i, c_i) \neq \mathbb{1}(c_i \leq \bar{c}_A(z_i))$. This implies that there exist two subsets of agents (with positive measure) with the same distribution of intensities z_i, such that agents' costs of voting are higher in the first subset than in the second, agents in the first subset vote, and agents in the second abstain. Consider any rule that makes some agents in the first subset abstain and an equal positive mass of agents with the same distribution of intensities z_i in the second subset vote, i.e. such that the mass of votes cast by the group remains the same. Such a rule decreases the expected cost of voting for the members z_i affected by the change, without changing the probability that the group wins, and therefore it yields a higher expected utility to those members than to the voting members unaffected by the change of rule. By a continuity argument, it is then possible to find another rule which results in the same mass of votes from the group but shifts voting duties from the members with lower expected utility to the members with higher expected utility (by decreasing the expected cost of voting for the first and increasing it for the second) in a way that equalizes the expected utility at higher level than with the original rule, which contradicts the fact that the original rule is optimal. Hence, the optimal rule takes the form of a threshold cost function $\bar{c}_A(z_i)$. Then, the expected cost $E[c_i \mathbb{1}(c_i \leq \bar{c}_A(z_i))]$ is increasing in $\bar{c}_A(z_i)$. Hence, for the first constraint in (11) to hold, $\bar{c}_A(z_i)$ must be increasing in z_i.

Proof of Proposition 4:

Given $\tau_A = \bar{c}_A$ and $\tau_B = \bar{c}_B$, an equilibrium must solve the following system of equations:

$$\begin{cases} \frac{w}{2} \frac{(\bar{c}_B)^2}{(\bar{c}_A + \bar{c}_B)^3} = \frac{\chi}{2} \bar{c}_A \\ \frac{w}{2} \frac{(\bar{c}_A)^2}{(\bar{c}_A + \bar{c}_B)^3} = \frac{\chi}{2} \bar{c}_B \end{cases}$$

By taking the ratio of the members on the same side of the equal sign, we obtain $(\bar{c}_A)^3 = (\bar{c}_B)^3$, hence $\bar{c}_A = \bar{c}_B$ and, by solving either equation after substituting, we obtain $\bar{c}_A = \bar{c}_B = \sqrt{\frac{w}{8\chi}}$.

Proof of Proposition 5:

First, I derive equation (17). The threshold on γ_A for $\sqrt{2\gamma_A z_i}$ to be lower than 1 for all $z_i \in [0, 1]$ is $\frac{1}{2}$. If $\gamma_A \leq \frac{1}{2}$, equation (16) becomes

$$\tau_A = \sqrt{2\gamma_A} \left[\frac{2}{3} (z_i)^{\frac{3}{2}} \right]_0^1 = \frac{2}{3} \sqrt{2\gamma_A}$$

If $\gamma_A > \frac{1}{2}$, equation (16) becomes

$$\tau_A = \int_0^{\frac{1}{2\gamma_A}} \sqrt{2\gamma_A z_i}\, dz_i + \int_{\frac{1}{2\gamma_A}}^1 1\, dz_i = \sqrt{2\gamma_A} \left[\frac{2}{3} (z_i)^{\frac{3}{2}} \right]_0^{\frac{1}{2\gamma_A}} + (1 - \frac{1}{2\gamma_A}) = \frac{1}{3\gamma_A} + 1 - \frac{1}{2\gamma_A} = 1 - \frac{1}{6\gamma_A}$$

An identical expression as (16) holds in group B for τ_B as a function of γ_B. The first order condition in problem (18) is

$$\frac{\frac{\partial \tau_A^*(\gamma_A)}{\partial \gamma_A} \tau_B}{(\tau_A^*(\gamma_A) + \tau_B)^2} w = \chi$$

After substituting $\tau_B = \tau_A^*(\gamma_A)$, if $\tau_A^*(\gamma_A) = \frac{2}{3}\sqrt{2\gamma_A}$, we obtain $\gamma_A = \frac{w}{8\chi}$. If $\tau_A^*(\gamma_A) = 1 - \frac{1}{6\gamma_A}$, we obtain the following equation of second degree

$$24\chi\gamma_A^2 - 4\chi\gamma_A - w = 0$$

whose positive solution is $\gamma_A = \frac{\chi+\sqrt{\chi^2+6\chi w}}{12\chi}$. The condition for γ_A higher or lower than $\frac{1}{2}$ becomes a condition for w higher or lower than 4χ. Substituting the solutions for γ_A in the threshold cost in equation (15) and in the expressions for the turnout rate τ_A yields the result in the Proposition.

Proof of Proposition 6:

First, I derive equation (23). If $\sqrt{\frac{2}{\chi}\frac{\tau_A}{\tau_A+\tau_B}w(z_i-\underline{z})} \leq 1$ for all z_i, equation (22) becomes

$$\tau_A = \sqrt{\frac{2}{\chi}\frac{\tau_A}{\tau_A+\tau_B}w}\int_{\underline{z}}^{1}\sqrt{z_i-\underline{z}}\,dz_i = \sqrt{\frac{2}{\chi}\frac{\tau_A}{\tau_A+\tau_B}w}\left[\frac{2}{3}(z_i-\underline{z})^{\frac{3}{2}}\right]_{\underline{z}}^{1} = \sqrt{\frac{2}{\chi}\frac{\tau_A}{\tau_A+\tau_B}w}\left[\frac{2}{3}(1-\underline{z})^{\frac{3}{2}}\right]$$

whose solution for $\underline{z}$ is

$$\underline{z}^*(\tau_A) = 1 - \left(\frac{9\chi\tau_A(\tau_A+\tau_B)}{8w}\right)^{\frac{1}{3}}$$

Given the previous value for $\underline{z}$, the condition $\sqrt{\frac{2}{\chi}\frac{\tau_A}{\tau_A+\tau_B}w(z_i-\underline{z})} \leq 1$ becomes $\frac{{\tau_A}^2}{\tau_A+\tau_B} \leq \frac{\chi}{3w}$. If this condition does not hold, and there exist z_i for which $\sqrt{\frac{2}{\chi}\frac{\tau_A}{\tau_A+\tau_B}w(z_i-\underline{z})} > 1$, then equation (22) becomes

$$\begin{aligned}
\tau_A &= \sqrt{\frac{2}{\chi}\frac{\tau_A}{\tau_A+\tau_B}w}\int_{\underline{z}}^{\underline{z}+\frac{\chi(\tau_A+\tau_B)}{2\tau_A w}}\sqrt{z_i-\underline{z}}\,dz_i + \int_{\underline{z}+\frac{\chi(\tau_A+\tau_B)}{2\tau_A w}}^{1} 1\,dz_i \\
&= \sqrt{\frac{2}{\chi}\frac{\tau_A}{\tau_A+\tau_B}w}\left[\frac{2}{3}(z_i-\underline{z})^{\frac{3}{2}}\right]_{\underline{z}}^{\underline{z}+\frac{\chi(\tau_A+\tau_B)}{2\tau_A w}} + 1 - \underline{z} - \frac{\chi(\tau_A+\tau_B)}{2\tau_A w} \\
&= \sqrt{\frac{2}{\chi}\frac{\tau_A}{\tau_A+\tau_B}w}\left[\frac{2}{3}(\frac{\chi(\tau_A+\tau_B)}{2\tau_A w})^{\frac{3}{2}}\right] + 1 - \underline{z} - \frac{\chi(\tau_A+\tau_B)}{2\tau_A w} \\
&= \frac{\chi(\tau_A+\tau_B)}{3\tau_A w} + 1 - \underline{z} - \frac{\chi(\tau_A+\tau_B)}{2\tau_A w}
\end{aligned}$$

whose solution for $\underline{z}$ is

$$\underline{z}^*(\tau_A) = 1 - \tau_A - \frac{\chi}{6w}\frac{\tau_A+\tau_B}{\tau_A}$$

The first order condition in problem (24) is

$$\frac{\tau_A}{(\tau_A+\tau_B)^2}w\underline{z}^*(\tau_A) + \frac{\tau_A}{\tau_A+\tau_B}w\frac{\partial \underline{z}^*(\tau_A)}{\partial \tau_A} = 0$$

If $\underline{z}^*(\tau_A) = 1 - \left(\frac{9\chi\tau_A(\tau_A+\tau_B)}{8w}\right)^{\frac{1}{3}}$, the equation corresponds to

$$\frac{\tau_A}{(\tau_A+\tau_B)^2} w \left(1 - \left(\frac{9\chi\tau_A(\tau_A+\tau_B)}{8w}\right)^{\frac{1}{3}}\right) + \frac{\tau_A}{\tau_A+\tau_B} w \left(-\frac{1}{3}\left(\frac{9\chi\tau_A(\tau_A+\tau_B)}{8w}\right)^{-\frac{2}{3}} \frac{9\chi(2\tau_A+\tau_B)}{8w}\right) = 0$$

and, after substituting $\tau_B = \tau_A$, to

$$\frac{1}{4\tau_A}\left(1 - \left(\frac{9\chi 2(\tau_A)^2}{8w}\right)^{\frac{1}{3}}\right) = \frac{1}{6}\left(\frac{9\chi 2(\tau_A)^2}{8w}\right)^{-\frac{2}{3}} \frac{9\chi 3\tau_A}{8w}$$

$$\Rightarrow \quad 1 - \left(\frac{9\chi 2(\tau_A)^2}{8w}\right)^{\frac{1}{3}} = \left(\frac{9\chi 2(\tau_A)^2}{8w}\right)^{\frac{1}{3}} \quad \Rightarrow \quad \frac{1}{2} = \left(\frac{9\chi 2(\tau_A)^2}{8w}\right)^{\frac{1}{3}}$$

$$\Rightarrow \quad \frac{1}{8} = \frac{9\chi 2(\tau_A)^2}{8w} \quad \Rightarrow \quad \tau_A = \sqrt{\frac{w}{18\chi}}$$

If $\underline{z}^*(\tau_A) = 1 - \tau_A - \frac{\chi}{6w}\frac{\tau_A+\tau_B}{\tau_A}$, the equation corresponds to

$$\frac{\tau_A}{(\tau_A+\tau_B)^2} w \left(1 - \tau_A - \frac{\chi}{6w}\frac{\tau_A+\tau_B}{\tau_A}\right) + \frac{\tau_A}{\tau_A+\tau_B} w \left(-1 + \frac{\chi}{6w}\frac{\tau_B}{(\tau_A)^2}\right) = 0$$

and, after substituting $\tau_B = \tau_A$, to

$$\frac{1}{4\tau_A}(1 - \tau_A - \frac{\chi}{3w}) + \frac{1}{2}(-1 + \frac{\chi}{6w}\frac{1}{\tau_A}) = 0$$

$$\Rightarrow \quad \frac{1}{4\tau_A} - \frac{3}{4} = 0 \quad \Rightarrow \quad \tau_A = \frac{1}{3}$$

In equilibrium, the condition $\frac{{\tau_A}^2}{\tau_A+\tau_B}$ higher or lower than $\frac{\chi}{3w}$ becomes the condition w higher or lower than 2χ, and the value of $\underline{z}^*$ is equal to $\underline{z} = \frac{1}{2}$ if $w \leq 2\chi$ and to $\underline{z} = \frac{2}{3} - \frac{\chi}{3w}$ if $w > 2\chi$. Substituting it into equation (21) yields the result in the Proposition.