LEMMA Working Paper

n°2024-10

Inflation expectations in time and frequency

A wavelet analysis

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Abstract

This paper presents a novel perspective and analytical approach to a long-standing debate: the relationship between inflation expectations and household consumption and savings behavior. With the recent return of inflation instability among developed economies, understanding how households behave in the aggregate when faced with rising prices is key to monetary policy. A fundamental component of such behavior, households' inflation expectations remain a closely watched, yet little understood trend, one for which economic research continues producing inconsistent, at times conflicting, results. Despite some recognitions of the cyclicity in such trends, there has been little formal research into the cyclical nature of this expectations-behavior relationship. As such, to seek these possible cyclical natures, I explore an approach known as wavelet analysis. Wavelet analysis allows me to examine series in both the time and frequency (i.e. cyclical) domains. I apply this technique to US inflation expectations, nondurables and durables personal consumption, and personal savings data over a long period of time, from 1978 to 2024. Through the new perspective provided by the frequency domain, I show how the often-inconsistent aggregate relationships between expectations and consumption and savings behavior in macroeconomic data may in fact be consistent, the very result of the series' multi-scale cyclical natures.

1. Introduction

Inflation can produce pronounced negative impacts on households' financial well-being. Understanding how households anticipate inflation and ultimately behave is paramount for monetary policy and consumer protection strategies and ultimately economic stability.

Since the 1990s and prior to the COVID-19 pandemic, households in developed economies faced nearly no inflation. During the last significant inflationary period in developed economies, most notably in the 1970s and 1980s, macroeconomic data revealed pronounced

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shifts in households' savings and consumption behaviors as a function not only of the inflation rate they faced, but of the rate they anticipated facing. Research at the time found that increases in the rates of households' expected inflation correlated with increases in their consumption of nondurable goods, or "stocking up." Decreases in anticipated inflation similarly correlated with increased savings rates (Juster & Wachtel, 1972; Katona, 1974). In other words, when households expected prices to rise in the future, they would naturally make more purchases in the present; when they did not anticipate price hikes, rather, they would maintain or grow their savings.

Since then, however, the literature has been anything but conclusive on the relationship between households' inflation expectations and behavior not only from an empirical perspective, but even a theoretical one as well. Theoretical research does consistently show, however, that households' ability to accurately anticipate inflation is critical for their productive economic decision-making (D'Acunto et al., 2022; Gautier & Montornès, 2022). Unfortunately, empirical research consistently shows that households typically demonstrate quite inaccurate expectations of inflation (Abildgren & Kuchler, 2019; Cornand & Hubert, 2022; Jungermann et al., 2007), which ultimately confound their decision making.

During the 1970s and 1980s in developed economies, households regularly under-anticipated inflation; they would then save during periods of rising prices, unknowingly doing so at negative real interest rates (Stephens & Tyran, 2017). As Katona (1974) postulates, when consumers underestimated future inflation during this time period, they failed to recognize the role inflation was playing on their subsequently worsening financial state. Consumers, instead, misinterpreted their increasing financial hardship as being simply the result of a "bad economy" or poor personal financial management and, thus, believed they needed to save more and/or act more financially responsibly—as opposed to limiting exposure to losses of wealth in real value.

As such, when facing inflation, households' ability to accurately anticipate inflation and then time their behavioral changes accordingly ultimately defines whether such actions produce economic benefit or harm. Improper behavioral changes can reduce one's purchasing power, in particular, by either underexposing wealth to increases or overexposing to decreases in real value, such reducing (increasing) money held in savings accounts with positive (negative) real interest rates.

That said, contemporary research has produced inconclusive, and at times conflicting, empirical results on the relationships between expected inflation and household behavior (Andrade et al., 2023; Binder, 2017; D'Acunto et al., 2022).[2] For instance, Burke and Ozdagli (2021) find little impact on consumption behavior in the United States, while Dräger and Nghiem (2021), Ichiue and Nishiguchi (2015), and Andrade et al. (2023) find positive relationships in Germany, Japan, and France respectively between the inflation rate households expect and their consumption. Moreover, Coibion et al. (2021) find a positive and negative relationship between expectations and nondurables and durables respectively in the US. Nevertheless, as Coibion et al. (2021) point out, inflation expectations and consumption decisions can be endogenous and, therefore, difficult to disentangle through macroeconomic data.

Ramsey (2002) proposes that part of the endogeneity problem may arise from the cyclical nature of consumers' behavior, which is often overlooked in economic modeling. Not only do consumers make decisions at competing time horizons, but in the aggregate, these competing time horizons across heterogeneous agents can produce macroeconomic phenomena that are difficult to disentangle without analyzing the cyclical nature of the series themselves. In other words, we must analyze trends not just in the time domain, but the frequency[3] domain as well to understand the cyclical nature underlying this economic relationship.

In this paper, I present an original approach based on *wavelet analysis* that allows me to identify the economic trends' cyclical periods. This wavelet analysis provides us with a new perspective and method to disentangle the relationship between inflation expectation and consumption.

This paper proceeds as such: Section 2 first reviews the current literature on inflation expectations and consumer behavior then presents wavelet analysis, its theoretical foundation as well as application economics. Section 3 presents the wavelet analysis of inflation expectations, consumption, and savings data from the United States, while further explaining the wavelet techniques applied. Finally, Section 4 discusses the results.

[2] See D'Acunto et al. (2022) for a contemporary summary of research on inflation expectations.
[3] I.e. in terms of cyclical periods

2. Literature review

2.1. Expected inflation

There exists a broad literature emphasizing inflation expectations as a key factor in inflationary and monetary-policy outcomes; however, the underlying mechanisms through which these variables interact at the macroeconomic level remain little understood (Abildgren & Kuchler, 2019; D'Acunto et al., 2022). Furthermore, the very relationship between inflation and economic stability and growth has, itself, proven difficult to disentangle (Bernanke, 2007).

Even at the theoretical microeconomic level, the relationship between expected inflation and consumption is unclear. For example, all else equal, an increase in expected inflation suggests a decrease in expected real interest rates and, thus, implies an increase in consumption and decrease in savings—exemplified by the Euler consumption equation (Dräger & Nghiem, 2021). But, a similarly feasible mechanism is that an increase in households' expected inflation can imply a reduction in the real value of their assets, leading them to reduce consumption to protect wealth—in other words a *precautionary behavior* (Gautier & Montornès, 2022). When examined at the aggregate empirical level, results can vary widely and at times conflict (D'Acunto et al., 2022; Gautier & Montornès, 2022). From a theoretical perspective, though, we may categorize the two possible relationships as relating to either the *Euler-consistent* behavior—increases in expected inflation lead to increases in consumption—or *precautionary* behavior—increases in expected inflation lead to increases in savings.

There has been a significant amount of empirical recent research at the macroeconomic level. According to the detailed review of contemporary literature on inflation expectations by D'Acunto et al. (2022), the results obtained on the whole are conflicting or inconsistent. In particular, while, there is clear evidence that households' inflation expectations and consumption decisions correlate, the underlying mechanism remains unclear.

That said, contemporary empirical research has revealed consistency in regards to some notable stylized facts. Of note, household consumers consistently:

1) overestimate inflation;
2) offer estimates that vary distinctly, based on socio-demographic factors, such as age, gender, income, and education-level; and
3) align expectations with overall economic sentiment (Abildgren & Kuchler, 2019; Reiche & Meyler, 2022).

How these stylized facts relate to household behavior at the macroeconomic level, though, remains unclear.

One challenge that contemporary empirical research confronts is that households' inflation expectations and consumption behavior are elicited through survey methods, such as the Michigan Survey of Consumers or Survey of Consumer Expectations in the United States (D'Acunto et al., 2022). Both surveys are rotating panels; this complicates the research by limiting the time horizon across which the relationship can be analyzed. Further, for consumption behavior, the surveys rely on respondents accurately remembering and predicting their behavior, rather than tracking their actions and purchases.

The controlled experiments presented in Lawrence et al. (2024b) and Lawrence et al. (2024a) demonstrate at the micro-level that subjects' inflation expectations relate positively to their resulting consumption behavior. The results validate the survey methods employed to elicit inflation expectations at the macroeconomic level. Further, the results provide evidence that a positive relationship between expectations and consumption can be identified using actual consumption behavior data, rather than the qualitative responses upon which current survey methods rely. Building upon this work, the aim of this paper is now to compare these results to macroeconomic data.

Given the challenges of survey-elicited consumption behavior, I choose compare inflation expectations directly to personal expenditure on consumer goods and personal savings data. Using this macro-level "behavioral" data, though, risks introducing only greater inconsistency in the trends. As Ramsey (2002) underscores:

- the endogeneity between expectations and behavior may arise from underlying cyclical, wave-like, patterns in consumption, and
- consumption at competing time horizons across a multitude of heterogeneous agents can produce seemingly unintelligible macroeconomic phenomena.

The difficulty this cyclical nature poses to econometric analysis can be summarized using a simple "cartoon" example shown in Figure 1. Here, we see two basic sinusoidal functions. The top panel shows the functions "in-phase," moving in a completely synchronized manner; the middle panel shows an "anti-phase" relationship, where the functions move in a completely oppositional manner; and the bottom panel shows the function "out of phase" by a quarter cycle ($\frac{\pi}{2}$). As a result, there are periods in the latter's case where the two functions demonstrate a positive relationship and others, negative; however, this apparent inconsistency in the relationship is, in fact, the result of the cyclicity inherent to their behavior. Therefore, studying the cyclical nature of series can help uncover the underlying phenomena producing apparently inconsistently macroeconomic behavior.

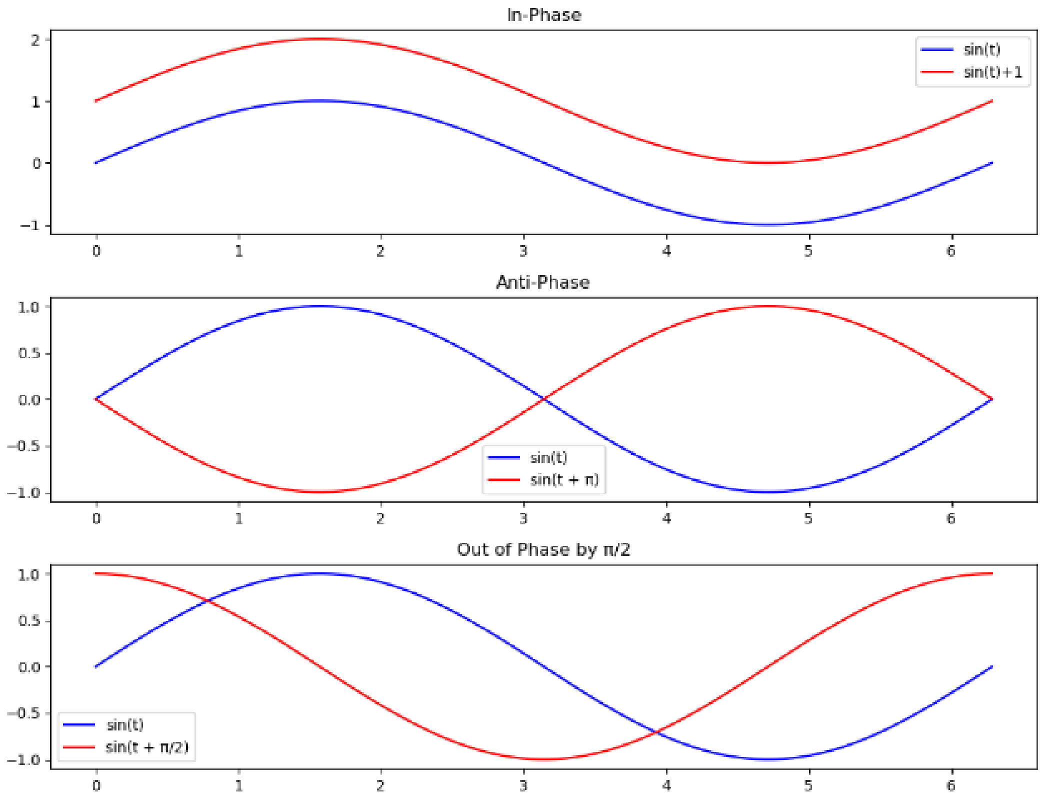

Figure 1 - Example: The impact of phase on the relationship between cyclical trends

To my knowledge, the relationship between inflation expectations and consumption has yet to be studied from the perspective of cyclicity, in terms of periodic variation. One method to gain such a perspective is through the use of wavelet analysis.

The next section presents this method and explains why it could be an efficient solution to address this gap. In Section 3. Analysis, we apply this analysis to examine the cyclical behavior of inflation expectations and consumption and savings and compare the relationship between them across different frequencies (i.e. cyclical periods or time-horizon windows).

2.2. Wavelet analysis

2.2.1. Theoretical framework

Normally, to analyze the cyclicity of a series, we would use the Fourier transform, which allows us to translate a series in the time domain to the frequency domain. From a theoretical perspective, essentially any periodic function can be decomposed into a series of sine and cosine waves. Joseph Fourier originally proved this in relation to thermodynamics, and over the ensuing two centuries, the applications of the Fourier transform have extended over many fields of study (Bracewell, 1989).

By transforming a series from the time to the frequency domain, the Fourier transform produces a frequency *spectrum* like in Figure 2 below. This spectrum acts like a fingerprint of the composite frequencies that make up a given series. Figure 2 shows two sets of time series in the left panels. The upper-left panel shows four sinusoidal functions with differing frequencies, or periodic oscillations in terms of time t. The Fourier transform calculates the frequency of each series in terms of $\frac{1}{t}$.[4], allowing us to plot the identified frequencies in the upper-right panel. Similarly, we can apply a Fourier transform to the series in the lower-left panel. In doing so, we produce a frequency spectrum identical to that of the upper-right. This result implies that the series in the lower-left panel is, in fact, resulting aggregation of the combined series in the upper-left panel.

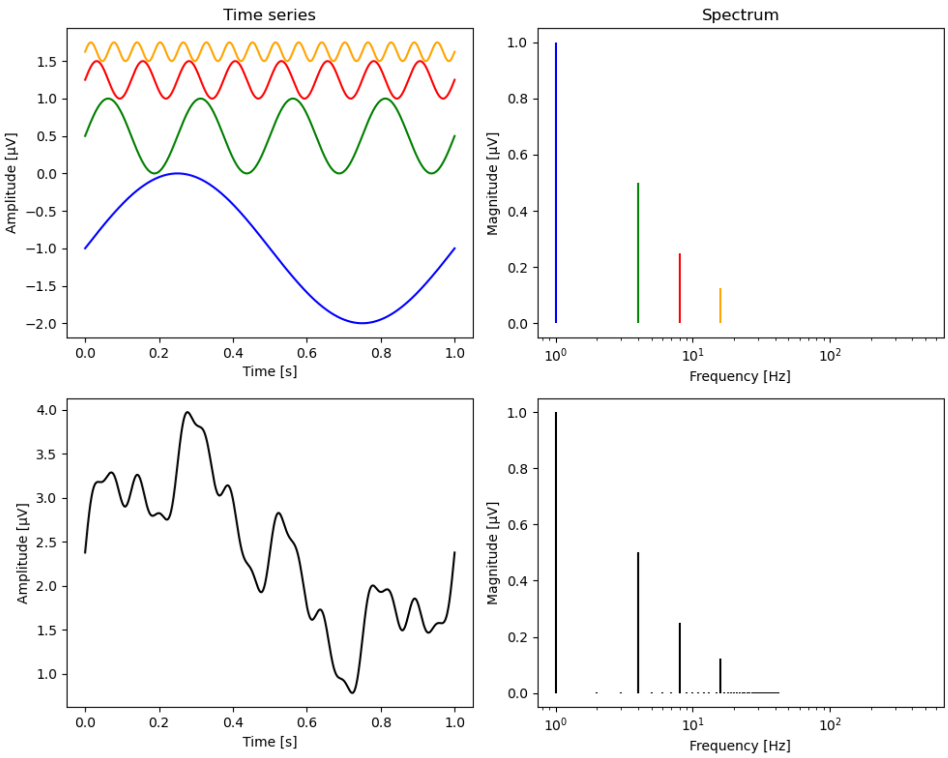

Figure 2 - Fourier transform[5]

[4] The example measures time t in terms of seconds (s). Hertz (Hz) is simply one cycle per second, $\frac{1}{s}$. Therefore, as frequency increases, the number of cycles per unit time increase.
[5] This example is based on the explanation by Bach and Meigen (1999).

This process of converting from time to frequency offers clear advantages. The frequency domain is a unique and important perspective without which it can be difficult to identify underlying trends. Even in the simple example in Figure 2, identifying the composite frequencies—of which there are merely four—is nearly impossible when viewed purely from the time-domain perspective on the left-hand side. In the frequency domain on the right-hand side, though, the relationship is immediately obvious.

That said, this fingerprint that the frequency spectrum generates also demonstrates a key drawback of the Fourier transform in that it requires series that are stationary, for which the same composite frequencies persist (theoretically, to infinity). This requirement makes economic time series especially difficult to analyze through Fourier transforms.

Wavelet analysis offers a conveniently flexible alternative. Based off the Fourier transform, the wavelet transform uses "wavelet" functions that are localized in both the time and frequency domain, rather than using sine and cosine functions that are infinite in time (Schleicher, 2002). Wavelets' coexistent time and frequency localization allows us to analyze nonstationary time series in the frequency domain through the wavelet transform, which is otherwise impossible with the Fourier transform. In other words, *with wavelets, we can measure how the frequency spectrum changes over time*.

While the bulk of the theory behind wavelets is beyond our present scope, to understand the fundamental concept underpinning our analysis, the following key concepts of wavelets are presented below.[6]

First, as mentioned above, wavelets are localized in both time and frequency. A wavelet may be essentially any function ψ that behaves like a wave (i.e. goes up and down as a function of time t), such that $\int_{-\infty}^{\infty} \psi(t)dt = 0$; in other words, the area under the curve at $y > 0$ is equal to the area at $y < 0$. Different than the Fourier transforms' infinite sines and cosines, though, $\psi(t)$ must also satisfy $\int_{-\infty}^{\infty} \psi(t)^2 dt = 1$ and $\int_{-T}^{T} \psi(t)^2 dt = 1 - \epsilon$, where $\epsilon > 0$—the latter implying that the wavelet peters out beyond $-T \leq t \leq T$ (Aguiar-Conraria & Soares, 2010; Ramsey et al., 2010). Figure 3 provides examples of wavelet functions and their respective frequency spectra via the Fourier transform.[7]

[6] For further information, one should consult Percival and Walden (2000) and Daubechies (1992) for detailed mathematical guides and Aguiar-Conraria and Soares (2010), Gallegati and Semmler (2014), Rua (2010), and Schleicher (2002) for guides on applications in economics.
[7] See Torrence and Compo (1998) for additional examples.

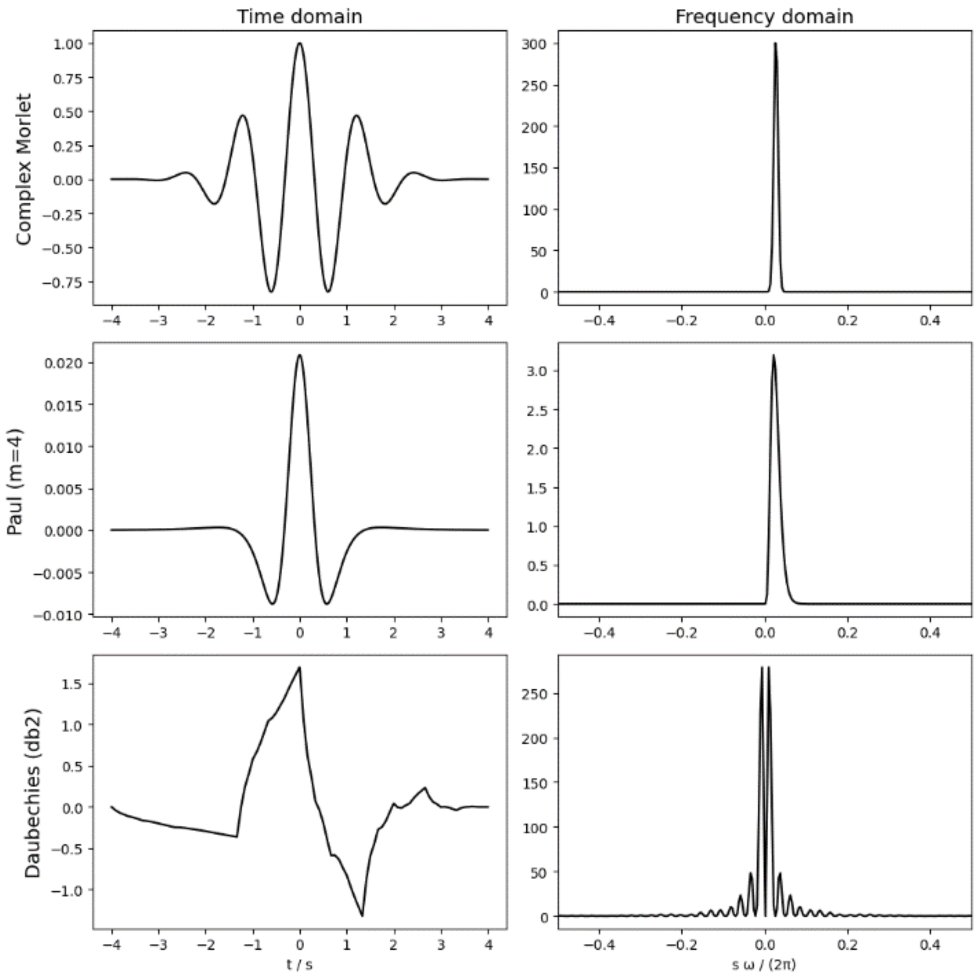

Figure 3 - Example Wavelet Functions

Second, the wavelet can be scaled and translated to produce "daughter" wavelets $\psi_{\tau,s}$ from the original "mother" function ψ:

$$\psi_{\tau,s}(t) := \frac{1}{\sqrt{|s|}}\psi\left(\frac{t-\tau}{s}\right), \qquad s,\tau \in \mathbb{R}, s \neq 0 \tag{1}$$

where s is a scaling factor that stretches or compresses ψ along the time axis and τ is a translation factor that shifts ψ along the time axis. Each daughter wavelet $\psi_{\tau,s}$ corresponds to a particular frequency and is projected onto the original series $x(t)$ to identify the frequency components. Generally speaking, the wavelet transform is:

$$W(s,\tau) = \int_{-\infty}^{\infty} \psi_{s,\tau}(t)x(t)dt \tag{2}$$

Practically speaking, this function represents the process of iteratively stretching (or shrinking) the mother wavelet and projecting each iteration onto the series being analyzed along the time axis to measure how the frequency spectrum changes over time.

Third, the wavelet transform of $x(t)$ in equation 2 represents a "continuous wavelet transform" (CWT). There exists a computationally more efficient "discrete wavelet transform" (DWT) with daughter wavelets:

$$\psi_{jk}(t) = 2^{-\frac{j}{2}}\psi\left(\frac{t - 2^j k}{2^j}\right) \tag{3}$$

at scales $s = 2^j$, $j = 1,2, \dots J$ and time index $k = 1,2, \dots N/2^j$, where N is the number of observations in series $x(t)$ (Ramsey et al., 2010). By projecting $\psi_{j,k}(t)$ on $x(t)$, we obtain a coefficient for the $d_{j,k} \approx \int \psi_{j,k}(t)x(t)dt$ for each scale and time index (j,k). This discretization leaves us with the ability to reconstruct $x(t)$:

$$x(t) \approx S + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + \cdots + \sum_k d_{1,k}\psi_{1,k}(t) \tag{4}$$

where S represents the averaged series at each scale, calculated with the scaling function "father wavelet" ϕ by:

$$\phi_{J,K}(t) = 2^{-\frac{J}{2}}\phi\left(\frac{t - 2^J k}{2^J}\right) \tag{5}$$

With the scaling coefficients $s_{J,k} \approx \int \phi_{J,k}(t)x(t)dt$, we can reconstruct the signal:

$$x(t) \approx \sum_k s_{J,k}\psi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + \cdots + \sum_k d_{1,k}\psi_{1,k}(t) \tag{6}$$

Rewriting the summations of coefficients in equation 6, we state $x(t)$ in terms of a smooth component and series of detail component vectors, where the frequency scale increases as $J \to 1$:

$$x(t) \approx S_J + D_J + D_{J-1} + \cdots + D_1 \tag{7}$$

where S_J shows the zoomed-out road map and D_1 shows the potholes (Ramsey et al., 2010).

With this set of smooth and component vectors, we can deconstruct and reconstruct the original $x(t)$ series and analyze it across and at different, specific frequencies.

2.2.2. Wavelets in economics

Within economics, the CWT is especially useful for exploratory analysis. In particular, we can compare the CWTs of two distinct economic time series to identify the time and frequency intervals where they interact. The ability to decompose series into component frequencies makes the DWT useful within existing econometric applications, such as denoising and regression (Gallegati et al., 2014, 2019).

As described by Ramsey and Lampart (1998), the DWT allows us to conduct *time scale regression*, where we regress output variables on input variables at each j frequency level individually within a set of models:

$$y[S_J]_t = \alpha_J + \beta_J x[S_J]_t + \epsilon_t \quad (8)$$

and

$$y[D_j]_t = \alpha_j + \beta_j x[D_j]_t + \epsilon_t \quad (9)$$

for the smooth component's independent and dependent variables, $y[S_J]_t$ and $x[S_J]_t$, respectively and for the j-level detail scale's independent and depend variables, $y[D_j]_t$ and $x[D_j]_t$, respectively for $j = 1, 2, \ldots, J$.

The literature on wavelets in economics is still relatively small, but there have been noteworthy findings regarding other areas of economic study with similarly conflicting results in the literature. For example, Ramsey and Lampart (1998) use time scale regression to find variation in the relationship between income and consumption as a function of frequency scale. Gallegati et al. (2014) and Aguiar-Conraria et al. (2020) both find similar variations in the relationship between productivity and unemployment at different frequency scales. More relevant to our present analysis of inflation, Gallegati et al. (2011) find evidence supporting the Phillips Curve across frequency scales. Rua (2012) finds evidence that the relationship between the money supply and inflation exists primarily at lower frequencies (i.e. longer time-cycle horizons), while Gallegati et al. (2019) similarly find supporting evidence for the Quantity Theory of Money at longer time-cycle horizons of 16 to 24 years. Further, Martins and Verona (2023) find evidence that inflation expectations exhibit strong influence on headline inflation dynamics at lower frequencies, whereas energy price inflation is a more prominent determinants at higher frequencies.

Considering the challenges in identifying a relationship between inflation expectations and consumption, applying a wavelet analysis seems worthwhile to disentangle their dynamics as well.

3. Analysis: Inflation expectations, consumption, and savings

My objective is to analyze the relationships between expected inflation and consumption using wavelet techniques, assessing the approach's effectiveness. To maintain a reference point within the literature, I compare the trends observed in the analysis to a benchmark model. Coibion et al. (2021) provide a particularly recent model with US data, breaking the relationship down between both nondurables and durables. Further, their survey includes actual purchase data via the Kilts-Nielsen Consumer Panel.[8]

Through their survey, Coibion et al. (2021) find that inflation expectations have a positive relationship with nondurables (and services) consumption and a negative relationship with durables consumption. A 1% increase in inflation expectations correlates with a 1.8% increase in nondurables consumption and 1.5% decrease in durables consumption.

My objective is not to replicate their exact model and results but rather compare the aggregate behavioral trends that they observe through recent survey data in our aggregate data. Whereas their model directly uses individuals' survey responses, mine uses time series of the corresponding aggregated indicators. I use the discrete wavelet, continuous wavelet, and cross-wavelet transforms as well as time scale regression.

The analysis consists of three parts. First, I present descriptive statistics of the series in the time domain. Then, I apply wavelet transforms to gain new perspectives on the data. These transforms include the discrete wavelet transform (DWT) to observe each frequency component separately; the continuous wavelet transform (CWT) to track the evolution of the frequency components' explanatory power over time; and the cross-wavelet transform, described below, to analyze the evolution over time of the relationships between inflation expectations and nondurable, durables, and savings at each frequency. Finally, I conduct a time scale regression to quantify the relationships between inflation expectations and nondurable, durables, and savings at each frequency.

[8] The Kilts-Nielsen Consumer Panel tracks roughly 80,000 households' consumption data, registered through the use of in-home barcode scanners that panelists use to register the products they purchase (Coibion, Gorodnichenko, et al., 2021; *NielsenIQ: Consumer Panel and Retail Scanner Data*, 2024).

All analysis is conducted in Python, using the statsmodels, PyWavelets, and PyCWT libraries (Krieger & Freij, 2023; Lee et al., 2019; Perktold et al., 2024). The code is freely available for use at https://github.com/o-nate/inflation-wavelets (Lawrence, 2024).[9]

3.1. Descriptive statistics

For my analysis, I use the one-year inflation expectations indicator generated by the University of Michigan's Surveys of Consumers, and the personal consumption expenditures for nondurable and durable goods, and personal savings between January 1, 1978 and July 1, 2024.[10] All time series provide observations on a monthly basis. I retrieve all data through the FRED API from the Federal Bank of St. Louis (*St. Louis Fed Web Services: FRED® API Overview*, n.d.).

Figure 4 graphs the four series over time. The bottom three panels compare inflation expectations to the percent change in personal nondurables and durables consumption as well as the personal savings rate;[11] the top panel compares inflation expectations to CPI inflation (University of Michigan, 2024; U.S. Bureau of Economic Analysis, 2024b, 2024a, 2024d; U.S. Bureau of Labor Statistics, 2024). Visually inspecting the series, it proves very difficult to identify any sort of relationship between expectations and consumption and savings.

[9] At the current time of writing, the code is not "production" ready, meaning it has not been fully tested on other computers. My intention is to make this a standalone program, freely available online, where people can generate wavelet transforms of their own time series data.

[10] Personal consumption expenditures: Nondurable goods (PCEND), Personal consumption expenditures: Durable goods (PCEDG), and Personal saving (PMSAVE)

[11] Here, I graph the Personal Saving Rate (PSAVERT) to simplify the visualization, but in my analysis, I use the gross Personal Saving (PMSAVE).

Figure 4 - Time series: Inflation, Inflation Expectations, Nondurables Consumption, Durables Consumption (US)

Table 1 shows descriptive statistics for the CPI inflation; inflation expectations; and the percent change in nondurables consumption, durables consumption, and savings (University of Michigan, 2024; U.S. Bureau of Economic Analysis, 2024a, 2024b, 2024c; U.S. Bureau of Labor Statistics, 2024), similar to those used by Kim and In (2005). CPI inflation and inflation expectations share nearly identical means, but CPI inflation has varied more over the same time period. Nondurables and durables consumption also share similar mean percent changes; however, durables consumption has also varied much more over time. The percent change in savings is significantly higher on average and varies much more than the other variables. All variables are fairly skewed to the right, except for nondurables. Although the percent change in nondurables has close to normal skewness and kurtosis, Jarque-Bera and Shapiro-Wilk tests confirm that none of the variables are normally distributed. The Ljung-Box tests further show that the data are autocorrelated as well.

Table 1 - Descriptive statistics (1978-2024)

	CPI inflation	Expectations	Nondurables (% change)	Durables (% change)	Savings (% change)
Observations	559	559	559	559	559
Mean	3.60	3.59	5.11	5.55	10.94
Standard deviation	2.78	1.62	3.83	7.25	42.54
Skewness	1.80	2.51	0.39	2.33	2.89
Kurtosis	3.65	6.27	3.43	23.37	16.80
Jarque-Bera	601.64***	1480.98***	280.92***	12987.82***	7224.63***
Shapiro-Wilk	0.83***	0.67***	0.95***	0.84***	0.76***
Ljung-Box	7891.17***	8192.97***	2628.26***	1462.64***	1156.48***

For the wavelet analysis, rather than the percent change in the series, I use the logarithmic difference. The reasoning for this is partly that logarithmic differences are additive, where a decrease of 0.01 represents an equal change in magnitude to that of an increase of 0.01, whereas a 10% increase or decrease are not equal in absolute magnitude. Additionally, households are indeed notoriously imprecise with their point estimates (Abildgren & Kuchler, 2019; Cornand & Hubert, 2022; Jungermann et al., 2007). Their qualitative expectations (i.e. stating whether they expected prices to increase, decrease, or stay the same), however, can prove more accurate and a better predictor of subsequent behavior (Andrade et al., 2023). In fact, we validate this greater predictive power of qualitative expectations experimentally in Lawrence et al. (2024a). Therefore, I choose to use the logarithmic difference—rather than the direct inflation rate expected in percentage terms—to compare how changes in households' expectations relate to their consumption and savings behavior. That is to say that a positive (negative) logarithmic difference implies an increase (a decrease) in expectations, so I compare that change in outlook to the corresponding change in behavior.

Being the case, though, the interpretation of logarithmic difference can be less intuitive, so I spare us the descriptive statistics here. They are essentially the same, though, aside from the distribution of nondurables appearing much less normal. See Table 7 in Appendix: Additional descriptive statistics for the logarithmic difference results.

Subsequently, Table 2 presents a correlation matrix of the series' logarithmic differences. There are clear positive correlations between CPI inflation and both inflation expectations ($p \leq 0.01$) and nondurables ($p \leq 0.01$) as well as a negative correlation with savings ($p \leq 0.05$). Inflation expectations, however, only demonstrate a statistically significant correlation with nondurables ($p \leq 0.01$); with durables and savings, the correlation is essentially zero.

Further, nondurables and durables are positively correlated ($p \leq 0.01$), and we also confirm that savings do indeed correlate negatively with both consumption series ($p \leq 0.01$).

Table 2 - Correlation matrix: Logarithmic differences of series (1978-2024)

	CPI inflation	Expectations	Nondurables	Durables	Savings
CPI inflation	—				
Expectations	0.24***	—			
Nondurables	0.42***	0.16***	—		
Durables	0.03	-0.03	0.39***	—	
Savings	-0.12**	0.04	-0.27***	-0.31***	—

3.2. Exploratory wavelet analysis: New perspectives

3.2.1. Frequency decomposition

As originally suggested by Ramsey (2002), decomposing two series into their corresponding detail and smooth components individually and comparing them visually provides a helpful initial exploratory step, allowing us to identify variations in synchronicity and changes in lead-lag relationships (i.e. which series appears to lead or follow the other). This decomposition requires the discrete wavelet transform (DWT).

I apply the DWT to the inflation expectations and nondurables, durables, and savings at each frequency series. Similar to Kim and In (2005), I use the Daubechies 4 asymmetric wavelet since it is effective at localizing features within a series (Bruzda, 2011; Daubechies, 1992). Figure 5 shows the decomposition of inflation expectations, with the lowest frequency (i.e. largest cycle) S_6 smooth component in the top panel and the highest frequency D_1 detail component in the bottom panel. This figure is the wavelet-transform equivalent of the upper-left panel in Figure 2 above.

Each level j represents a time scale interval 2^j, such that the detail components D_1, D_2, D_3, D_4, D_5, D_6 contain cycles of 2-4, 4-8, 8-16, 16-32, 32-64, and 64-128 months respectively. S_6 represents the long-term trend, while each component D_j represents the deviations from this trend in the cyclical interval (Gallegati et al., 2014).

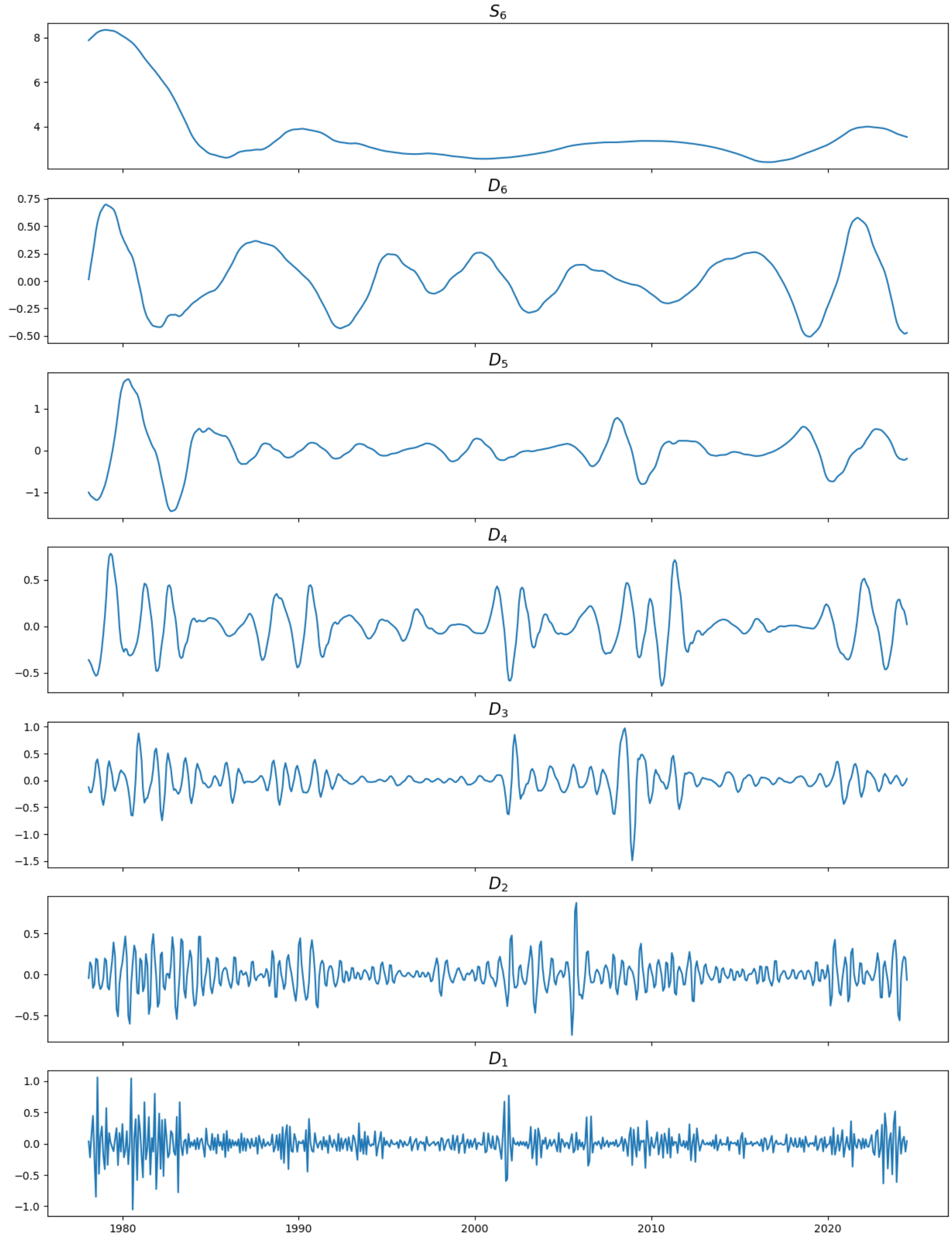

Figure 5 - Frequency decomposition of inflation expectations

Of immediate note, the highest frequency components, D_1 and D_2, contain the most noise. In fact, by decomposing the series into discrete frequencies, we can denoise the series by reconstructing it with an inverse DWT and simply removing the noisiest detail components, D_1 and D_2 for instance. Given equation 7, where the DWT of a series $x(t)$ of scale J produces components such that

$$x(t) \approx S_J + D_J + D_{J-1} + \cdots + D_1$$

we can remove (add) detail components to the smooth component S_J to produce increasingly smooth (detailed) approximations S_{J-k} of $x(t)$ (Gallegati et al., 2014; Ramsey et al., 2010), where k is the number of detail components included. Figure 6 visualizes this possibility of approximating additional smooth components,

$$x_{exp}(t) \approx S_6 + D_6 + \cdots + D_1$$

$$S_1 = S_6 + D_6 + D_5 + D_4 + D_3 + D_2$$

$$S_2 = S_6 + D_6 + D_5 + D_4 + D_3$$

$$\ldots$$

$$S_5 = S_6 + D_6$$

$$S_6 = S_6,$$

where the S_1 smooth component approximates the inflation expectations series without the D_1 detail component. The top panel shows the approximation of the S_1 smooth component. Moving from the top to bottom panel, we iteratively remove the next level detail component.

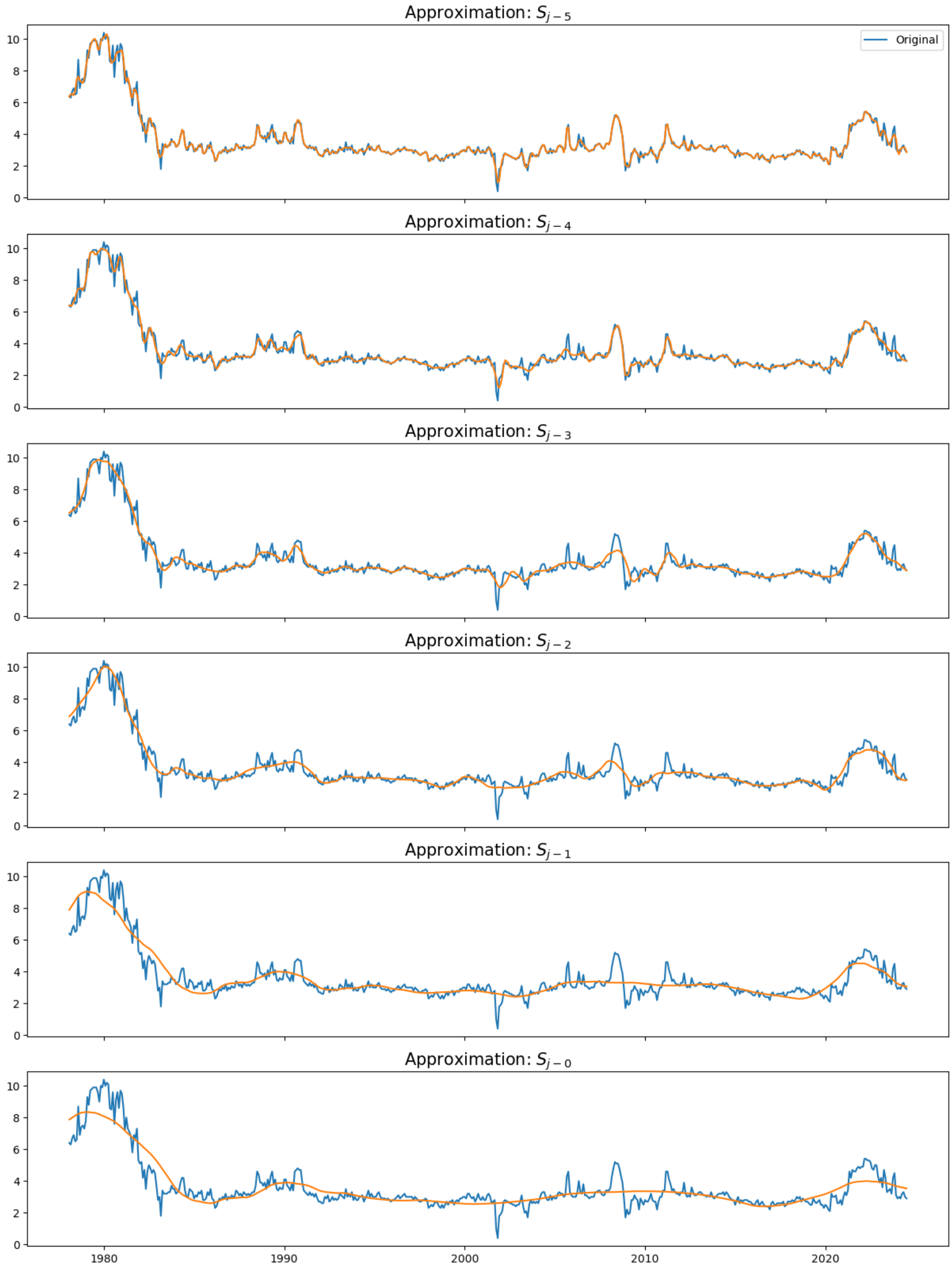

Figure 6 - Smoothing of inflation expectations through inverse DWT and detail-component removal

Moreover, this decomposition via the DWT allows us to compare series at each frequency too. Figure 7 presents the comparative frequency decompositions of inflation expectations and nondurables consumption, juxtaposing in terms of percent expected and percent change as originally presented against the logarithmic differences of the two. First, comparing percent and logarithmic differences, we remark a distinct spike in inflation expectations in terms of logarithmic difference in the D_1 and D_2 detail components around 2001. The D_5 and D_6 detail components reveal distinct cyclical behavior in the series as well as synchronization between them. But their amplitudes differ greatly and, in fact, such that the amplitude of nondurables percentage change is much larger than that of inflation expectations, while the opposite is true for logarithmic difference. We should note that there is a difference in interpretation between the left-hand and right-hand panels; whereas the left-hand depicts how nondurables consumption is changing at a given moment in time with the corresponding inflation expectation level, the right-hand shows how nondurables consumption is changing (in terms of logarithmic difference) at a given moment with the corresponding *change* in inflation expectations in terms of logarithmic difference.

Also, of note, the two series do appear primarily in-phase. But, this co-movement disperses in terms of percentage, given relatively stable inflation expectations between 1990 and the early 2000s, at which point the series' amplitudes become more pronounced from 2008 through 2013. The relationship again disperses, given the flattening of inflation expectations, until right before 2020, when their amplitudes again increase. In terms of logarithmic difference, though, we observe in the D_5 and D_6 components that the sensitivity of households' inflation expectations may have been greater than is immediately apparent from the raw estimations. Generally speaking, the series exhibit in-phase behavior, rising and falling together, in both D_5 and D_6. Such cyclical behavior should, indeed produce a positive, *Euler-consistent* relationship, like we see both in the correlations in Table 2 and Table 8 (in Appendix: Additional descriptive statistics) as well as in Coibion et al. (2021).

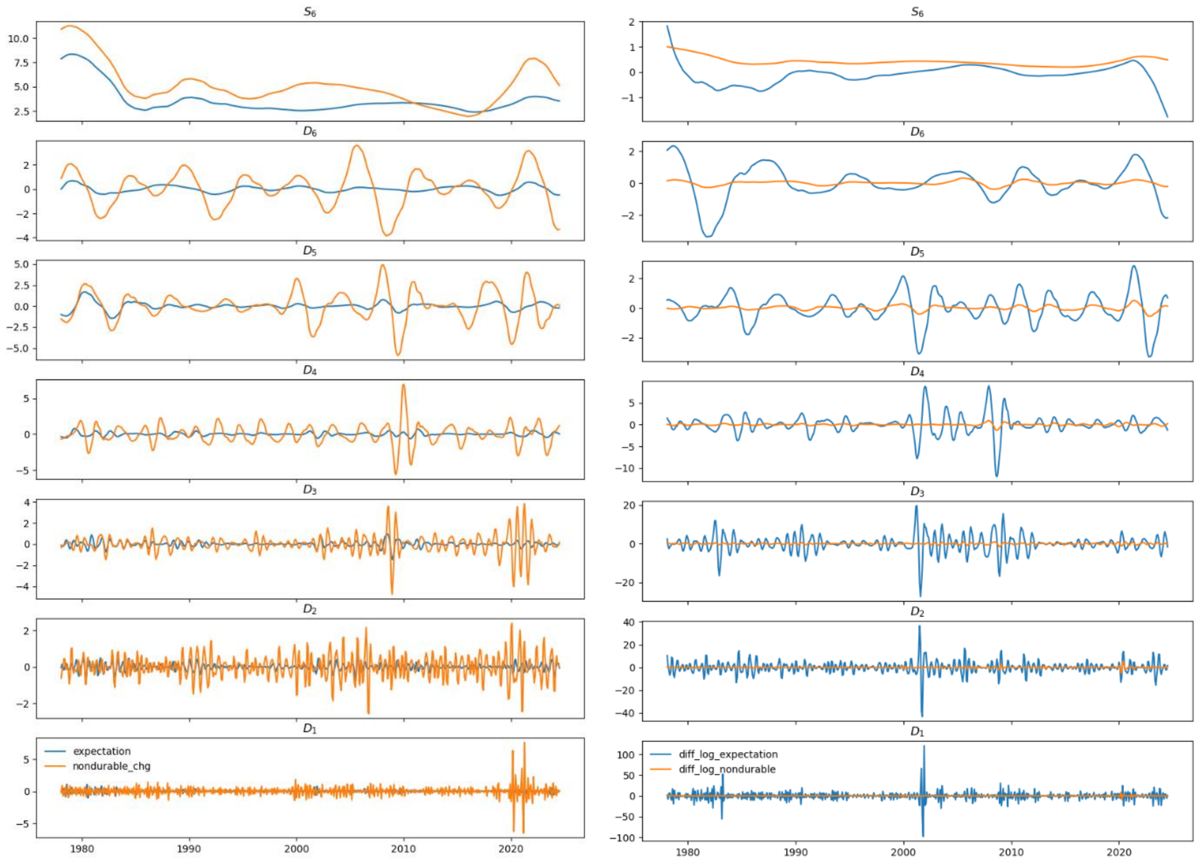

Figure 7 - Frequency decompositions: Inflation expectations and nondurables consumption

Next, we examine the smooth S_6 and detail components $D_j, j = 1 \ldots 6$ of inflation expectations and durables consumption in Figure 8 in terms of percent expected and percent change on the left and logarithmic differences on the right.

Interestingly, there are clear time intervals of an anti-phase relationship (i.e. one rises while the other falls), especially in the S_6 component. This is consistent with the findings of Coibion et al. (2021), whereby individuals demonstrate a negative relationships between expectations and durables consumption—*precautionary* behavior. Nevertheless, the high degree of shifting between in- and anti-phase in the detail components suggests that Euler-consistent aggregate behavior may also arise—particularly in periods of instability, such as the 2008 financial crisis and start of the COVID-19 pandemic in 2020. These intervals of in-phase relationship are particularly apparent in the D_5 and D_6 components.

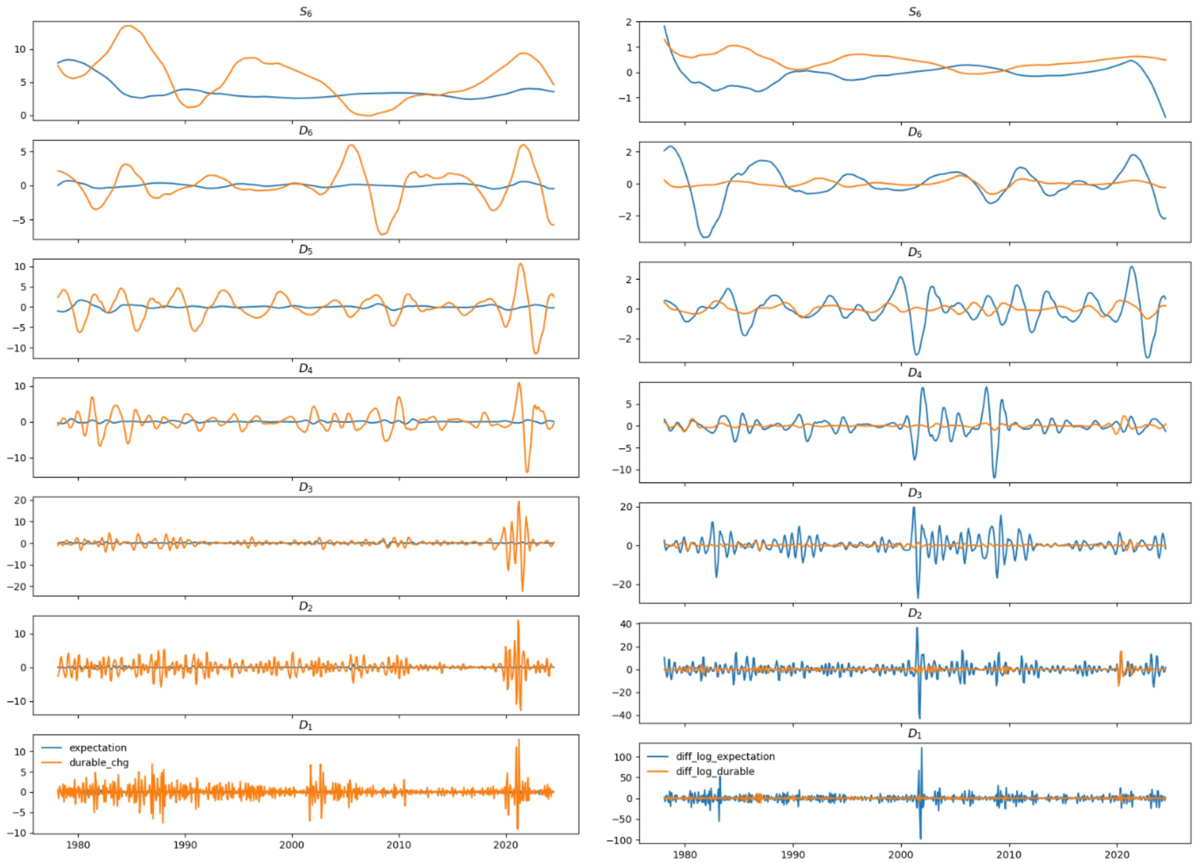

Figure 8 - Frequency decompositions: Inflation expectations and durables consumption

Finally, Figure 9 compares inflation expectations and savings, with the left-hand decomposition showing the percent expected and the savings rate and the right-hand, the logarithmic differences of expectations and savings.

Perhaps the most striking feature at first glance is the utterly linear decrease in savings rate in the S_6 component from the end of the 1970s to right before the 2008 financial crisis. In addition, inflation expectations and savings—both in rate and logarithmic difference—appear quite in-phase in S_6. Detail components D_6 and D_5, however, present mainly anti-phase behavior, while D_4 and D_3 seem to contain intervals of in-phase and intervals of anti-phase. Taken together, we might suspect a pattern of precautionary behavior at least over the longest time horizon (i.e. lowest frequency, S_6 component) and one of Euler-consistent behavior of the medium-term, as per D_6 and D_5.

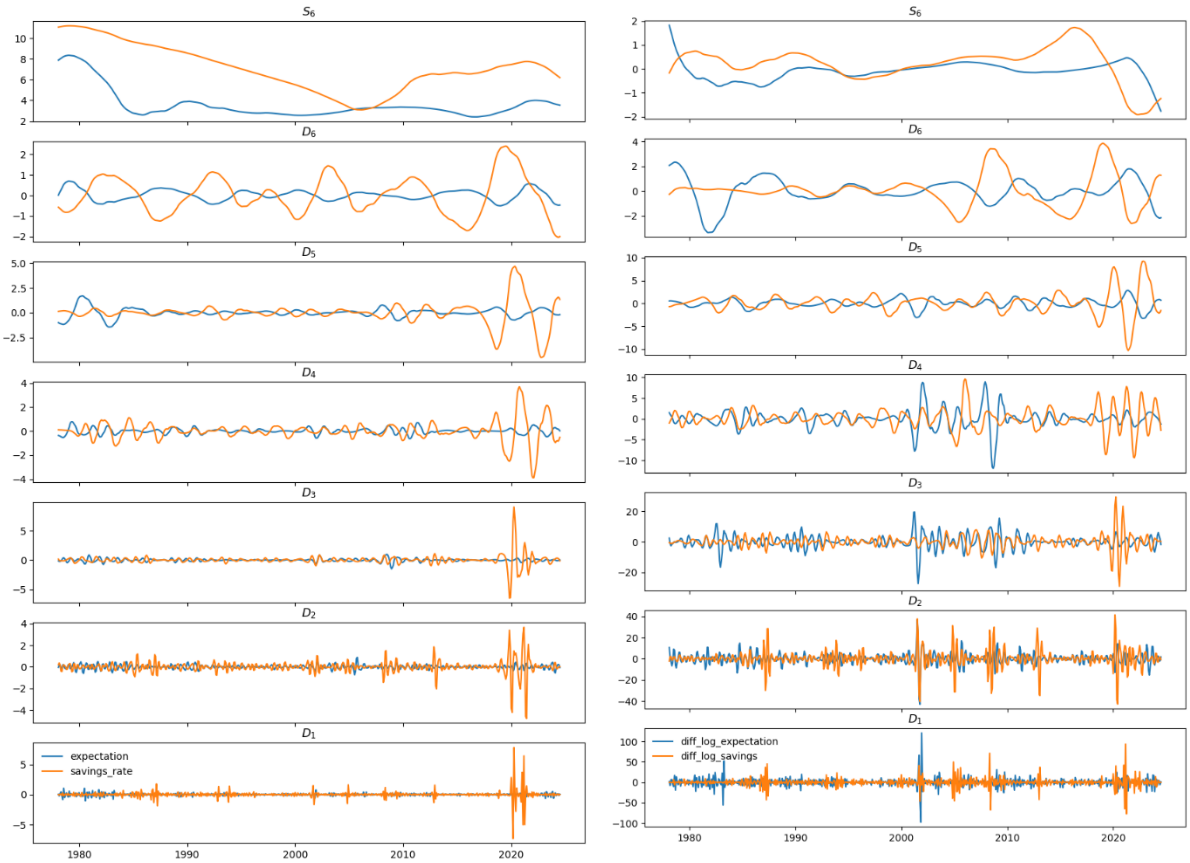

Figure 9 - Frequency decompositions: Inflation expectations and savings

This preliminary analysis, based purely on visual inspection of the components, though, does not allow us to determine the phase and lead-lag between the variables in a consistent manner. A consistent manner requires the continuous wavelet transform (CWT), cross-wavelet transform (XWT), and wavelet phase-difference, which will allow us to extract additional information from each series that is otherwise undetectable in the time domain.

3.2.2. Individual time series: Continuous wavelet transforms

Summarizing the detailed explanation by Aguiar-Conraria and Soares (2010), the continuous wavelet transform (CWT) allows us to plot how our variables' spectral characteristics evolve over time through a wavelet power spectrum.[12,13] The CWT and wavelet power spectrum, therefore, offer a more detailed method for us to analyze the component changes in a series over time.

The CWT of a series $x(t)$ is defined as:

[12] The wavelet power spectrum is sometimes referred to as a scaleogram or wavelet periodogram.

[13] In essence, this is like mapping how the frequency spectrum generated by the Fourier transform evolves over time.

$$W_{x;\psi}(\tau, s) = \langle x, \psi_{\tau,s} \rangle = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{|s|}} \psi^* \left(\frac{t - \tau}{s} \right) dt \tag{10}$$

Subsequently, the Fourier transform allows us to represent W_x in terms of frequency ω as well:[14]

$$W_x(\tau, s) = \frac{\sqrt{|s|}}{2\pi} \int_{-\infty}^{\infty} \Psi^*(s\omega) X(\omega) e^{i\omega t} d\omega \tag{11}$$

This duality allows us to map shifts in frequencies' amplitude, or "power", within our series over time through the wavelet power spectrum:

$$(WPS)_x(\tau, s) = |W_x(\tau, s)|^2 \tag{12}$$

Applying the CWT to our target series produces the four wavelet power spectra below.

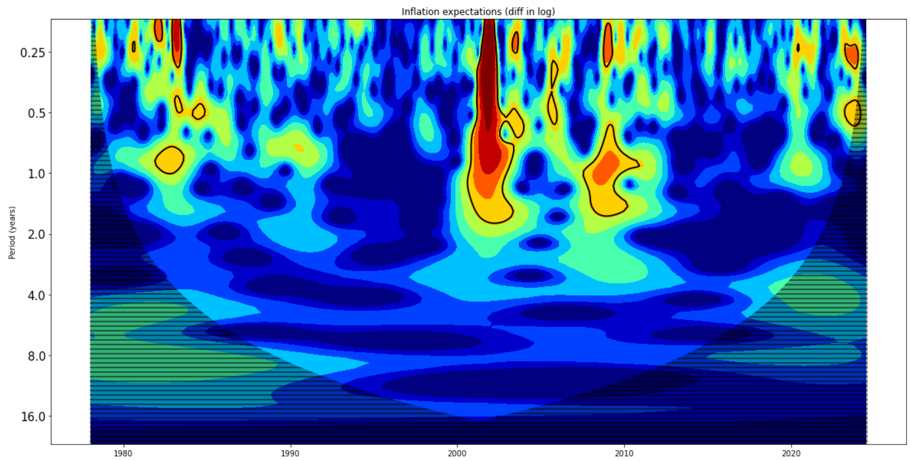

Figure 10 - Power spectrum: Inflation expectations

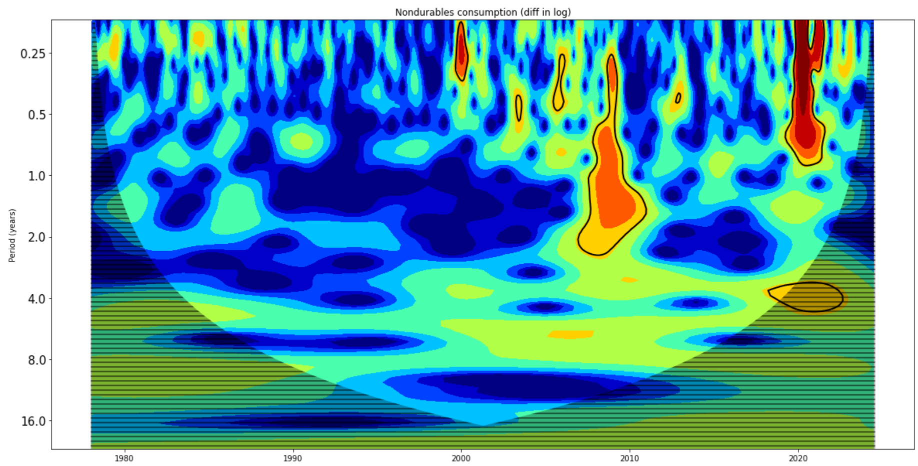

Figure 11 - Power spectrum: Nondurables consumption

[14] Note that common notation for $W_{x;\psi}$ is simply W_x.

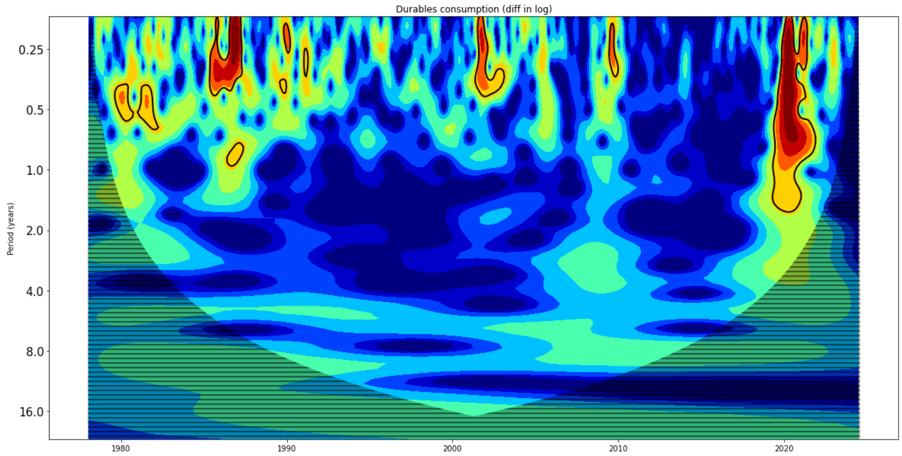

Figure 12 - Power spectrum: Durables consumption

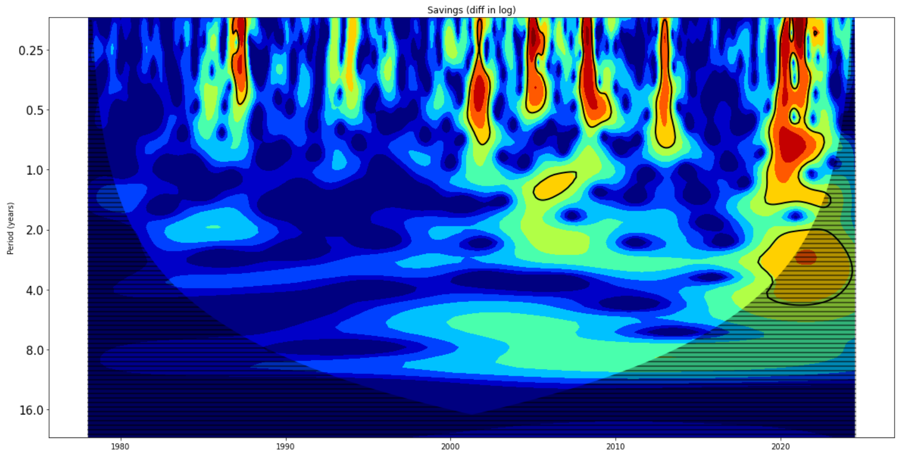

Figure 13 - Power spectrum: Savings

The power spectra in Figure 10, Figure 11, Figure 12, and Figure 13 show the changes in explanatory power of frequencies within each series over time, in terms of logarithmic difference.[15] The y axis displays the frequencies in terms of periods (where frequency and period are inverses) so as to frame the units in years; a frequency of $\frac{1}{2}$ has a period of 2 years. Areas in blue are low-power and red high-power. Black contours encircle areas of statistical significant ($p < 0.05$) using Monte Carlo simulation and chi-square distributions (Torrence & Compo, 1998). Finally, the shaded region along the edges and bottom represent areas where edge affects arise from the CWT, known as the "cone of influence" (COI). The cone of influence occurs because the series must be padded with zeros at the beginning and end to fit

[15] See Appendix: Continuous wavelet transforms of series in percentage terms for the CWT power spectra of the series in percentage terms, similar to the time series and frequency decompositions in percentage terms.

the complete cycle of each daughter wavelet ψ_s.[16] Results within the COI should be interpreted with caution since they include artificially padded zeros.

In Figure 10, we see that the logarithmic differences in inflation expectations primarily demonstrate more power over shorter periods (higher frequencies) between three months an and one year. There are, however, notable and statistically significant jumps in power at higher frequencies in the early 1980s and around more recent periods of crisis: the 2001 burst of the tech bubble, the 2008 financial crisis, and the 2020 start of the COVID-19 pandemic.

In Figure 11 and Figure 12, nondurables and durables consumption logarithmic differences also demonstrate consistently higher power spectra at higher frequencies of up to one year in period with jumps in power around the 2001 tech bubble, 2008 financial crisis, and COVID-19 pandemic. Both series also seem to contain an underlying, lower-power frequency at a four- to eight-year cycle, roughly in-line with the business cycle (Addo et al., 2014; Aguiar-Conraria et al., 2011).

Interestingly, in Figure 13, we see no underlying power at the business-cycle level until around 2000. Rather, the logarithmic difference of savings shows bursts of high-power at high frequencies during periods of economic turmoil but otherwise appears inconsistent—possibly a reflection of the steady trend down in savings in the United States since the 1980s.

Taken together, we observe that most power within the four series exists at higher frequencies, with jumps in power around crises. Next, I compare these observed trends between each CWT $W_x(\tau, s)$ using the cross-wavelet transform.

3.2.3. Time series co-movements: Cross wavelet transforms and phase difference

While the CWT of each series $W_x(\tau, s)$ provides a detailed view through the power spectrum, we cannot directly compare them. Direct comparison requires the cross-wavelet transform (XWT) and resulting cross-wavelet power spectrum.

Given two time series $x(t)$ and $y(t)$, the cross-wavelet transform $\left|W_{xy}\right| = |W_x W_y|$ represents the covariance between x and y at each scale and frequency (Gallegati et al., 2014). We can produce a cross-wavelet power spectrum to identify the time-frequency regions where x and y show commonly high power.

[16] Note that this implies that the cone of influence is a function of the wavelet function chosen (Torrence & Compo, 1998).

Further, wavelet functions ψ and their corresponding CWTs $W_x(\tau, s)$ can be real or complex. In the latter case, the real $\Re\{W_x(\tau, s)\}$ and imaginary components $\Im\{W_x(\tau, s)\}$ capture the amplitude $|W_x(\tau, s)|$ and phase $\phi_x(\tau, s)$: $W_x(\tau, s) = |W_x(\tau, s)|e^{i\phi_x(\tau,s)}$ respectively.[17] By extracting these two sets of information in the real and imaginary components, we can determine the phase as (Aguiar-Conraria & Soares, 2010):

$$\phi_x(\tau, s) = \text{Arctan}\left(\frac{\Im\{W_x(\tau, s)\}}{\Re\{W_x(\tau, s)\}}\right) \tag{13}$$

Given a XWT W_{xy}, we can similarly calculate the difference between two series' phases ϕ_x and ϕ_y, known as the "phase difference" ϕ_{xy} through:

$$\phi_{xy} = \text{Arctan}\left(\frac{\Im(W_{xy})}{\Re(W_{xy})}\right) \tag{14}$$

which is simply $\phi_{xy} = \phi_x - \phi_y$ (Aguiar-Conraria & Soares, 2010). We then map this phase difference onto the cross-wavelet power spectrum over time and frequency in the form of vector arrows.

Figure 14 provides a key for interpreting the arrows. An arrow pointing in the right or left direction, with an angle of either $\theta \approx 0°$ or $\theta \approx 180°$, implies in- or anti-phase relationship respectively. Right and up ($0° < \theta < 90°$) represents x leading y in-phase, left and down ($90° < \theta < 180°$) x leading y anti-phase, right and down ($180° < \theta < 270°$) y leading x in-phase, and left and up ($270° < \theta < 360°$) y leading x anti-phase.[18]

[17] Note that the use of ϕ is not related to the father wavelet function, as mentioned in Section 2.2.1.
[18] As can be seen in Figure 14, the angle θ can equally be represented in terms of radians, where $90° = \frac{\pi}{2}$.

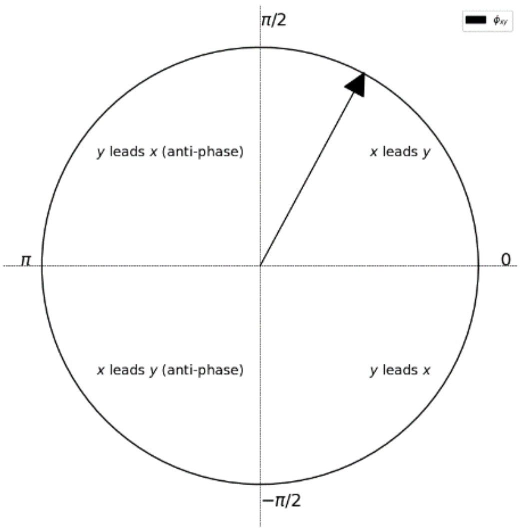

Figure 14 - Key, Phase Difference

For example, within the context of inflation expectations and nondurables consumption, an in-phase relationship (i.e. an arrow pointing to the right as represented in Figure 14) would reflect a positive relationship with expectations and consumption moving in the same direction, and thus suggesting Euler-consistent behavior. Conversely, an anti-phase relationship (i.e. pointing leftward) would represent precautionary behavior.

The XWTs of logarithmic difference in inflation expectations with nondurables consumption, durables consumption, and savings produce the three cross-wavelet power spectra shown below. For the spectra in percentage terms, see Appendix: Cross-wavelet transforms of series in percentage terms.

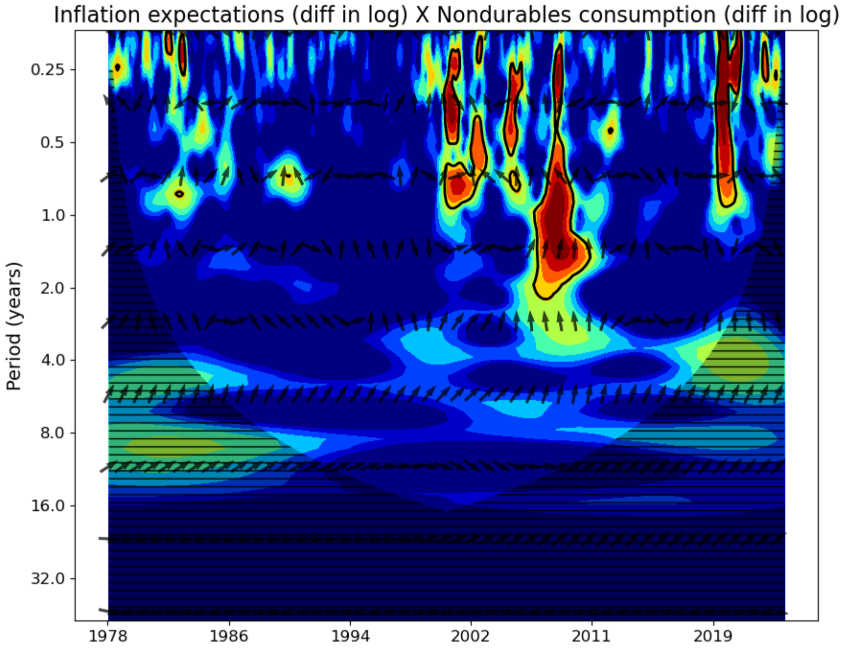

Figure 15 - Cross-wavelet power spectrum: Logarithmic differences of inflation expectations and nondurables consumption

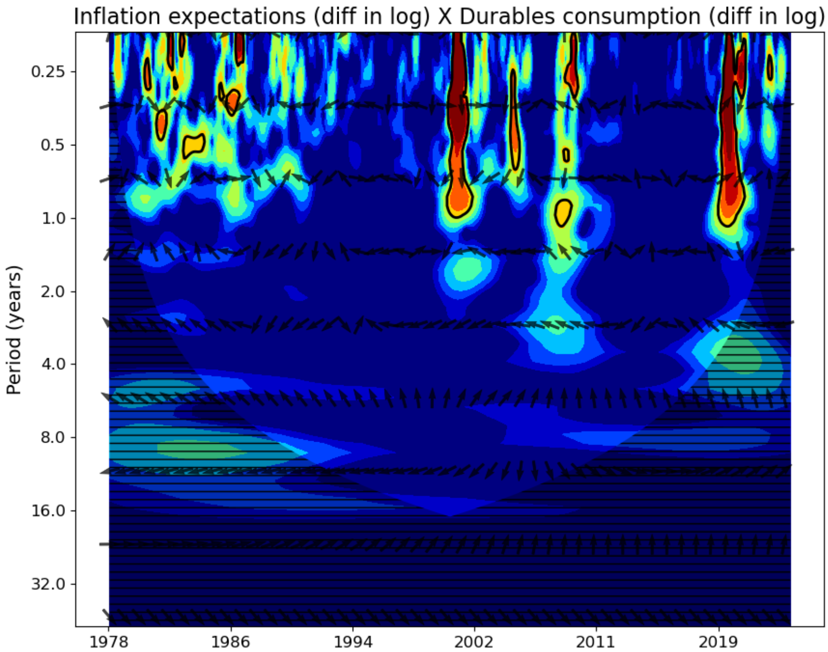

Figure 16 - Cross-wavelet power spectrum: Logarithmic differences of inflation expectations and durables consumption

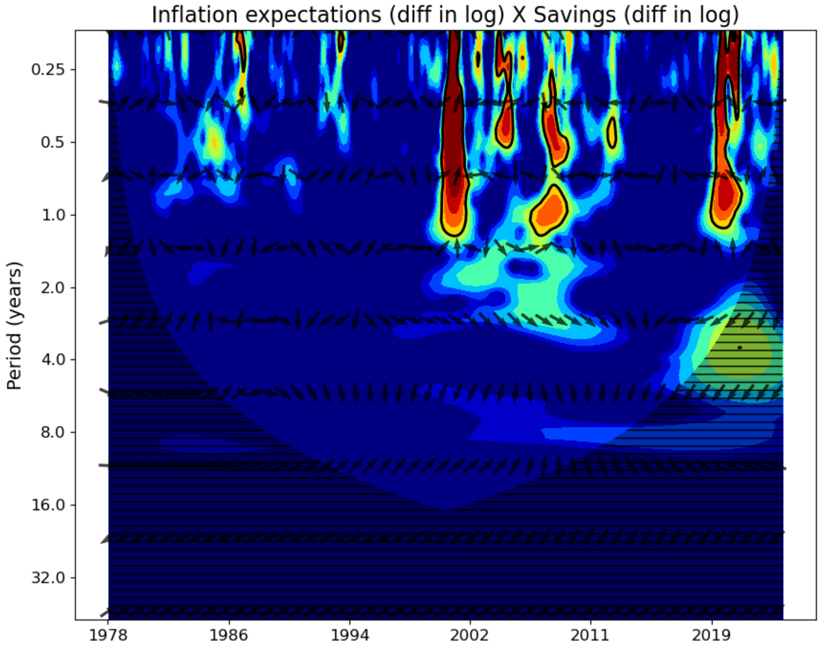

Figure 17 - Cross-wavelet power spectrum: Logarithmic differences of inflation expectations and savings

First, in Figure 15, we observe that co-movement between inflation expectations and nondurables spikes in higher frequencies at times of economic turmoil, the late 1970s, the 2001 tech bubble-burst, the 2008 financial crisis, and the beginning of the COVID-19 pandemic. For the most part, at periods of high power, either inflation expectations lead nondurables in an in-phase pattern (i.e. an arrow point at angle $0° < \theta < 90°$) or nondurables lead expectations in an anti-phase ($90° < \theta < 180°$). In the case of the former, this translates as a positive relationship, in line with an Euler-consistent behavior of US households, in which an increase (decrease) in the logarithmic difference of inflation expectations precedes a similar increase (decrease) in the logarithmic difference of nondurables consumption. For the latter, this signifies that an increase (a decrease) in the logarithmic difference of nondurables consumption precedes an opposite decrease (increase) in the logarithmic difference of inflation expectations, suggesting on the contrary a precautionary behavior of US households.

Figure 16 presents the cross-wavelet power spectrum between the logarithmic differences in inflation expectations and durables consumption. Here, we observe a similar pattern to that displayed in the power spectrum with nondurables, particularly with high power around the 2001 tech bubble across periods from three months to one year; however, there is much less power around the 2008 financial crisis. In fact, the phase difference during this period reveals anti-phase relationships (precautionary behavior) at periods of two years or shorter. Conversely, around the start of the COVID-19 pandemic, there is a clear shift to in-phase (Euler-consistent behavior), with expectations leading durables consumption. During the inflationary interval in the 1970s and 1980s, we see quite erratic shifts between in- and anti-phase, lead and lag behavior. This appears consistent with CPI inflation's significantly higher variance during this time interval, compared to the 2000s and 2010s,[19] complicating households' ability to not only anticipate inflation, but settle on a consumption behavior—be it Euler-consistent or precautionary. Nevertheless, as with nondurables, most of the cross-wavelet power seems to exist between logarithmic differences in expectations and durables consumptions at periods of less than two years.

Lastly, Figure 17 displays the cross-wavelet power spectrum between the logarithmic differences in inflation expectations and savings. Compared to the previous spectra, we again see a similar pattern with high power in times of economic turmoil, particularly the 2001 tech bubble, 2008 financial crisis, and COVID-19 pandemic at periods from three months to one year. In contrast, there is little explanatory power during the interval from the 1970s to 1990s. The phase differences are also erratic, like for durables consumption, exhibiting periods of both in-phase (precautionary behavior) and anti-phase (Euler-consistent behavior).

Holistically, our primary take-away may be that the logarithmic differences in the series demonstrate greatest co-movement during times of economic turmoil: the 1970s and 1980s inflationary interval, the 2001 tech bubble-burst, the 2008 financial crisis, and the 2020 start of the COVID-19 pandemic. Moreover, this co-movement between the logarithmic differences is at high frequencies (i.e. shorter cyclical periods), suggesting that the mechanism—through which changes in inflation expectations relate to changes in behavior—operates at higher frequency as well (i.e. within shorter time horizon). This pattern appears consistent as well with the literature on inflation expectations and attention, whereby consumers pay greater attention to information on inflation during a phase change, when in a high-inflation environment (Cavallo et al., 2017; Weber, Candia, et al., 2023).

Interestingly, the cross-wavelet power spectra in percentage terms (expected inflation rate and percentage change in behavior) shows high power of co-movement at lower frequencies; see Appendix: Cross-wavelet transforms of series in percentage terms.

One interpretation of this difference is that while the mechanism of changes in inflation expectations relating to changes in behavior may operate over a short time horizon, the mechanism through which the indicator data relate (i.e. the expected inflation rate and percentage changes in consumption and savings) may operate over a longer time horizon.

[19] See Table 9 in Appendix: Additional descriptive statistics for a CPI inflation trends in each decade.

Indeed, this difference between mechanisms and time horizons may be further reflected in the fact that in percentage terms, the phase relationships and low frequencies appear much more consistent and stable. For percentage change in nondurables, the relationship appears mainly anti-phase (precautionary) at low frequencies with periods greater than two years, where consumption leads expectations, but in-phase (Euler-consistent) at higher frequencies, during which expectations lead. The relationship with percentage change in durables seems mainly anti-phase (precautionary) but with shifts to in-phase (Euler-consistent) starting in the early 2000s in the range of four- to eight-year cycles. This also seems noteworthy considering a significant portion of durables may be purchased on a four- to eight-year basis. Expectations and the percentage change in savings demonstrate pronounced in-phase (precautionary) relationships in the range of one- to four-year cycles but anti-phase (Euler-consistent) and the eight-year.

Although observation of the cross-wavelet power-spectra do not provide conclusive results, they allow us to observe the changes in the relationships through additional dimensions beyond the one dimension provided in the original time series or even the two dimensions provided by the frequency decomposition. Through the cross-wavelet power spectra, we can in fact perceive the shifts in the relationships across both time and cyclical period.

Returning to the relationships observed by Coibion et al. (2021), whereby inflation expectations and nondurables are Euler-consistent while expectations and durables are precautionary, the power spectra may shed some light. In particular, the researchers conduct their survey between June and December 2018. The power spectra in percentage terms (see Figure 22 and Figure 23 in Appendix: Cross-wavelet transforms of series in percentage terms) reveal in-phase arrow indicators for nondurables in the range of three- to six-month cycles between 2018 and 2019—Euler-consistent behavior. For durables, though, there are both in- and anti-phase indicators across these same frequency and time intervals, which may produce either Euler-consistent or precautionary behavior in the aggregate. Coupled with the much clearer anti-phase relationship in the same time interval across the range of lower frequencies, of one year or more, an aggregate precautionary behavior of US households seems then quite plausible.

3.3. Regression analysis

The exploratory analysis in the previous section provides us with new perspectives on the series and their relationships, particularly through the CWT and XWT. Frequency decomposition via the DWT, however, also offers a means to regress the series across the cyclical components, known as time scale regression (Gallegati et al., 2014; Kim & In, 2005; Ramsey & Lampart, 1998). This section aims to time-scale regress the behavioral series on inflation expectations.

3.3.1. Baseline model

First, to establish a baseline, we conduct "aggregate" ordinary least square (OLS) regressions of the logarithmic differences in nondurables and durables consumption and savings on the logarithmic difference in inflation expectations respectively.

The aggregate OLS regressions follow:

$$y_{non,t} = \alpha + \beta x_{exp,t} + \epsilon_t \tag{15}$$

$$y_{dur,t} = \alpha + \beta x_{exp,t} + \epsilon_t \tag{16}$$

$$y_{sav,t} = \alpha + \beta x_{exp,t} + \epsilon_t \tag{17}$$

As shown in Table 3, only $y_{non,t}$ demonstrates a statistically significant relationship with inflation expectations, and in fact, all three aggregate models provide very little explanatory power. That said, in the aggregate, we do observe positive relationships between the logarithmic differences in expectations and both nondurables (Euler-consistent) and savings (precautionary) as well as a negative relationship with durables (precautionary).

Additionally, we conduct the aggregate OLS regressions in percentage terms (see Table 10 in Appendix: Regressions in percentage terms) to compare to Coibion et al. (2021), who finds nondurables increases 1.8% with a 1% increase in expectations while durables consumption decreases by 1.5%. Our aggregate regressions show that nondurables similarly increase 1.5% but reveals no relationship involving durables.

Table 3 - OLS regressions: Behavioral series on inflation expectations, logarithmic differences

	$y_{non,t}$	$y_{dur,t}$	$y_{sav,t}$
α	0.4077***	0.4346***	0.3196
	(0.0475)	(0.1221)	(0.6718)
β	0.0134***	-0.0060	0.0437
	(0.0034)	(0.0088)	(0.0487)
r^2	0.0267	0.0008	0.0015
r^2 Adj.	0.0249	-0.0010	-0.0003

Standard errors in parentheses.

* p<.1, ** p<.05, *** p<.01

3.3.2. Time scale regression

As mentioned above, time scale regression involves regressing the component vector of our output variable at a given scale on the corresponding component vector of our input variable (Ramsey & Lampart, 1998). Following the approach first laid out by Ramsey and Lampart (1998), we conduct six regressions for:

$$y_{non}[S_J]_t = \alpha_J + \beta_J x_{exp}[S_J]_t + \epsilon_t \tag{18}$$

$$y_{non}[D_j]_t = \alpha_j + \beta_j x_{exp}[D_j]_t + \epsilon_t \tag{19}$$

$$y_{dur}[S_J]_t = \alpha_J + \beta_J x_{exp}[S_J]_t + \epsilon_t \tag{20}$$

$$y_{dur}[D_j]_t = \alpha_j + \beta_j x_{exp}[D_j]_t + \epsilon_t \tag{21}$$

$$y_{sav}[S_J]_t = \alpha_J + \beta_J x_{exp}[S_J]_t + \epsilon_t \tag{22}$$

$$y_{sav}[D_j]_t = \alpha_j + \beta_j x_{exp}[D_j]_t + \epsilon_t \tag{23}$$

where $y_{non}[S_J]_t$, $y_{dur}[S_J]_t$, $y_{sav}[S_J]_t$, and $x_{exp}[S_J]_t$ are the smooth components of the logarithmic difference of nondurables, durables, savings, and expectations respectively and $y_{non}[D_j]_t$, $y_{dur}[D_j]_t$, $y_{sav}[D_j]_t$, and $x_{exp}[D_j]_t$ are the corresponding detail components at scale j of each series.

The time scale regression of logarithmic differences in nondurables consumption on inflation expectations is shown in Table 4. Here, we see positive relationships across all components, suggestive of Euler-consistent behavior among US households at all frequencies.

Table 4 - Time scale regression: Nondurables consumption on inflation expectations, logarithmic differences

	S_6	D_6	D_5	D_4	D_3	D_2	D_1
α_j	0.4205***	0.0013	-0.0007	-0.0034	-0.0000	-0.0005	0.0001
	(0.0073)	(0.0035)	(0.0025)	(0.0068)	(0.0123)	(0.0235)	(0.0360)
β_j	0.0817***	0.1020***	0.1407***	0.0606***	0.0352***	0.0343***	0.0018
	(0.0183)	(0.0034)	(0.0026)	(0.0031)	(0.0028)	(0.0042)	(0.0031)
r^2	0.0346	0.6230	0.8448	0.4070	0.2170	0.1051	0.0006
r^2 Adj.	0.0328	0.6224	0.8446	0.4059	0.2156	0.1035	-0.0012

Standard errors in parentheses.

* p<.1, ** p<.05, *** p<.01

Table 5 provides the results of the time scale regression of logarithmic differences in durables consumption on inflation expectations. At the lowest frequency S_6 component as well as higher frequency D_3 component, we see a negative relationship, whereas at the D_6, D_5, and D_4 component frequencies, there is a positive relationship. This translates to Euler-consistent behavior within the range of periodic cycles between 16 and 128 months and precautionary behavior at the long-term cycle of over ten years and short-term of eight to 16 months. Further, although not statistically significant, the precautionary behavior appears plausible in the D_1 and D_2 components. In other words, within the business-cycle range, there is Euler-consistent behavior, but at higher and lower frequencies, the behavior could rather be precautionary.

Table 5 - Time scale regression: Durables consumption on inflation expectations, logarithmic differences

	S_6	D_6	D_5	D_4	D_3	D_2	D_1
α_j	0.4319***	-0.0086	-0.0059	-0.0047	0.0018	0.0006	-0.0000
	(0.0116)	(0.0079)	(0.0100)	(0.0172)	(0.0260)	(0.0718)	(0.0916)
β_j	-0.2131***	0.0555***	0.0837***	0.0400***	-0.0157***	-0.0105	-0.0072
	(0.0293)	(0.0077)	(0.0102)	(0.0079)	(0.0060)	(0.0129)	(0.0079)
r^2	0.0871	0.0861	0.1078	0.0444	0.0123	0.0012	0.0015
r^2 Adj.	0.0855	0.0845	0.1062	0.0427	0.0105	-0.0006	-0.0003

Standard errors in parentheses.

* p<.1, ** p<.05, *** p<.01

Subsequently, Table 6 shows the results of the time scale regression of logarithmic differences in savings on inflation expectations. The S_6 component and D_2 and D_3 components demonstrate a positive relationship (precautionary behavior), whereas at the D_6 and D_5 show a negative relationship (Euler-consistent behavior). Of note, the shift in behavior between precautionary at high frequencies to Euler-consistent at business-cycle frequencies back to precautionary at the lowest frequency is the same pattern we observe in the time scale regression of durables.

Table 6 - Time scale regression: Savings on inflation expectations, logarithmic differences

	S_6	D_6	D_5	D_4	D_3	D_2	D_1
α_j	0.2392***	0.0049	0.0389	0.0299	-0.0027	-0.0012	-0.0006
	(0.0318)	(0.0532)	(0.0869)	(0.1040)	(0.1803)	(0.3528)	(0.5179)
β_j	0.2258***	-0.3421***	-1.1523***	0.0650	0.1067**	0.1783***	0.0190
	(0.0799)	(0.0516)	(0.0884)	(0.0476)	(0.0415)	(0.0636)	(0.0449)
r^2	0.0142	0.0734	0.2341	0.0034	0.0118	0.0139	0.0003
r^2 Adj.	0.0124	0.0717	0.2328	0.0016	0.0100	0.0122	-0.0015

Standard errors in parentheses.

* p<.1, ** p<.05, *** p<.01

Finally, to again compare to the results of Coibion et al. (2021), we repeat the time scale regressions in terms of percentage in Table 11, Table 12, and Table 13 in Appendix: Regressions in percentage terms. Coibion et al. (2021) find that the 1.8% increase in nondurables and 1.5% decreases in durables consumption occurs within a six-month window following the inflation-expectation elicitation. As such, we can compare their results to those of the D_2 detail component in our time scale regression, which corresponds to a four- to eight-month cyclical period. Indeed, we find that nondurables consumption demonstrates a positive relationship ($p \leq 0.01$) in D_2 and durables a negative relationship ($p \leq 0.05$). Broadening to the D_1 and D_3 components, these relationships hold, although, only to a statistically significant degree for the D_3 detail component of nondurables ($p \leq 0.01$).

4. Discussion

Defining a clear empirical relationship between inflation expectations and consumption has remained elusive for economists. As our present findings suggest, the ambiguity in the literature may very well be the result of more complex underlying relationships.

Through wavelet analysis, we project otherwise one-dimensional time series data onto two- and then three-dimensional space and study their cyclical natures over time and in relation to each other. Further, we decompose and then regress the time series by frequency scale to better understand how their aggregate observable behavior arises from their different—and often competing—cyclical components. Indeed, this method does reveal underlying complexity that can produce apparently inconsistent patterns in the aggregate.

Through initial discrete wavelet transforms (DWTs), we observe each series' frequency components and can compare inflation expectations to the consumption and savings trends on a component-by-component basis.

Then, continuous wavelet transforms (CWTs) reveal how the underlying periodic oscillations interact and compete amongst each other over time to produce each aggregate time series. In particular, we find that periods of crisis correlate with time intervals of greater explanatory power for each series in terms of their logarithmic difference.

With these CWTs, we can next analyze the co-movement and, thus, relationship between expectations and consumption and savings through cross-wavelet transforms (XWTs) of the logarithmic differences. Through the resulting power spectra, we identify not only the spikes in co-movement during times of economic turmoil and in high-frequency ranges, but we also uncover the phase difference between each combination. This phase difference allows us to relate these patterns back to the concept first highlighted regarding cyclicity in Figure 1, in which we first see how phase difference between two cyclical series can impact our interpretation of their relationship in the time domain. By extracting and visualizing this additional information, we establish objective indicators to show from where the inconsistency in the aggregate relationships may originally arise.

Finally, we conduct a time scale regression using the frequency components generated by the DWTs to quantify the relationship between inflation expectations and consumption and savings at each cyclical period. These results provide clear evidence of the positive, Euler-consistent relationship between inflation expectations and nondurables. But perhaps more crucially, the regressions on durables consumption and savings offer explanations as to how their relationships with inflation expectations have seemed so inconsistent at the aggregate level. Essentially, the time scale regressions suggest that durables and savings may relate in an Euler-consistent manner at the business cycle-range of frequencies but in a precautionary manner at both higher and lower frequency ranges than this range.

The decision to analyze the relationships through the series' logarithmic differences allows us to observe the impact of changes in inflation expectations on the behavioral patterns, but interpreting the results is admittedly less intuitive. The additional wavelet analysis we conduct using the more "headline" format or percentages—the direct inflation rate expected and the percentage change in consumption and savings—provides a means to connect the results back to the original data and existing literature. As we see, the results in percentage terms appear in line with our benchmark model by Coibion et al. (2021). Further, the corresponding CWT and XWT power spectra (in Appendix: Continuous wavelet transforms of series in percentage terms and Appendix: Cross-wavelet transforms of series in percentage terms respectively) show that the relationships in these headline terms have higher power at lower frequencies and that they maintain greater consistency in their phase differences. This is an interesting distinction and may suggest that while the sensitivity of these relationship (i.e. logarithmic difference), and especially that of their inflation expectations, may have greater power at higher frequencies, the more headline format—more present in our daily lives—may adhere more to the larger cyclical periods.

Indeed, this distinction between the logarithmic and headline results also connects to the difference in roles that qualitative and quantitative inflation expectations play in consumption and savings decisions uncovered in Lawrence et al. (2024a). Just as qualitative estimates are a better predictor of short-term decisions in the Savings Game than quantitative estimates, the logarithmic differences in expectations (i.e. our proxy for macro-level qualitative estimates) exhibit a stronger relationship with consumption and savings at shorter cycles. Further, the positive relationship identified in our present analysis between expectations and nondurables consumption is also consistent with the positive relationship identified in Lawrence et al. (2024a).

Naturally, there are points of contention with our approach. One key point is that we only investigate households' inflation expectations on their consumption and savings behavior, ignoring all other variables. Incorporating other variables offers an intriguing direction to take this research. Income-related variables, such as personal income or even gross domestic product and/or gross national product, certainly contribute to households' decision-making. But, this mechanism too could depend on how households perceive and anticipate inflation as well as vary at different time-cycle horizons. Interest rates offer another key variable from the perspective of both savings incentive as well as the cost of borrowing for durables (or even nondurables) consumption.

Another point is that we apply essentially no data processing, cleaning, normalizing, or detrending. This is intentional so as to test wavelet's ability to inherently bring clarity to noisy data, which our results generally suggest that the technique does.

Being so, one might also argue that these results could be potentially biased by the use of nominal consumption and savings data. As a result of the nominal data, the analyses could overstate the relationships with inflation expectations since expectations correlate quite strongly with headline inflation (Bignon & Gautier, 2022; Reiche & Meyler, 2022; Weber, Gorodnichenko, et al., 2023). In other words, inflation increases both expectations as well as nominal consumption and savings, so this could inherently bias the results. To address this objection, we conduct the same analyses on the series in real terms. Based on the results presented in Appendix: Continuous wavelet transforms of series in real terms, Appendix: Cross-wavelet transforms of series in real terms, and Appendix: Regressions in real terms, no such bias appears evident.[20]

Moreover, our objective in this study is to provide a new perspective to a long-standing and—for better or worse—recently revived economic conundrum. Through this new perspective, we find that changes in inflation expectations demonstrate the clearest relationship to consumption and savings behaviors during times of higher inflation as well as economic turmoil, particularly in shorter cyclical periods. This finding suggests that inflation expectations as an indicator may, therefore, be most important as inflationary periods and economic crises unfold. Such results seem consistent with the literature on inflation expectations and attention, whereby consumers pay greater attention to information on inflation when in a high-inflation environment (Cavallo et al., 2017; Weber, Candia, et al., 2023).

Furthermore, through time scale regression, this new perspective that wavelet analysis affords us offers quantitative evidence of the differing relationships with each behavioral series across time-cycle horizons, which can produce seemingly inconsistent patterns in the aggregate. Specifically, while inflation expectations and nondurables consumption exhibit Euler-consistent behavior across all cycles, durables and savings appear Euler-consistent within the business-cycle range of cyclical periods but demonstrate precautionary behavior at shorter and longer terms. This shift in behavioral model at the business-cycle range is notable, given that many durables are purchased at a two- to eight-year cycle, meaning that at shorter time-cycle horizons, households may focus more on nondurables and savings.

With this quantitative evidence, we can more clearly reason about how consumption and savings patterns may proceed in relation to inflation expectations as inflationary conditions

[20] I use constant 2017 US dollars.

ease in the United States. Particularly, if inflation, and thus expectations, continues to decrease, we may expect to see a slowing of nondurables consumption and shift to both durables and savings in the short-term, with households ultimately growing their savings over the course of the longer-term business cycle. We may also expect that in the aggregate, these patterns may not be immediately obvious.

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Appendix: Additional descriptive statistics

Table 7 - Descriptive statistics, logarithmic difference

	CPI inflation	Expectations	Nondurables	Durables	Savings
Observations	558	558	558	558	558
Mean	0.29	-0.10	0.41	0.44	0.32
Standard deviation	0.37	13.81	1.14	2.88	15.87
Skewness	-0.13	1.13	-2.87	0.85	0.45
Kurtosis	2.95	34.63	47.23	17.50	17.94
Jarque-Bera	198.36***	27482.91***	51673.53***	7048.12***	7361.04***
Shapiro-Wilk	0.97***	0.75***	0.75***	0.82***	0.72***
Ljung-Box	1048.53***	74.34***	58.10**	62.76**	84.32***

Table 8 - Correlation matrix: Logarithmic differences of series in nominal and real terms

	CPI inflation	Expectations	Nondurables	Durables	Savings	Nondurables (Real)	Durables (Real)	Savings (Real)
CPI inflation	1.0	—	—	—	—	—	—	—
Expectations	0.24***	1.0	—	—	—	—	—	—
Nondurables	0.36***	0.16***	1.0	—	—	—	—	—
Durables	0.0	-0.03	0.36***	1.0	—	—	—	—
Savings	-0.1**	0.04	-0.27***	-0.31***	1.0	—	—	—
Nondurables (Real)	0.03	0.09**	0.94***	0.38***	-0.25***	1.0	—	—
Durables (Real)	-0.13***	-0.06	0.31***	0.99***	-0.3***	0.38***	1.0	—
Savings (Real)	-0.12***	0.03	-0.28***	-0.31***	1.0***	-0.25***	-0.3***	1.0

Table 9 – Non-stationarity of CPI inflation over decades

Decade	Observations	Mean	Standard deviation	Minimum	25%	50%	75%	Maximum
1910	72.00	10.05	7.46	-0.98	2.02	12.56	17.42	20.74
1920	120.00	0.15	7.08	-15.79	-2.24	-0.58	2.38	23.67
1930	120.00	-1.95	5.04	-10.74	-6.45	-0.70	2.20	5.56
1940	120.00	5.66	5.47	-2.87	1.70	3.36	9.29	19.67
1950	120.00	2.07	2.44	-2.08	0.37	1.71	3.12	9.36
1960	120.00	2.33	1.48	0.67	1.31	1.64	3.14	6.20
1970	120.00	7.09	2.72	2.71	5.27	6.53	9.31	13.29
1980	120.00	5.56	3.53	1.10	3.64	4.25	6.46	14.76
1990	120.00	3.00	1.12	1.38	2.46	2.81	3.18	6.29
2000	120.00	2.57	1.44	-2.10	1.96	2.73	3.51	5.60
2010	120.00	1.77	0.86	-0.20	1.24	1.75	2.22	3.87
2020	55.00	4.35	2.56	0.12	2.55	3.67	6.43	9.06

Appendix: Continuous wavelet transforms of series in percentage terms

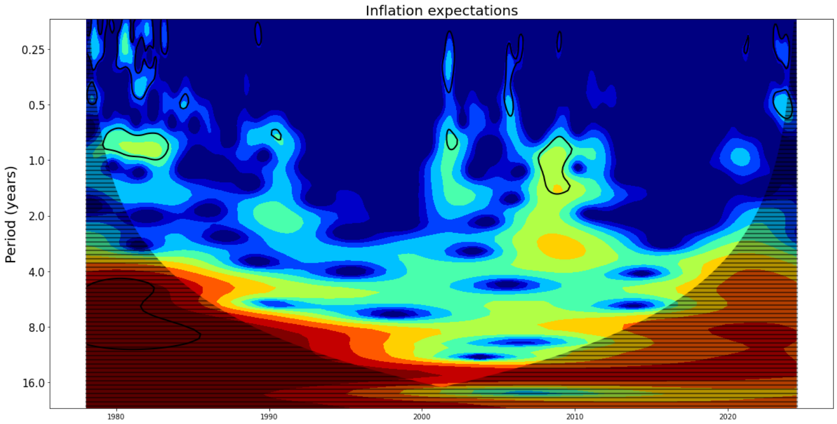

Figure 18 - Power spectrum: Inflation expectations

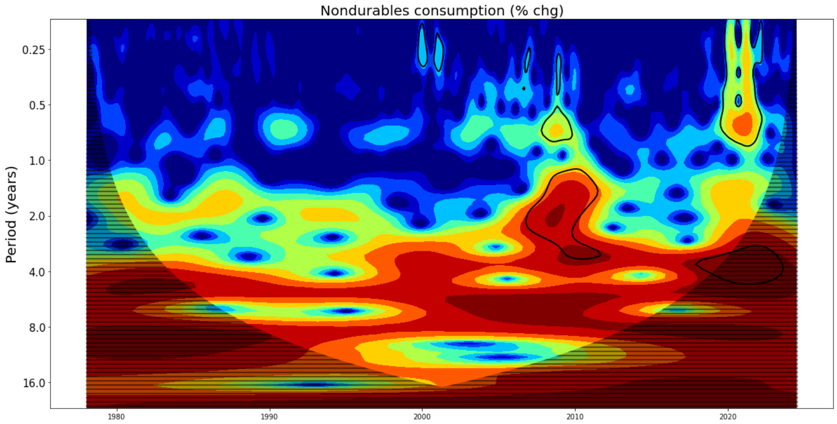

Figure 19 - Power spectrum: Nondurables consumption

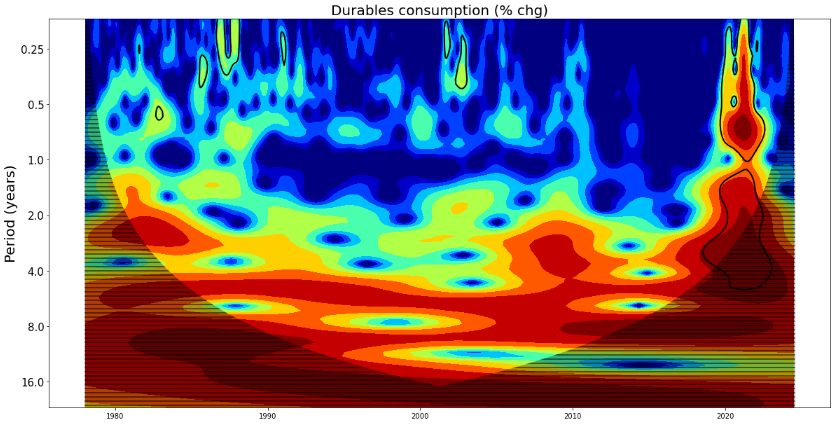

Figure 20 - Power spectrum: Durables consumption

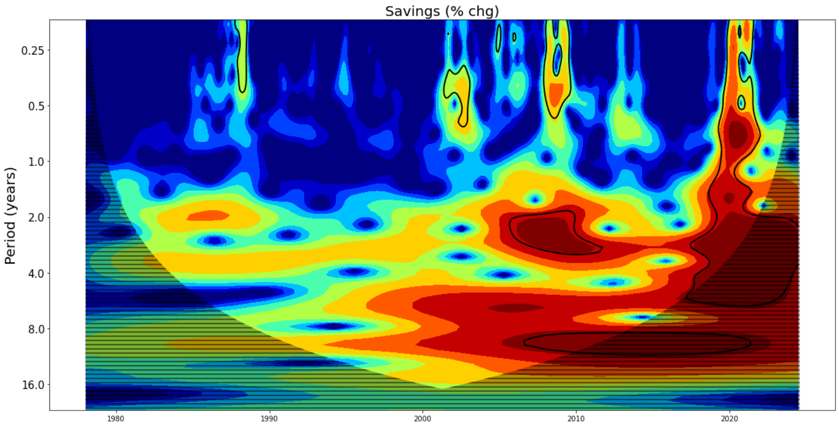

Figure 21 - Power spectrum: Savings

Appendix: Cross-wavelet transforms of series in percentage terms

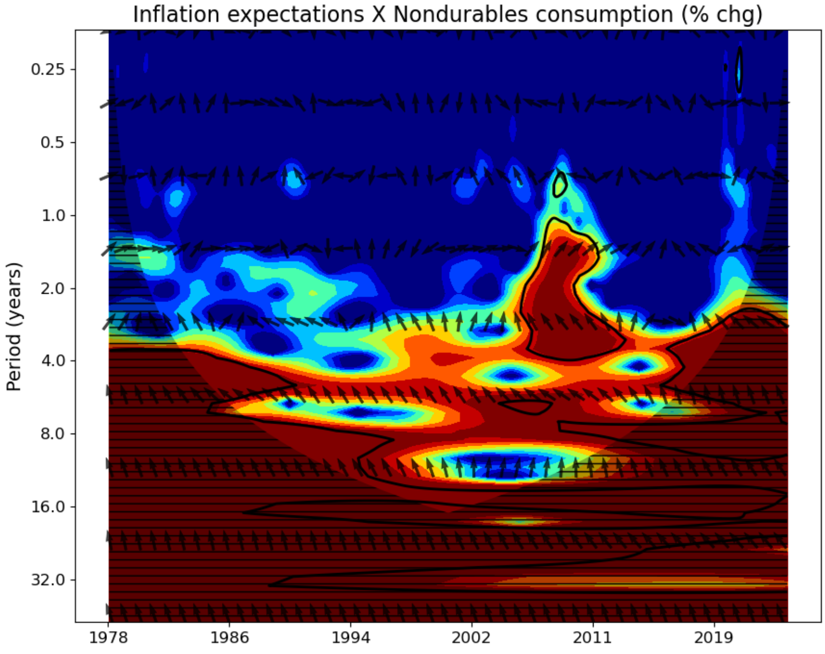

Figure 22 - Cross-wavelet power spectrum: Inflation expectations and nondurables consumption

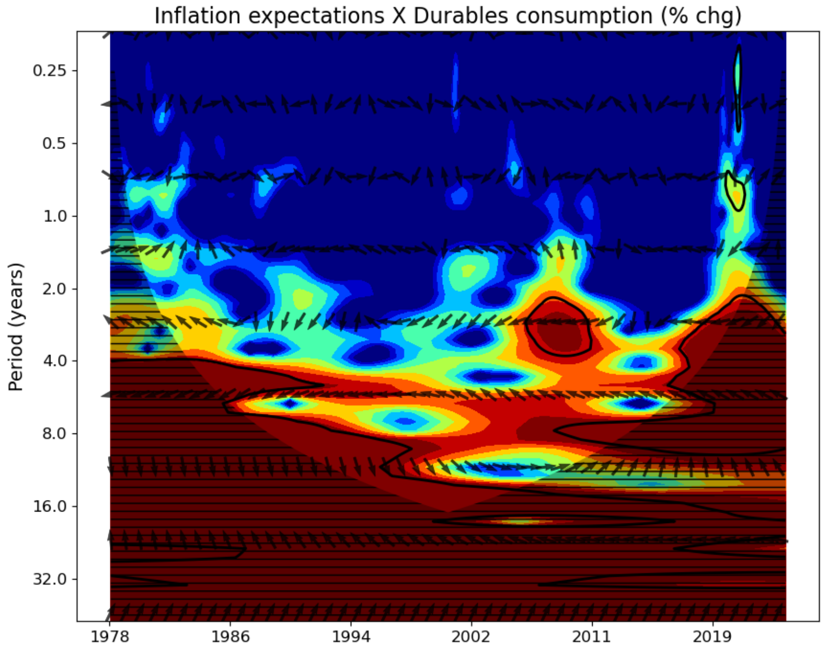

Figure 23 - Cross-wavelet power spectrum: Inflation expectations and durable consumption

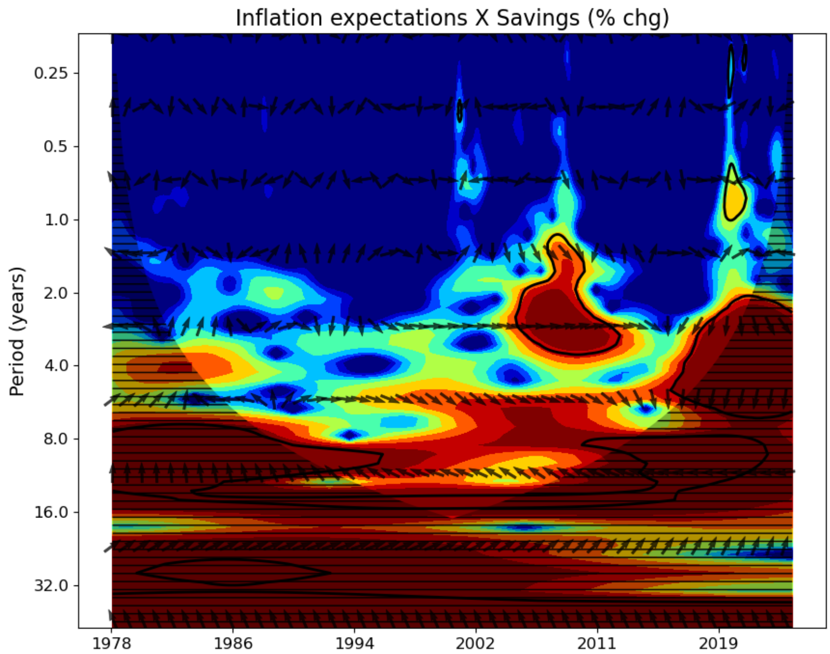

Figure 24 - Cross-wavelet power spectrum: Inflation expectations and savings

Appendix: Regressions in percentage terms

Table 10 - Aggregate OLS regressions: Behavioral series on inflation expectations, percentage

	$y_{non,t}$	$y_{dur,t}$	$y_{sav,t}$
α	-0.2712	5.5920***	14.2253***
	(0.3039)	(0.7459)	(4.3745)
β	1.5009***	-0.0138	-0.9243
	(0.0772)	(0.1896)	(1.1118)
r^2	0.4044	0.0000	0.0012
r^2 Adj.	0.4033	-0.0018	-0.0006

Standard errors in parentheses.

* p<.1, ** p<.05, *** p<.01

Table 11 - Time scale regression: Nondurables consumption on inflation expectations, percentage

	S_6	D_6	D_5	D_4	D_3	D_2	D_1
α_j	0.4030***	-0.0230	-0.0306	-0.0041	-0.0008	-0.0006	0.0000
	(0.1149)	(0.0496)	(0.0581)	(0.0505)	(0.0340)	(0.0272)	(0.0352)
β_j	1.3176***	3.7535***	2.5668***	2.7433***	1.2438***	1.0014***	0.1028
	(0.0299)	(0.1905)	(0.1295)	(0.2227)	(0.1419)	(0.1485)	(0.1744)
r^2	0.7769	0.4111	0.4140	0.2144	0.1214	0.0756	0.0006
r^2 Adj.	0.7765	0.4100	0.4130	0.2130	0.1198	0.0739	-0.0012

Standard errors in parentheses.

* p<.1, ** p<.05, *** p<.01

Table 12 - Time scale regression: Durables consumption on inflation expectations, percentage

	S_6	D_6	D_5	D_4	D_3	D_2	D_1
α_j	4.9316***	-0.0453	-0.0536	-0.0063	-0.0053	-0.0022	0.0002
	(0.3828)	(0.0986)	(0.1286)	(0.1097)	(0.1106)	(0.0931)	(0.0822)
β_j	0.1867*	3.5314***	-1.5062***	-3.5007***	-0.5135	-1.0715**	-0.5427
	(0.0998)	(0.3791)	(0.2865)	(0.4836)	(0.4617)	(0.5077)	(0.4077)
r^2	0.0063	0.1350	0.0473	0.0861	0.0022	0.0079	0.0032
r^2 Adj.	0.0045	0.1334	0.0456	0.0845	0.0004	0.0062	0.0014

Standard errors in parentheses.

* p<.1, ** p<.05, *** p<.01

Table 13 - Time scale regression: Savings on inflation expectations, percentage

	S_6	D_6	D_5	D_4	D_3	D_2	D_1
α_j	3.5985***	0.0506	-0.0257	-0.0005	0.0005	0.0004	-0.0000
	(0.1930)	(0.0311)	(0.0450)	(0.0328)	(0.0378)	(0.0259)	(0.0310)
β_j	0.9834***	-2.2379***	-0.9584***	-1.1126***	0.0391	-0.2479*	0.0372
	(0.0503)	(0.1194)	(0.1003)	(0.1444)	(0.1578)	(0.1413)	(0.1539)
r^2	0.4076	0.3871	0.1411	0.0965	0.0001	0.0055	0.0001
r^2 Adj.	0.4066	0.3860	0.1396	0.0948	-0.0017	0.0037	-0.0017

Standard errors in parentheses.

* p<.1, ** p<.05, *** p<.01

Appendix: Continuous wavelet transforms of series in real terms

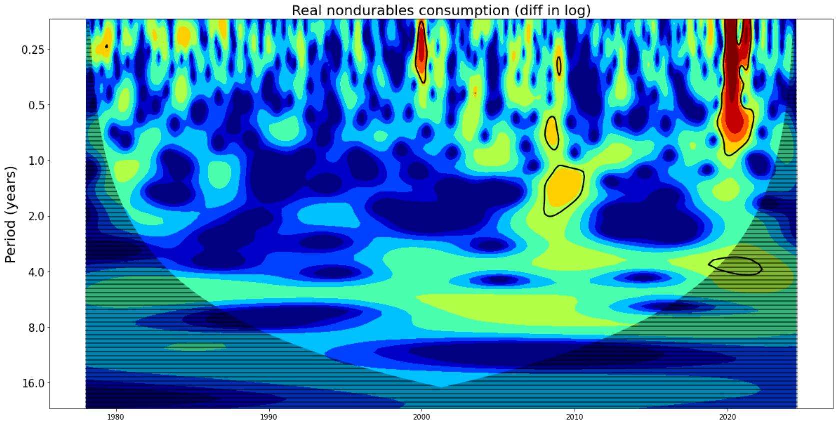

Figure 25 - Real nondurables consumption

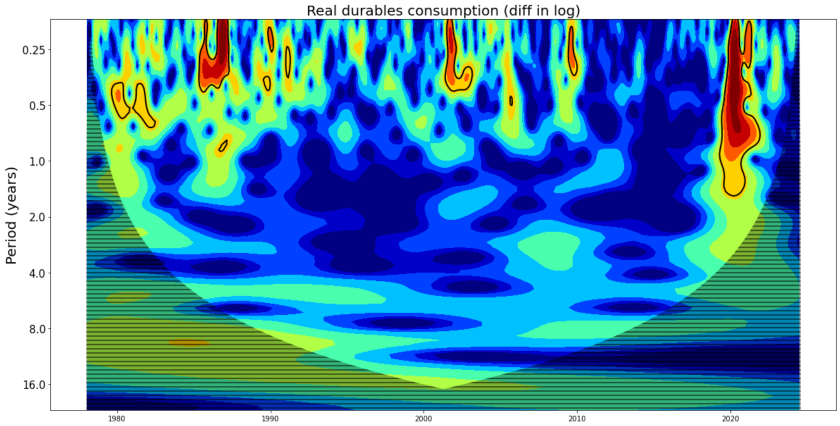

Figure 26 - Power spectrum: Real durables consumption

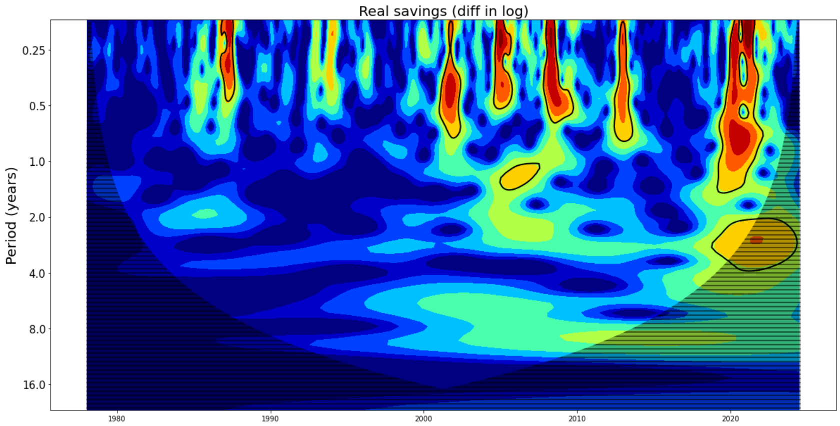

Figure 27 - Power spectrum: Real savings

Appendix: Cross-wavelet transforms of series in real terms

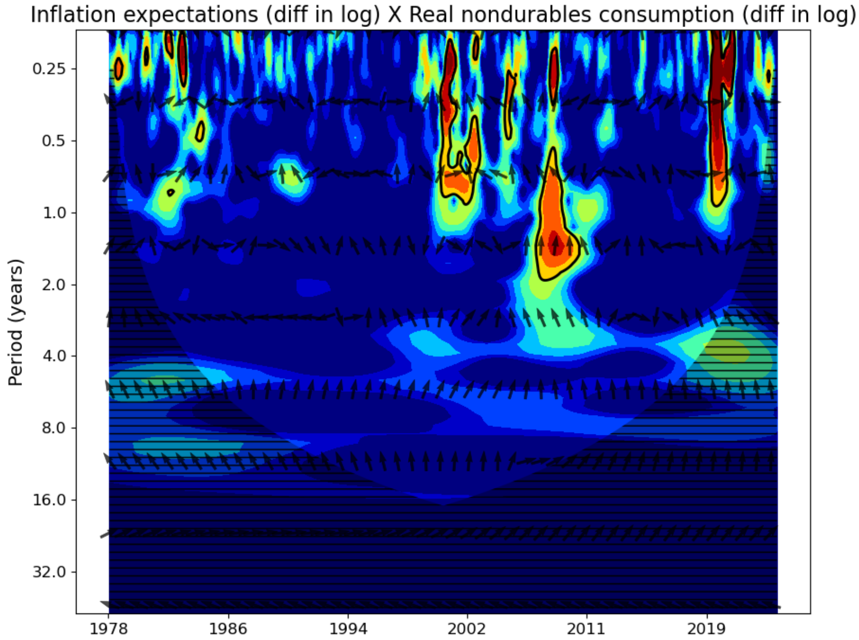

Figure 28 - Cross-wavelet power spectrum: Inflation expectations and real nondurables consumption

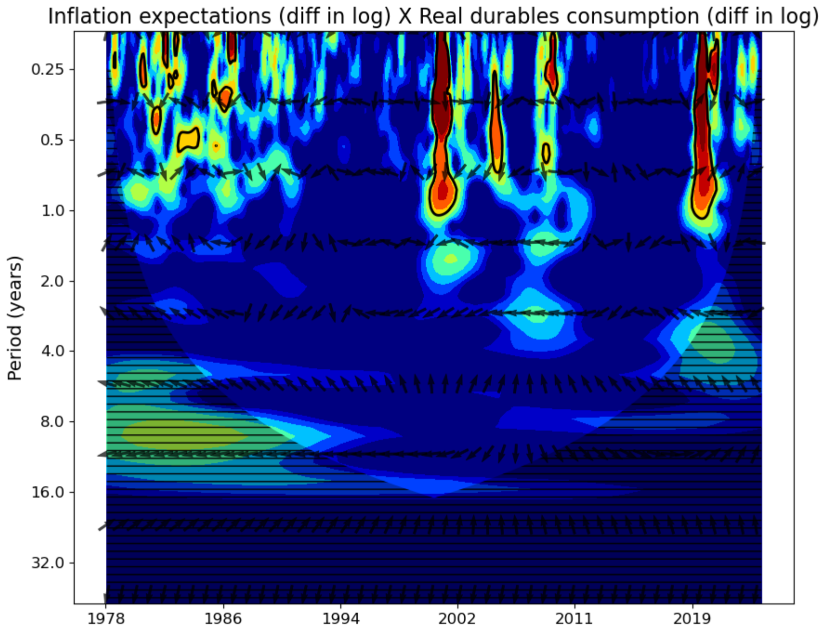

Figure 29 - Cross-wavelet power spectrum: Inflation expectations and real durables consumption

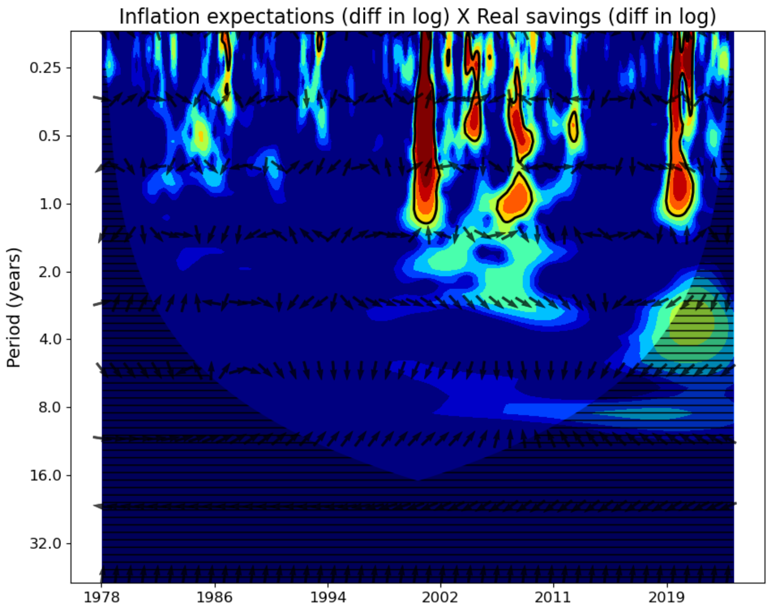

Figure 30 - Cross-wavelet power spectrum: Inflation expectations and real savings

Appendix: Regressions in real terms

Table 14 - Aggregate OLS regressions: Real behavioral series on inflation expectations, logarithmic differences

	$y_{non,t}$	$y_{dur,t}$	$y_{sav,t}$
α	0.1175***	0.1443	0.0293
	(0.0437)	(0.1226)	(0.6741)
β	0.0069**	-0.0125	0.0372
	(0.0032)	(0.0089)	(0.0488)
r^2	0.0085	0.0036	0.0010
r^2 Adj.	0.0067	0.0018	-0.0008

Standard errors in parentheses.

* p<.1, ** p<.05, *** p<.01

Table 15 - Time scale regression: Real nondurables consumption on inflation expectations, logarithmic differences

	S_6	D_6	D_5	D_4	D_3	D_2	D_1
α_j	0.1261***	-0.0004	-0.0004	-0.0035	-0.0007	-0.0001	0.0001
	(0.0026)	(0.0027)	(0.0028)	(0.0053)	(0.0109)	(0.0214)	(0.0349)
β_j	0.0234***	0.0611***	0.1106***	0.0373***	0.0129***	0.0234***	-0.0005
	(0.0066)	(0.0026)	(0.0029)	(0.0024)	(0.0025)	(0.0039)	(0.0030)
r^2	0.0219	0.4918	0.7259	0.2981	0.0455	0.0622	0.0000
r^2 Adj.	0.0201	0.4909	0.7254	0.2968	0.0438	0.0605	-0.0018

Standard errors in parentheses.

* p<.1, ** p<.05, *** p<.01

Table 16 - Time scale regression: Real durables consumption on inflation expectations, logarithmic differences

	S_6	D_6	D_5	D_4	D_3	D_2	D_1
α_j	0.1375***	-0.0103	-0.0055	-0.0048	0.0012	0.0010	-0.0000
	(0.0111)	(0.0080)	(0.0122)	(0.0180)	(0.0264)	(0.0714)	(0.0918)
β_j	-0.2714***	0.0145*	0.0536***	0.0166**	-0.0380***	-0.0214*	-0.0094
	(0.0280)	(0.0078)	(0.0124)	(0.0082)	(0.0061)	(0.0129)	(0.0080)
r^2	0.1447	0.0062	0.0325	0.0073	0.0660	0.0049	0.0025
r^2 Adj.	0.1431	0.0045	0.0308	0.0055	0.0643	0.0032	0.0007

Standard errors in parentheses.

* p<.1, ** p<.05, *** p<.01

Table 17 -Time scale regression: Real savings on inflation expectations, logarithmic differences

	S_6	D_6	D_5	D_4	D_3	D_2	D_1
α_j	-0.0552	0.0032	0.0392	0.0298	-0.0034	-0.0008	-0.0006
	(0.0339)	(0.0542)	(0.0874)	(0.1052)	(0.1818)	(0.3548)	(0.5183)
β_j	0.1675**	-0.3831***	-1.1824***	0.0417	0.0845**	0.1675***	0.0168
	(0.0852)	(0.0525)	(0.0889)	(0.0481)	(0.0418)	(0.0640)	(0.0449)
r^2	0.0069	0.0874	0.2415	0.0013	0.0073	0.0122	0.0003
r^2 Adj.	0.0051	0.0858	0.2401	-0.0004	0.0055	0.0104	-0.0015

Standard errors in parentheses.

* p<.1, ** p<.05, *** p<.01

Appendix: Wavelet analysis of CPI inflation

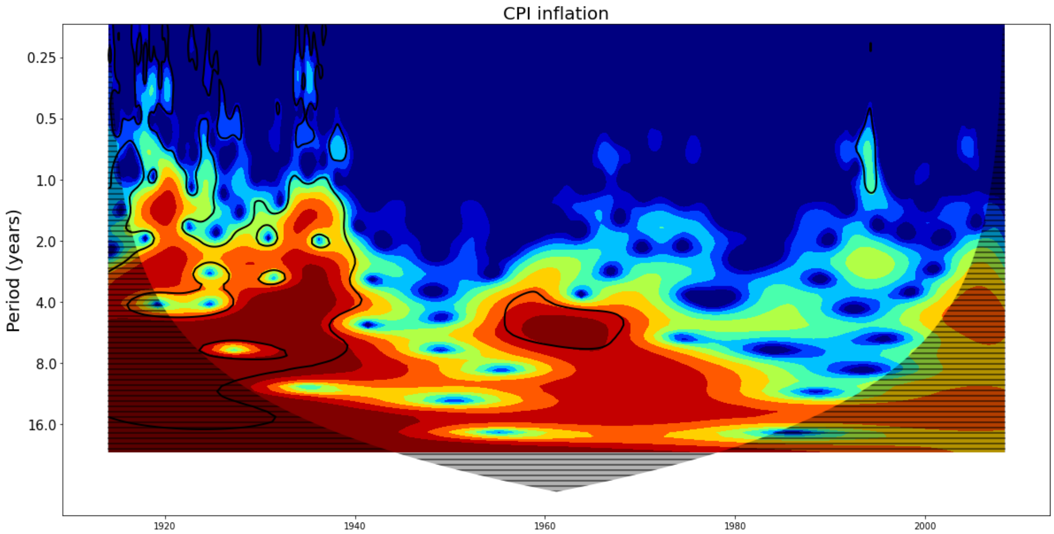

Figure 31 - Power spectrum: CPI inflation (all-time)

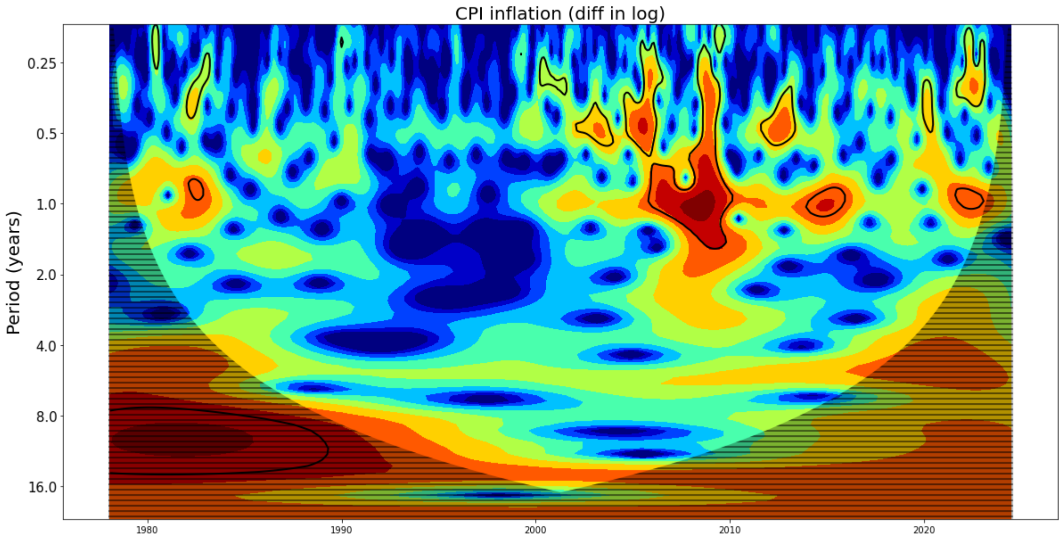

Figure 32 - Power spectrum: CPI inflation