Marital Preferences and Stable Matching in Cultural Evolution

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Abstract

Building on previous literature that examines the influence of intergenerational transmission in cultural evolution, we highlight the importance of the marriage market in the determination of cultural homogeneity ("melting pot") versus heterogeneity ("diversity"). To do so, we characterize cultural evolutionary processes under different distributions of marital preferences and stable matching schemes. In our setting, cultural substitutability (Bisin and Verdier, 2001) is neither sufficient nor necessary for cultural heterogeneity. We introduce a new concept, elasticity of cultural substitution, to capture the degree of increase in vertical socialization efforts in response to minority population decline. With perfect or inelastic vertical transmission in homogamous families, cultural heterogeneity is sustained only if all proposers are homophilic or all members of a cultural group are homophilic. With imperfect vertical transmission in homogamous families, the presence of heterophilic agents may destabilize cultural heterogeneity, and the proportion of heterophilic agents and elasticity of cultural substitution determine whether cultural heterogeneity can be sustained. We discuss the model's implications for the long-lasting impact of temporary gender imbalance on cultural evolution as well as the cultural assimilation and preservation of minorities and immigrants under distinct governmental and religious attitudes toward intermarriage.

Keywords: cultural evolution, cultural integration, cultural diversity, marital preferences, homophily, heterophily, Gale-Shapley matching, intergenerational transmission

JEL Classification: C73; C78; D10; Z10

1   Introduction

Culture plays a crucial role in many economic choices and outcomes at the individual and national level (Landes, 1998; Guiso et al., 2006; Fernández, 2008, 2011; Alesina and Giuliano, 2015). Which cultures or cultural traits survive and thrive and what familial and societal factors influence their success are important questions. A growing literature seeks to disentangle the mechanisms that drive the evolution of culture across generations; see Bisin and Verdier (2011, 2023) for surveys.

Children are primarily cultivated in families. Familial transmission of preferences in cultural evolution has attracted biologists and anthropologists since Cavalli-Sforza and Feldman (1981) and Boyd and Richerson (1985) and economists since Bisin and Verdier (2000, 2001). In this paper, we demonstrate that family formation—what marital preferences individuals possess and how parents are matched—is also of prime importance for our understanding of cultural evolution. Empirical evidence and historical examples point to the important roles of the marriage market and marital preferences in cultural evolution. For instance, cultural norms differ in patriarchal and matriarchal societies (Andersen et al., 2008; Gneezy et al., 2009; Andersen et al., 2013; Giuliano, 2017; Lowes, 2020; Brulé and Gaikwad, 2021; Tène, 2021); factors that affect the respective positions of men and women in the marriage market, such as a temporary gender imbalance, could have a long-run impact on cultural evolution (Grosjean and Khattar, 2019; Gay, 2019; Teso, 2019; Alix-Garcia et al., 2020; Baranov et al., 2021); and government policies and religious practices with respect to intermarriage lead to different cultural assimilation patterns of ethnic minorities and immigrants (Silcock, 1963; Bisin and Verdier, 2000; Skinner, 2008; Dien and Knapp, 2020).

A central question regarding cultural evolution is under what conditions cultural integration and preservation occur. Our paper contributes to this line of inquiry by studying cultural evolution when agents have heterogeneous marital preferences and family formation is determined by stable matching (Gale and Shapley, 1962). Whereas most models in the literature sidestep the marriage market or assume homogeneous marital preferences and exogenous matching, we consider heterogeneous marital preferences and endogenous matching.[1]

The primitives of the Gale-Shapley matching model are sets of men and women and preference orderings of individuals over agents of the opposite sex. These elements are determined in our model as follows. At the beginning of each discrete period, a mass of women and a mass of men become adults. Each adult wants to match with an adult of the opposite sex to form a family. Each participant in the matching market has a cultural trait, acquired during childhood, and marital preferences over the cultural trait of their partner. In addition to the usually considered *homophilic* marital preferences—individuals prefer partners with the same trait the most—we incorporate the possibility that some men and women are *heterophilic*: Individuals prefer partners with a different cultural trait. Stable matching depends on the distribution of cultural traits and the distribution of marital preferences in both populations such that no positive mass of individuals of opposite sexes would both rather have each other than their current mates. in the case of

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[1]As argued by Pollak (2019) and supported by empirical evidence, the Gale-Shapley matching model is the appropriate framework for analyzing marriage-market equilibrium under the assumption that bargaining in marriage determines allocation within marriage. As we will see, the Gale-Shapley model is especially appropriate when agents have heterogeneous marital preferences—e.g., preferences other than homophily—as considered in this paper.

multiplicity, the set of stable matchings forms a lattice, and among them there is one *men-optimal stable matching* (MOSM) and one *women-optimal stable matching* (WOSM), with MOSM (resp., WOSM) being the stable matching the most preferred by men (resp., women) and the least preferred by women (resp., men), achieved by men (resp., women) acting as the proposers through the so-called deferred acceptance (DA) algorithm, which Baïou and Balinski (2002) generalize to the setting with a continuum of agents. We assume that, either MOSM or WOSM is chosen in every period. One can imagine that marriages in a society are arranged through the DA algorithm, and the society either has a convention that only men propose or one that only women propose.

Once matched, each couple has two children, one son and one daughter. Children acquire the cultural trait they will retain when they become adults. Through the cultural transmission process, stable matching in one period will determine the joint distribution of cultural traits among populations of both men and women in the next period. Since *homogamies*—marriages in which spouses have the same cultural trait—have a well-defined cultural trait to transmit, they have a more efficient socialization technology than other families. We consider two transmission technologies of homogamies. We first consider *perfect vertical transmission*: Homogamies transmit this trait to their children with probability one. Then, we consider *imperfect vertical transmission* that is *culturally substitutable* to societal transmission: Homogamies transmit their culture with a probability that is strictly less than one and strictly decreasing in the proportion of that trait in the population. If they fail to transmit their cultural trait, their children are socialized by the society at large (*oblique transmission*): They adopt the trait of a randomly chosen adult role model. Children of *heterogamies*—marriages in which two spouses have different cultural traits—do not have a well-defined familial model to follow, and thus we assume they are socialized by the society at large.

Broadly speaking, we analyze how cultural evolution is influenced by the determinants of stable matching—namely, the distribution of marital preferences and the side of the market favored by the matching procedure—under common transmission technologies. In this regard, we provide a unified and generalizable model to investigate the effects of different forms of matching and intergenerational transmission on cultural evolution. More specifically, we characterize the conditions under which cultural heterogeneity is sustained in the long run, which refines and advances prior results (Cavalli-Sforza and Feldman, 1981; Boyd and Richerson, 1985; Bisin and Verdier, 2000, 2001; Della Lena and Panebianco, 2021). In particular, we identify two key factors that affect the extent of cultural diversity in the stable steady state. First, the fraction of homophilic proposers. Since this fraction influences the distribution of homogamies in the population and homogamies possess a better transmission technology, it crucially shapes cultural evolution and the long-run distribution of traits. Second, our new concept, *elasticity of cultural substitution*, which captures the strength of substitutability between oblique and vertical transmission. It describes the percentage increase in the chance of vertical transmission for the minority group—the crucial source of cultural preservation for minorities—as the minority population decreases. Our main results, listed below, illustrate the interactions between these two factors.

We start by demonstrating that if all proposers are homophilic, the stable steady-state distribution of traits is culturally heterogeneous. This holds true regardless of the cultural transmission technology (with or without cultural substitutability) and regardless of the distribution of receivers' marital preferences.

Indeed, if all proposers are homophilic, as many homogamies as possible are formed. Then, in each cultural trait, the proportion of homogamies is determined by the short side of the marriage market. Since homogamies have a better transmission technology, this creates a tendency toward the equal distribution of traits between the populations of men and women, and the society ends up in a situation in which all marriages are homogamous so that each cultural group can maintain its legacies. We reach exactly the same conclusion when one cultural group is strongly opposed to intermarriage, such that all members of this group (both men and women) are homophilic.

With heterophilic proposers, the long-run distribution of traits crucially depends on the transmission technology under consideration. Under either perfect or inelastic vertical transmission (i.e., in the absence of cultural substitutability), an arbitrarily small fraction of heterophilic proposers is sufficient for cultural diversity to disappear so that cultural homogeneity is the generic long-run outcome. Moreover, the surviving culture is the initially dominant one in population size. This is because although heterophilic preferences facilitate the formation of heterogamies, the majority group manages to preserve a larger fraction of homogamies. Since homogamous couples are more effective in transmitting their traits, this creates an evolutionary advantage for the majority group, which ultimately drives out the minority group.

These results illustrate the crucial role played by the distribution of proposers' marital preferences. For instance, under either perfect or inelastic vertical transmission, diversity is preserved if all proposers are homophilic, but this is not the case if some proposers are heterophilic. This implies that when one side of the matching market is fully homophilic and the other side is not, the matching institution, which determines the side of the market that will play the role of proposers, might have dramatic consequences for the long-run cultural composition of the society. We demonstrate that because of this feature, even though MOSM is the outcome men prefer from a static point of view, this is not necessarily true from a dynamic point of view: Cultural evolution under MOSM (resp., WOSM) might lead to a path considered to be suboptimal by men (resp., women). This result also implies that factors that influence the selection of one particular stable matching (MOSM or WOSM) might have crucial consequences for the long-run distribution of traits. The choice between MOSM and WOSM might also be related to gender imbalance. The distribution of marital preferences of agents on the shorter side of the marriage market, even if they are not proposers, determines the matching outcome. Hence, a slightly skewed gender ratio can result in a dramatic change in the matching outcome and, consequently, the cultural evolution. We introduce gender imbalance in our model and show that a short-run change in gender ratio leads to a long-run change in the cultural distribution; this is in line with empirical findings of long-lasting impacts of temporary gender imbalance (Grosjean and Khattar, 2019; Gay, 2019; Teso, 2019; Alix-Garcia et al., 2020; Baranov et al., 2021).

Under imperfect vertical transmission, cultural substitutability helps maintain cultural diversity except in cases in which either all proposers and/or receivers are heterophilic or an entire cultural group is heterophilic with the other group being nonhomophilic. In other words, as long as there is a positive fraction of homophilic proposers cultural diversity is sustainable in the long run. The extent of cultural diversity (i.e., the size of the minority group) in the stable steady state increases with the fraction of homophilic proposers and the elasticity of cultural substitution. We apply our results to connect intermarriages with cultural assimilation and preservation, and provide examples of how a more open attitude toward inter-

marriages due to government or religion leads to cultural homogeneity and a more restricted attitude leads to cultural heterogeneity (Silcock, 1963; Bisin and Verdier, 2000; Skinner, 2008; Dien and Knapp, 2020).

Most of the cultural evolution literature (Cavalli-Sforza and Feldman, 1981; Boyd and Richerson, 1985; Bisin and Verdier, 2001; Cheung and Wu, 2018) considers asexual reproduction models in which cultural transmission is the result of vertical parental socialization and oblique society socialization. Since a child is socialized by one parent, couple formation does not play a role in cultural evolution. We depart from these foundational models by considering a two-sex cultural transmission model in which marital preferences are heterogeneous and the matching between spouses is endogenous. Whereas the literature highlights the fact that cultural substitutability is key for the preservation of cultural diversity, we show the central role played by the interactions between cultural substitutability and preference for homophily. In particular, cultural heterogeneity may arise even in the absence of cultural substitutability, and cultural homogeneity might be the long-run outcome even in the presence of cultural substitutability. For some matching structures, cultural substitutability is neither necessary nor sufficient for cultural heterogeneity.

Some papers consider socialization by two parents. Bisin and Verdier (2000) propose a cultural transmission model with a marriage market. Individuals might be one of two types and prefer that their children have their trait. Agents must enter a frictional marriage market to marry and reproduce. The marriage market consists of two restricted matching pools exclusive to the two types, respectively, and a common matching pool. Entering a restricted matching pool is costly. The authors assume that homogamous parents enjoy more efficient socialization for their shared type than heterogamous parents. As a result, individuals prefer to and do marry their own type (homophily). They also assume that daughters and sons are socialized in the same way, such that the cultural distribution is the same across gender. In contrast, we propose a two-sex cultural transmission model and allow for heterophilic preferences. These features separate the consideration of marital preferences and the socialization of cultural traits, and allow consideration of the joint cultural evolution of men and women.

Recently, some two-sex cultural evolution models have been developed. Hiller and Baudin (2016) and Baudin and Hiller (2019) propose models in which parents may socialize their sons and daughters differently. However, their analysis considers random matching; the effects of stable matching on the evolution of preferences are not considered. Wu and Zhang (2021) allow for random or assortative matching of spouses, but implicitly assume homophily (because heterophilic individuals are not distinct from homophilic ones when stable matching is not considered). In contrast, we consider stable matching with heterophilic individuals so that there could exist multiple stable matches; as a result, multiple cultural equilibria may arise.

The rest of the paper is organized as follows. Section 2 presents the general setup of the model. Section 3 shows that uniformly homophilic proposers or an entirely homophilic cultural group leads to culturally heterogeneous states. Section 4 shows that under perfect or inelastic vertical transmission in homogamies, even an arbitrarily small fraction of heterophilic proposers is sufficient for cultural homogeneity. Section 4 also presents the implications of having gender-differential distributions of marital preferences. Section 5 presents the results for imperfect vertical transmission with cultural substitutability in homogamies, and Section 6 concludes. Appendices collect omitted proofs and details.

2 Model

There is mass $1 + \lambda^t$ of men (m) and a unit mass of women (w) in every period $t \in \{0, 1, \dots \}$. We assume equal population size ($\lambda^t = 0$) except in Section 4.3.2, which discusses the implications of imbalanced sex ratio. Each person lives for two periods: *childhood* and *adulthood*. Each adult has cultural trait/type a or b. Let p^t denote the mass of type-a men in period t, and q^t the mass of type-a women in period t. In the remainder of the paper, we will frequently use θ to denote a trait/type, i.e., θ equals either a or b.

2.1 Marital preferences

Let $U_{\theta_m \theta_w}$ denote a type-θ_m man's utility from marrying a type-θ_w woman and let $V_{\theta_w \theta_m}$ denote a type-θ_w woman's utility from marrying a type-θ_m man. We assume that for any θ and θ', $U_{\theta \theta'} > 0$ and $V_{\theta \theta'} > 0$, and we normalize the utility from remaining single to 0. This implies that each woman considers each man to be an acceptable match and vice versa. Each individual belongs to one of two marital preference groups: *homophilic* or *heterophilic*. For any types θ and $\theta' \neq \theta$, a type-θ man is *homophilic* if $U_{\theta \theta} > U_{\theta \theta'}$ and *heterophilic* if $U_{\theta \theta} < U_{\theta \theta'}$. Women's marital preferences are similarly defined. We assume that each adult of gender $g \in \{m, w\}$ and cultural type $\theta \in \{a, b\}$ is homophilic with an independent probability denoted by $h_{g\theta} \in [0, 1]$. For a given time t, we define $M^t_{\theta_i}$ as the set of type-θ men having group $i = 1$ (homophilic) or group $i = 2$ (heterophilic) preferences; $W^t_{\theta_i}$ is similarly defined for women.

2.2 Stable matching

In each period, men and women match to form marriage pairs. A type-θ_m husband and a type-θ_w wife form a $\theta_m \theta_w$ couple, which is *homogamous* if $\theta_m = \theta_w$ and *heterogamous* otherwise. Let $\mu^t_{\theta_m \theta_w}$ denote the mass of $\theta_m \theta_w$ couples in period t. A (feasible) matching is described by a 2-by-2 matrix $\mathsf{M}^t = (\mu^t_{\theta_m \theta_w})$ that satisfies $\mu^t_{\theta_m \theta_w} \in \mathbf{R}_+$ for any θ_m and θ_w, and feasibility conditions $\mu^t_{aa} + \mu^t_{ab} = p^t$, $\mu^t_{ba} + \mu^t_{bb} = 1 - p^t$, $\mu^t_{aa} + \mu^t_{ba} = q^t$, and $\mu^t_{ab} + \mu^t_{bb} = 1 - q^t$.[2]

Since we are looking at continuums of individuals with finite types, we adopt the definition of stable matching of Echenique et al. (2013). A type-θ_m man and a type-θ_w woman form a blocking pair for M^t if there exist types $\theta'_m \neq \theta_m$ and $\theta'_w \neq \theta_w$ such that $U_{\theta_m \theta_w} > U_{\theta_m \theta'_w}$, $V_{\theta_w \theta_m} > V_{\theta_w \theta'_m}$, $\mu^t_{\theta_m \theta'_w} > 0$ and $\mu^t_{\theta'_m \theta_w} > 0$. A matching M^t is stable if there are no blocking pairs for it.[3]

We further assume that men and women match according to either *men-optimal stable matching* (*MOSM*)—the stable matching most preferred by men—or *women-optimal stable matching* (*WOSM*). Baïou and Balinski (2002) show the existence of these two matchings for continuous populations with finite types.

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[2]If there are any singles, the masses of singles who possess different cultural traits can be backed out from the matching matrix. Note that since we are interested in the joint evolution of p^t and q^t, and since—as will become clear—this evolution depends on the composition of couples in terms of cultural types, we define a matching in terms of only those types rather than in terms of both cultural types and preference groups.

[3]A stable matching must also be individually rational, which means that no individual should prefer remaining single to retaining their match. This is always the case given our earlier assumption, since the utility derived from celibacy is normalized to 0 and the utility of a match is strictly positive. This assumption, along with the fact that there are as many women as men, implies that at any stable matching, all individuals are matched. In Section 4.3.2 we study the setting in which the two populations are not balanced, so that some individuals remain single.

MOSM (resp., WOSM) can be achieved by the generalized Gale-Shapley deferred acceptance algorithm if men (resp., women) are proposers. Hence, we refer to the side that has implemented their most preferred stable matching as the proposing side and to the opposite side as the receiving side.

2.3   Intergenerational transmission

Each couple has two children, one son and one daughter; equivalently, each child is male or female with equal probabilities. Children are born without a cultural trait and will acquire, during childhood, a trait they will hold during their entire adulthood. As is usual in the literature, the cultural transmission process entails two steps. Children are first socialized by their parents. If this stage of *vertical transmission* fails, children are socialized by the society at large in the second stage of *oblique transmission*. We assume that only parents from homogamies can directly transmit their culture during the vertical transmission stage. Or, stated differently, the probability of vertical transmission by heterogamous couples is zero: All heterogamies have oblique transmission.[4] Since heterogamous couples do not have a well-defined cultural type to transmit, it is natural to assume that a homogamy has a more superior transmission technology than does a heterogamy. Bisin and Verdier (2000) and Hiller and Baudin (2016) consider this transmission technology, for example, and Dohmen et al. (2012) provide empirical support. Hence, the probability that a child adopts a particular cultural type depends on the cultural types of both parents. We let $P_{\theta\theta'}^t$ and $Q_{\theta\theta'}^t$ denote the probability that the son and daughter of trait-θ father and trait-θ' mother possess trait a at date $t + 1$, respectively. Their expressions depend on the vertical transmission process in homogamies. Note that although we only explicitly model the transmission of cultural traits, marital preferences might also be transmitted across generations over time because marital preferences are correlated with cultural traits in our model.[5]

Oblique transmission in heterogamies.   Children of heterogamous couples acquire their trait during the oblique socialization stage: Each child randomly searches for a role model in their respective gender and adopts the cultural type of this role model with probability one. This corresponds to the benchmark assumption in the literature (Cavalli-Sforza and Feldman, 1981; Boyd and Richerson, 1985; Bisin and Verdier, 2001, 2011). This assumption involves a simple conformist component in cultural evolution.

Perfect vertical transmission in homogamies.   Homogamous couples transmit their cultural type to their children with probability one. In contrast, as previously discussed, the probability of vertical transmission is zero for heterogamous couples and their children are directly subject to societal transmission. The combination of perfect vertical transmission in homogamies and oblique transmission in heterogamies can be seen as a relaxation of the first generation of cultural transmission models (Cavalli-Sforza and Feldman, 1981; Boyd and Richerson, 1985). In those models, the vertical transmission probability is exogenous and constant. Instead, we consider the setting in which this probability depends on the homogamous or

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[4]This assumption is relaxed when we consider an alternative transmission technology for heterogamies, imitative logit, in an earlier version of the paper (Hiller et al., 2021). The current paper focuses on cultural homogeneity versus cultural heterogeneity when all heterogamies have oblique transmission.

[5]For instance, if $h_{ma} > h_{mb}$ and $h_{wa} > h_{wb}$, culture a is less tolerant of intermarriage than culture b.

heterogamous type of couple—but, for a given type of couple, it is exogenous and constant. In summary, transmission probabilities are

$$P_{aa}^t = Q_{aa}^t = 1, \quad P_{bb}^t = Q_{bb}^t = 0, \quad P_{ab}^t = P_{ba}^t = p^t, \quad Q_{ab}^t = Q_{ba}^t = q^t.$$

Imperfect vertical transmission in homogamies.   We relax the perfect vertical transmission assumptions in homogamous couples such that for these couples, the probability of direct transmission of the parental trait is not always one. Let this vertical transmission probability be $d(r)$, where r is the mass of individuals of the same gender and type, and $1 - d(r)$ is the probability that the transmission is oblique. We assume that $d(r)$ is continuous, differentiable, and decreasing in r. This captures the *cultural substitutabil-ity* between vertical and oblique transmission (Bisin and Verdier, 2001). That is, the vertical transmission probability of a trait is higher if there is a lower proportion of the trait. This property can be rationalized in a model in which homogamies have the option to make costly effort to transmit their traits and exhibit a form of cultural intolerance, as in Bisin and Verdier (2001).[6] In such kind of model, when r is high, parents anticipate that the probability for their children to adopt their own cultural traits, if they fail to transmit it directly, will be high. Consequently, they will have a lower incentive to exert efforts to directly transmit their traits, making the probability of direct transmission $d(r)$ low.

As we will see, the strength of cultural substitutability plays a critical role in the possibility of persistence of cultural diversity. We introduce the notion of *elasticity of cultural substitution* $\varepsilon_d(r) = -rd'(r)/d(r)$ to account for this strength. In the limit case in which the probability of direct transmission is constant, $d(r) = d \in (0, 1)$, we have $\varepsilon_d(r) = 0$. This corresponds to the assumption made in the first generation of cultural transmission models (Cavalli-Sforza and Feldman, 1981; Boyd and Richerson, 1985). We refer to this case as *inelastic transmission.* Note that *perfect transmission* might be viewed as a limit case of *inelastic transmission* in which $d = 1$. When $d'(r) < 0$, $\varepsilon_d(r) > 0$ and we have cultural subsitutability, which is at the heart of the second generation of cultural transmission models (Bisin and Verdier, 2001).[7]

Transmission probabilities among heterogamies hold unchanged:

$$P_{ab}^t = P_{ba}^t = p^t, \quad Q_{ab}^t = Q_{ba}^t = q^t,$$

and transmission probabilities among homogamies become

$$P_{aa}^t = d(p^t) + (1 - d(p^t))p^t, \quad Q_{aa}^t = d(q^t) + (1 - d(q^t))q^t,$$
$$P_{bb}^t = (1 - d(1 - p^t))p^t, \quad Q_{bb}^t = (1 - d(1 - q^t))q^t.$$

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[6]We present a simple derivation of this result in Appendix A. Note that Bisin and Verdier (2001) also provide examples of transmission technologies that exhibit some degree of complementarity between vertical and oblique transmission. However, since cultural complementarity mechanically induces cultural homogenization, most of the subsequent literature has adopted the cultural substitutability assumption.

[7]As shown in Appendix A, within the framework of those models, the *elasticity of cultural substitution* is decreasing in the elasiticity of the parental socialization costs function. Put differently, if parents can increase the direct transmission probability without increasing the socialization costs too much, they will more easily adjust d in reaction to a change in r. Hence, the *elasticity of cultural substitution* will be higher.

For example, a boy from an *aa* family directly receives trait *a* from his parents with probability $d(p^t)$ and, if this vertical transmission fails, he has probability p^t to be influenced by a type-*a* role model. This is why $P_{aa}^t = d(p^t) + (1 - d(p^t))p^t$. A boy from a *bb* family will acquire trait *a* if the vertical socialization stage fails, which occurs with probability $1 - d(1 - p^t)$, and he picks a type-*a* role model. Hence, $P_{bb}^t = (1 - d(1 - p^t))p^t$. Q_{aa}^t and Q_{bb}^t are similarly obtained.

2.4 Cultural evolution and steady states

The distribution of traits in a period depends on the proportion of families of different pairs of traits and the intergenerational transmission in different types of families. Generally, cultural evolution is characterized by the following system of equations:

$$p^{t+1} = \mu_{aa}^t P_{aa}^t + \mu_{ab}^t P_{ab}^t + \mu_{ba}^t P_{ba}^t + \mu_{bb}^t P_{bb}^t; \tag{1}$$

$$q^{t+1} = \mu_{aa}^t Q_{aa}^t + \mu_{ab}^t Q_{ab}^t + \mu_{ba}^t Q_{ba}^t + \mu_{bb}^t Q_{bb}^t. \tag{2}$$

This system clearly demonstrates that cultural evolution is determined by intergenerational transmission ($P_{\theta\theta'}^t$ and $Q_{\theta\theta'}^t$) and matching ($\mu_{\theta\theta'}^t$), which depend on underlying marital preference and trait distributions as well as the stable matching scheme.

We consider the evolutionary outcome from any interior initial state $(p^0, q^0) \in (0,1)^2$ as $t \to \infty$. When the limits exist, we denote the steady-state masses of type-*a* men and women by $p^* = \lim_{t\to\infty} p^t$ and $q^* = \lim_{t\to\infty} q^t$, respectively. The steady state may not be unique and may depend on the initial state. Below we provide a definition of a stable and an attracting steady state.

Definition 1. *Steady state (p^*, q^*) is* stable *if for all $\varepsilon > 0$, there exists $\delta_\varepsilon > 0$ such that $E((p^0, q^0), (p^*, q^*)) < \delta_\varepsilon$ implies $E((p^t, q^t), (p^*, q^*)) < \varepsilon$ for all $t > 0$, where $E(x, y)$ indicates the Euclidean distance between points x and y. A steady state is* unstable *if it is not stable. Steady state (p^*, q^*) is* attracting *if there exists $\eta > 0$, such that $E((p^0, q^0), (p^*, q^*)) < \eta$ implies $\lim_{t\to\infty}(p^t, q^t) = (p^*, q^*)$. It is* globally attracting *if $\eta = \infty$. A steady state is* asymptotically stable *if it is stable and attracting. A steady state is* globally asymptotically stable *if it is stable and globally attracting; when such a steady state exists, it is the unique steady state. Stable set $\mathcal{S}$ of steady states is a nonsingleton connected set of steady states such that there exists an open neighborhood of the set (in $\mathbb{R}^2$), $\mathcal{N} \supseteq \mathcal{S}$, such that for any initial state $(p, q) \in \mathcal{N} \cap [0,1]^2$, the steady state reached is in the set $\mathcal{S}$.*

3 Special case: Homophilic proposers

We start by characterizing the long-run joint distribution of cultural traits when all agents on the proposing side of the market are homophilic, which might be viewed as a natural assumption at first sight. Without loss of generality, we assume that men are proposers so that $h_{ma} = h_{mb} = 1$. We do not make any assumption on the distribution of preferences among receivers; we consider any $(h_{wa}, h_{wb}) \in [0,1]^2$. We also show that when all members of a given culture are homophilic ($h_{ma} = h_{wa} = 1$ or $h_{mb} = h_{wb} = 1$), the

unique stable matching is identical to the matching obtained when all proposers are homophilic. Hence, our results also apply to this case.

After a brief description of stable matching (Section 3.1), we address the case of perfect or inelastic vertical transmission in Section 3.2.[8]

3.1 Stable matching

Suppose all men have homophilic preferences: $U_{\theta\theta} > U_{\theta\theta'}$ for any θ and $\theta' \neq \theta$ and consider MOSM (i.e., men are the proposers in the stable matching algorithm).[9] MOSM is given by mass q^t of aa couples, mass $1 - p^t$ of bb couples, and mass $p^t - q^t$ of ab couples if $p^t \geqslant q^t$ (Figure 1a); mass p^t of aa couples and mass $1 - p^t$ of bb couples if $p^t = q^t$ (Figure 1b); and mass p^t of aa couples, mass $1 - q^t$ of bb couples, and mass $q^t - p^t$ of ba couples if $p^t < q^t$ (Figure 1c). This matching pattern is independent of women's preference distribution, as long as they find every man to be acceptable.
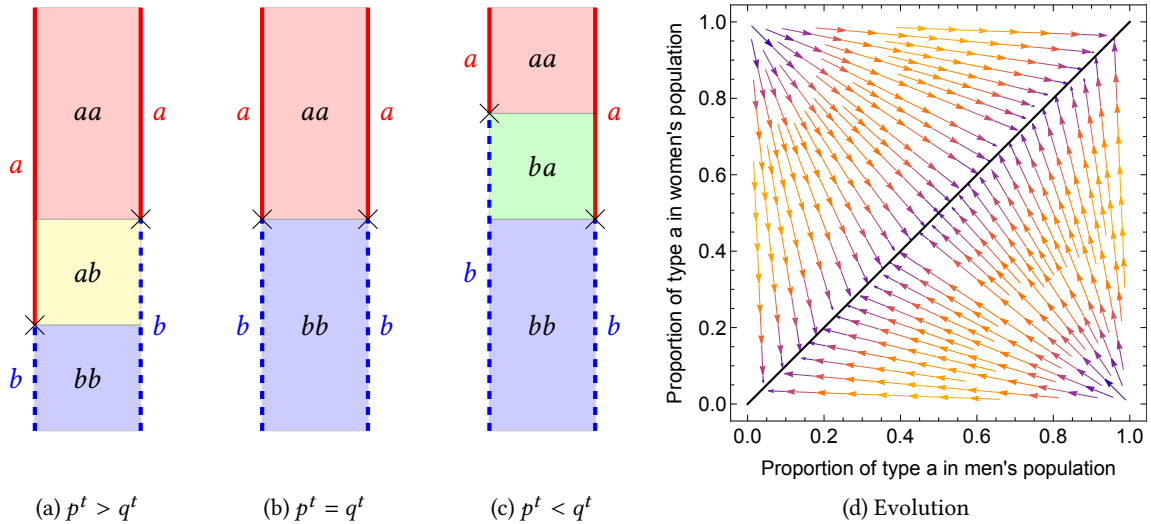

| (a) $p^t > q^t$ | (b) $p^t = q^t$ | (c) $p^t < q^t$ | (d) Evolution |

Figure 1: Matching and evolution with homophilic proposers under perfect or inelastic vertical transmission in homogamies

3.2 Cultural evolution

Substituting the values of $\mu_{\theta\theta'}^t$ that correspond to this stable matching and the values of $P_{\theta\theta'}^t$ and $Q_{\theta\theta'}^t$ that correspond to the benchmark transmission into equations (1) and (2), we get the cultural evolution when $p^t \geqslant q^t$ is characterized by the following dynamic system:

$$p^{t+1} = q^t + (p^t - q^t)p^t; \tag{3}$$

$$q^{t+1} = q^t + (p^t - q^t)q^t. \tag{4}$$

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[8]The case of imperfect vertical transmission with cultural substitutability is adressed in Section 5.1.
[9]Symmetrically, we could assume that women have homophilic preferences and are the proposers without any consequences for our results.

Here are three observations of the dynamic system. First, from equation (3), $p^{t+1} \leqslant p^t$ for any $p^t \in (0,1]$. The equality holds only when either $p^t = 1$ or $p^t = q^t$ (or both). Second, from equation (4), $q^{t+1} \geqslant q^t$ for any $q^t \in [0,1)$. The equality holds only when either $q^t = 0$ or $p^t = q^t$ (or both). Third, from equations (3) and (4), $p^{t+1} \geqslant q^{t+1}$ for any $0 \leqslant q^t \leqslant p^t \leqslant 1$. The equality holds only when $p^t = q^t$. Hence, for any initial state (p^0, q^0) that satisfies $0 \leqslant q^0 \leqslant p^0 \leqslant 1$ with either the first or the last inequality being strict or both (the southeast triangle in Figure 1d without the point $(1,0)$), $\lim_{t \to \infty} p^t = \lim_{t \to \infty} q^t = r$, for some $r \in (0,1)$. The dynamic system in the case of $p^t < q^t$ is symmetric to that in the case of $p^t > q^t$ (the northwest triangle in Figure 1d without the point $(0,1)$). Note that $\{(r,r)|r \in (0,1)\}$ constitutes a stable set of steady states in the sense that perturbations from a steady state in this set may result in a different steady state, but the new steady state falls in the set of steady states. These observations lead to the following characterization of steady states when all proposers are homophilic.

Lemma 1. *Suppose all proposers are homophilic and vertical transmission is perfect in homogamies. For any interior initial state (p^0, q^0), cultural evolution (p^t, q^t) converges to (r,r) for some $r \in (0,1)$; in other words, the set of steady states, $\{(r,r)|r \in (0,1)\}$, forms a stable set.*

Lemma 1 demonstrates that the dynamic system converges to steady states in which both types coexist in both genders and the distributions of types are balanced across genders. Hence, we have cultural heterogeneity (coexistence of both cultures) as the long-run outcome when all proposers are homophilic. The rationale is as follows. With homophilic proposers, the proportion of homogamies in each trait is determined by the short side of the marriage market (see Figure 1). This creates a tendency toward a balanced cultural distribution between sexes, which guarantees that people will maintain their legacies through homogamies. However, there is no tension between the two types in cultural transmission. Therefore, the dynamic can reach any steady state with a balanced sex ratio in traits.

In addition, consider the case in which one cultural group is homophilic. All type-a members—men and women—are homophilic: $h_{ma} = h_{wa} = 1$. We do not make specific assumptions regarding the preference distribution of trait b: $(h_{mb}, h_{wb}) \in [0,1]^2$. This configuration represents the case in which a culture or religion (without loss of generality, trait a) strongly opposes intermarriage. The unique stable matching is identical to the one obtained when all proposers are homophilic (see Section 3.1). To see this, it is sufficient to note that since all type-a men and women are homophilic, in any stable matching as many aa couples as possible must be formed such that the mass of aa couples is $\min\{p^t, q^t\}$. In such a case, the long-run steady state will be characterized by cultural heterogeneity (see Lemma 1).

Lemma 2. *Suppose vertical transmission is perfect in homogamies. If all members of at least one cultural group are homophilic, for any interior initial state, cultural evolution (p^t, q^t) converges to (r,r) for some $r \in (0,1)$; in other words, $\{(r,r)|r \in (0,1)\}$ forms a stable set.*

Finally, consider that vertical transmission in homogamies is imperfect (the probability that parents directly transmit their trait is lower than one) but inelastic (this probability is independent of the distribution of traits), i.e., $d(r) = d \in (0,1]$. Results obtained under perfect vertical transmission generalize to this case. The following proposition offers the most general claim.

Proposition 1. *Suppose vertical transmission is either perfect or inelastic in homogamies. If all proposers are homophilic or all members of one cultural group are homophilic, $\{(r, r)|r \in (0, 1)\}$ forms a stable set.*

Hence, when the proportion of homogamies in each trait is determined by the short side of the market, even a small advantage for homogamies in the transmission process (a small d) is sufficient to guarantee the coexistence of the two cultural groups at the stable steady state.[10]

4 Perfect or inelastic vertical transmission in homogamies

In this section, we relax the assumption that all proposers are homophilic (or all members of a given culture are homophilic). To identify the consequences of having heterophilic proposers, we will first use the extreme case in which all proposers are heterophilic (Section 4.1). Then, we use generic marital preference distributions to demonstrate that cultural homogeneity is the generic long-run outcome as long as a fraction of proposers (even small) is heterophilic (Section 4.2). Finally, we use this result to discuss in Section 4.3 the consequences of having an unbalanced distribution of marital preferences between sexes.

4.1 Heterophilic proposers

Suppose men have heterophilic preferences that favor heterogamies: $U_{\theta\theta} < U_{\theta\theta'}$ for any θ and $\theta' \neq \theta$. Again, we consider MOSM, which is independent of women's preference distribution. Stable matching is given by mass $1 - p^t$ of ba couples, mass $1 - q^t$ of ab couples, and mass $p^t + q^t - 1$ of aa couples if $p^t + q^t > 1$ (Figure 2a); mass p^t of ab couples and mass q^t of ba couples if $p^t + q^t = 1$ (Figure 2b); and mass p^t of ab couples, mass q^t of ba couples, and mass $1 - p^t - q^t$ of bb couples if $p^t + q^t < 1$ (Figure 2c).

(a) $p^t + q^t > 1$ (b) $p^t + q^t = 1$ (c) $p^t + q^t < 1$ (d) Evolution
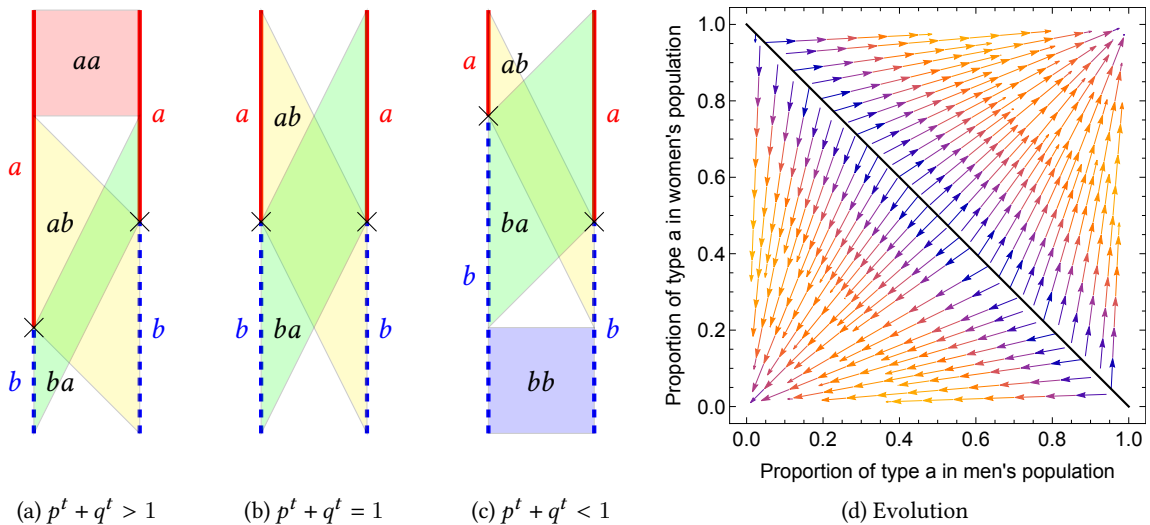
Figure 2: Matching and evolution with heterophilic proposers under perfect vertical transmission in homogamies

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[10]In Section 5.1 we show that this remains true under imperfect vertical transmission with cultural substitutability.

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For illustrative purposes, we assume perfect vertical transmission. Figure 2d depicts the cultural evolution. When $p^t + q^t > 1$, it is characterized by

$$p^{t+1} = (p^t + q^t - 1) + (2 - p^t - q^t)p^t; \tag{5}$$

$$q^{t+1} = (p^t + q^t - 1) + (2 - p^t - q^t)q^t. \tag{6}$$

Rearrange the equations:

$$p^{t+1} = p^t + (1 - p^t - q^t)(p^t - 1); \tag{7}$$

$$q^{t+1} = q^t + (1 - p^t - q^t)(q^t - 1). \tag{8}$$

Observe that (i) $p^{t+1} \geqslant p^t$, and the equality holds only when $p^t = 1$; (ii) $q^{t+1} \geqslant q^t$, and the equality holds only when $q^t = 1$. Hence, $\lim_{t\to\infty} p^t = \lim_{t\to\infty} q^t = 1$ for any initial state (p^0, q^0) that satisfies $p^0 + q^0 > 1$. When $p^t + q^t = 1$, the dynamic system is always in a steady state. The dynamic system in the case of $p^t + q^t < 1$ (the southwestern triangle in Figure 2d) is given by

$$p^{t+1} = (p^t + q^t)p^t; \tag{9}$$

$$q^{t+1} = (p^t + q^t)q^t. \tag{10}$$

It converges to $(0, 0)$.

Proposition 2. *Suppose vertical transmission is either perfect or inelastic in homogamies. If all proposers are heterophilic, both $(0,0)$ and $(1,1)$ are asymptotically stable. Cultural evolution (p^t, q^t) converges to $(0,0)$ if $p^0 + q^0 < 1$, (p^0, q^0) if $p^0 + q^0 = 1$, and $(1,1)$ if $p^0 + q^0 > 1$.*

Proposition 2 shows that with heterophilic proposers, in the long run, the entire society consists of only one type and cultural homogeneity is the robust long-run phenomenon. Importantly, this result holds regardless of the distribution of receivers' preferences. Hence, in the statement of Proposition 2, we do not specify the distribution of receivers' preferences.[11] The result directly comes from the nature of the stable matching. As shown in Figure 2, when all proposers are heterophilic, as many heterogamous couples as possible are formed. Hence, only a residual fraction of couples are homogamies and those homogamies are composed of spouses who belong to the majority culture.[12] Since homogamies have a better cultural transmission technology than heterogamies, this induces an evolutionary advantage for the majority. Hence, in the long run, the majority drives out the minority because the majority type manages to keep a fraction of homogamies due to sheer population size.

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[11]Note that the result also does not depend on the transmission technology in homogamies. In the proof of Proposition 2, the case with imperfect vertical transmission with cultural substitutability is also considered. For the general discussion on imperfect vertical transmission, please refer to Section 5.

[12]Type a is categorized as the majority type if $p^t + q^t > 1$. When this is the case, all homogamies are aa couples (see Figure 2).

4.2 Strict mixture of heterophilic and homophilic proposers

Now, we provide the most general setup by allowing both the populations of men and women to have a strict mixture of homophilic and heterophilic preferences. We allow the proportion of homophilic preferences to differ by cultural type. However, for the sake of clarity and, in particular, to limit the number of parametric configurations to be considered, we make the following assumption. Our subsequent characterization suggests that this assumption is not crucial for the result of long-run cultural homogeneity, but is made mainly to simplify exposition.

Assumption 1. *For $(h_{ma}, h_{mb}, h_{wa}, h_{wb}) \in (0,1)^4$,*

$$\frac{h_{wb}}{h_{wa}} = \frac{h_{mb}}{h_{ma}} =: \tau \in (0,1]; \tag{11}$$

$$\frac{h_{wb}}{h_{mb}} = \frac{h_{wa}}{h_{ma}} =: \rho \in (0,1]. \tag{12}$$

According to equation (11), the ratio of the probabilities of being homophilic conditional on belonging to cultural group a rather than b is independent of gender. In a similar spirit, equation (12) implies that the ratio of the probabilities of being homophilic conditional on being a woman rather than a man is independent of cultural trait. The assumptions $\tau \leqslant 1$ and $\rho \leqslant 1$ are made without loss of generality. They imply that, all else equal, the probability to be homophilic is higher for men and for individuals who belong to group a (culture a is less tolerant of intermarriage than culture b).

The full characterization of stable matching (into nine cases) is relegated to Appendix C.2. In Figure 3, we provide a graphic description of the stable matching for the special case in which all receivers are homophilic ($h_{wa} = h_{wb} = 1$). This case illustrates important (and general) features of the matching when the population of proposers is mixed. For very unbalanced cultural distribution between sexes—cases (a) and (e)—the stable matching looks like the one obtained when all proposers are homophilic (Figure 1) such that the mass of homogamies is determined by the short side of the market in each trait. However, for more balanced cultural distributions—cases (b) and (d)—the majority manages to preserve a relatively larger fraction of homogamies, as in the case in which all proposers are heterophilic (Figure 2a and 2c).[13] As we will see, this creates an evolutionary advantage for the majority and this advantage (even if it is small) will be sufficient for the majority to drive out the minority.

Figures 4a and 4b illustrate the phase diagrams for the cultural evolution under MOSM and WOSM, respectively. In Appendix C.3, we provide the formal construction of the phase diagrams. In these figures, the pp and qq curves, respectively, correspond to the stationary locus of p^t and q^t. As shown in these figures—and Proposition 3 states that this is a general result—under either matching scheme, we have two stable steady states $(0,0)$ and $(1,1)$, and one unstable steady state $(1/2, 1/2)$. By extension, even if we allow for a selection of stable matching between MOSM and WOSM (for example, by median stable matching), cultural evolution retains the same stable steady states.

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[13]For instance, in case (b) there are relatively few individuals of type a in the whole population such that some heterophilic b men will be matched with type b women.

(a) $q \in [0, \pi_{a_1}]$ (b) $q = (\pi_{a_1}, \pi_{a_1} + \pi_{b_2})$ (c) $q = \pi_{a_1} + \pi_{b_2}$ (d) $q \in (\pi_{a_1} + \pi_{b_2}, 1 - \pi_{b_1})$ (e) $q \in [1 - \pi_{b_1}, 1]$
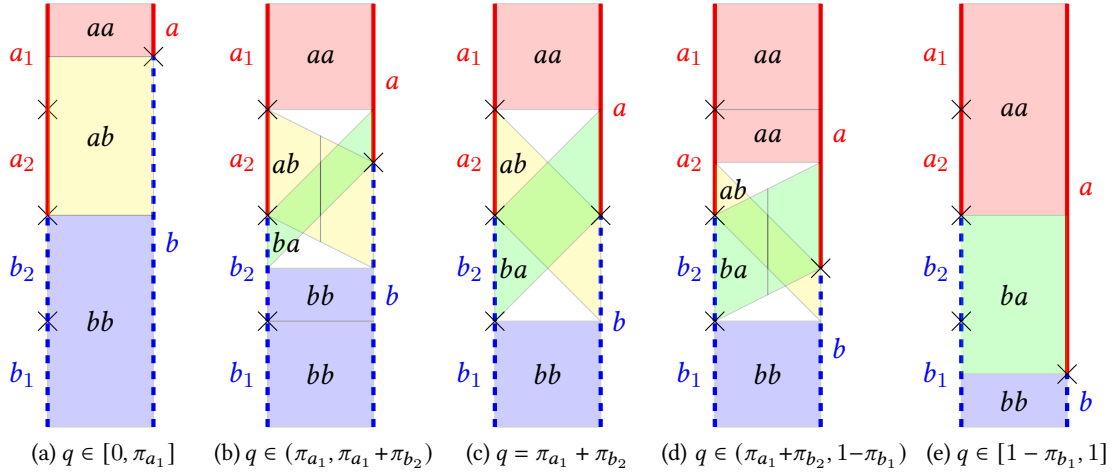
Figure 3: Stable matching with mixed proposing men and homophilic receiving women, by q

Note. $\pi_{\theta_i}^t$ is the size of the set $M_{\theta_i}^t$ (i.e., the mass of men of type $\theta \in \{a, b\}$ who belong to preference group $i \in \{1, 2\}$ at time t), so that we have $\pi_{a_1}^t = p^t h_{ma}$, $\pi_{b_1}^t = (1 - p^t)h_{mb}$, $\pi_{a_2}^t = p^t(1 - h_{ma})$, and $\pi_{b_2}^t = (1 - p^t)(1 - h_{mb})$.

Proposition 3. *Suppose there is a strict mixture of homophilic and heterophilic proposers, and vertical trans-mission is either perfect or inelastic in homogamies. Both $(0, 0)$ and $(1, 1)$ are asymptotically stable. Cultural distribution (p^t, q^t) converges to $(0, 0)$ if $p^0 + q^0 < 1$, $(1, 1)$ if $p^0 + q^0 > 1$, and $(1/2, 1/2)$ if $p^0 + q^0 = 1$.*

Hence, in the presence of mixed proposers, regardless of the preferences of receivers, the long-run distribution of preferences is fully homogeneous.[14] As detailed above, this comes from the fact that the presence of heterophilic proposers induces an evolutionary advantage for the majority when the cultural distribution becomes sufficiently balanced between sexes. Then, regardless of the intensity of homophilic preferences within each group (i.e., the values of $h_{wa}, h_{wb}, h_{ma}, h_{mb}$), the majority ultimately drives out the minority.[15] Put differently, as long as there is a positive mass of heterophilic men and women for every cultural type, regardless of the stable matching scheme, generically, cultural homogeneity is the long-run stable outcome. Even more strikingly, the respective basins of attraction of the steady states $(0, 0)$ and $(1, 1)$ are exactly the same under WOSM and MOSM even if, for some (p^t, q^t), the two mechanisms do not result in the same stable matching.

4.3 Gender-differential distributions of marital preferences

Propositions 1 and 3 illustrate the crucial role played by marital preferences among proposers in shaping the long-run cultural composition of the population. If all proposers are homophilic, cultural diversity is preserved in the long run (Proposition 1); this is no longer the case when there are some heterophilic pro-posers (Proposition 3). This result has striking implications when the distributions of marital preferences

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[14]Except in the knife-edge case in which $p^0 = 1 - q^0$. However, in this case, any small perturbation will induce a convergence to a homogeneous steady state.

[15]Note that, whatever those values are, the basin of attraction of $(0, 0)$ is $\{(p^t, q^t) \in [0, 1]^2 : p^t + q^t < 1\}$ and the basin of attraction of $(1, 1)$ is $\{(p^t, q^t) \in [0, 1]^2 : p^t + q^t > 1\}$.
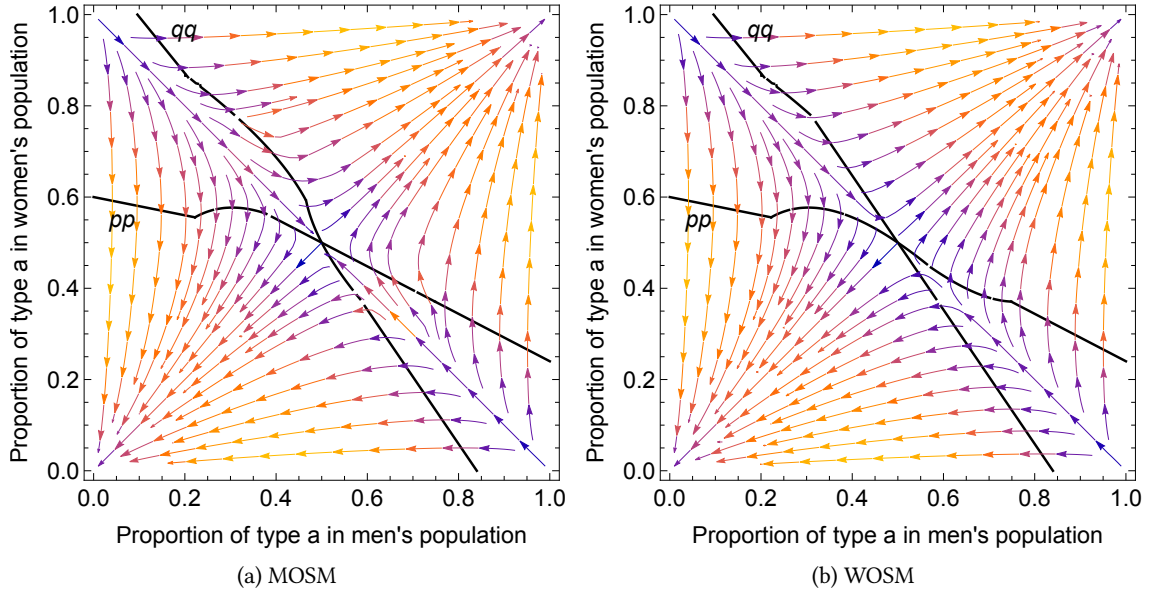
Figure 4: Evolution with mixed proposers and receivers under perfect vertical transmission in homogamies

Note. $h_{ma} = 0.4$, $\tau = 0.6$ and $\rho = 0.4$.

differ between the two sides of the matching market and, in particular, when one side is fully homophilic while some individuals from the other side are heterophilic. In this case, the choice of MOSM or WOSM, by determining which side of the market will play the role of proposers, might have dramatic consequences for the long-run distribution of cultural traits. For example, homophilic proposing men would result in cultural heterogeneity while heterophilic proposing women in the same environment would instead result in cultural homogeneity.[16]   Below, we show the additional result that because of this feature, short-run optimal stable matching may lead to a long-run loss (Section 4.3.1). Then we discuss the role played by the presence of a biased sex ratio (Section 4.3.2) on cultural evolution, as it might influence the selection of one particular stable matching.

4.3.1   Short-run optimal stable matching may lead to long-run loss

Since men are proposers under MOSM and women are proposers under WOSM, Propositions 1 and 3 imply that when one side is homophilic and the other side is not, the long-run distribution of cultural traits depends on the selected stable matching. This result compels us to reassess the optimality of MOSM and WOSM from a dynamic point of view. By definition, the distribution of cultural traits being given, all men prefer MOSM to WOSM and the reverse is true for women. However, the cultural transformation induced by the choice of MOSM might lead to a situation in which men get a lower expected utility than if WOSM had been chosen (while the reverse might be true for women).

We formally state this result in Proposition 4 in which we assume that no men are heterophilic but

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[16]It is interesting to consider the possibility that men are proposing in one cultural group and women are proposing in the other, but since the outcome may not be MOSM or WOSM, it is beyond the scope of the current analysis.

Expected utility of men (left panel) / Expected utility of women (right panel)
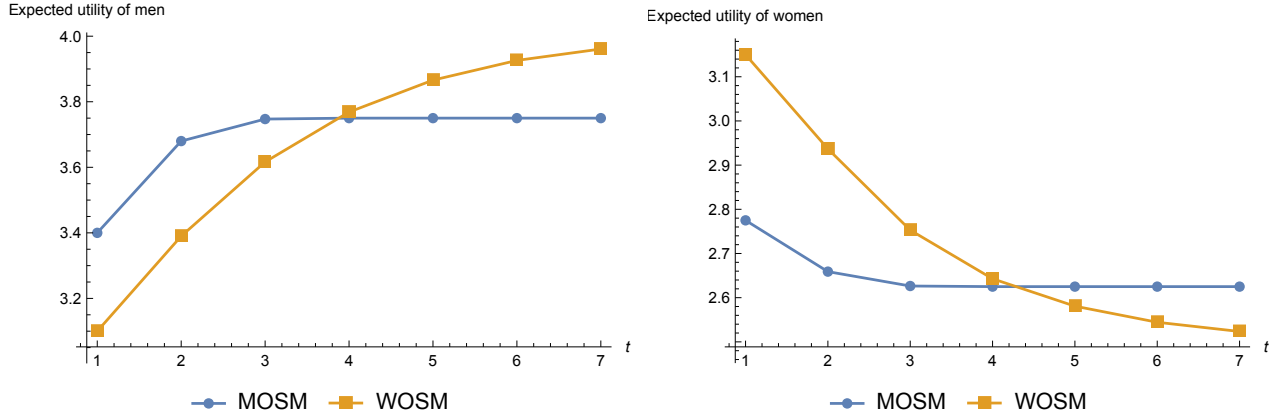
(a) Men are worse off in MOSM than in WOSM

(b) Women are worse off in WOSM than in MOSM

Figure 5: Expected payoffs of homophilic men and heterophilic women under MOSM and WOSM

Note. For the illustrations, we use $U_{aa} = 4$, $U_{bb} = 3$, $U_{ab} = U_{ba} = 2$, $V_{ab}^2 = V_{ba}^2 = 4$, $V_{bb}^2 = V_{aa}^2 = V_{ab}^1 = V_{ba}^1 = 2$, $V_{aa}^1 = 3$, $V_{bb}^1 = 4$, $h_{wa} = h_{wb} = 0.5$, $p^0 = 0.6$, and $q^0 = 0.8$.

some women are. In this proposition, we denote by $V_{\theta\theta'}^1$ (resp., $V_{\theta\theta'}^2$) the utility of a homophilic (resp., heterophilic) type-θ woman matched with a type-θ' man.

Proposition 4. *Suppose all men are homophilic, there is a mixture of homophilic and heterophilic women, and vertical transmission is either perfect or inelastic in homogamies. When $p^0 + q^0 > 1$, men are strictly better off (resp., worse off) in the long run under MOSM than under WOSM if and only if $U_{aa} < U_{bb}$ (resp., $U_{aa} > U_{bb}$); and women are strictly better off (resp., worse off) in the long run under WOSM than under MOSM if and only if $h_{wa}V_{aa}^1 + (1 - h_{wa})V_{aa}^2 > h_{wb}V_{bb}^1 + (1 - h_{wb})V_{bb}^2$ (resp., $h_{wa}V_{aa}^1 + (1 - h_{wa})V_{aa}^2 < h_{wb}V_{bb}^1 + (1 - h_{wb})V_{bb}^2$). When $p^0 + q^0 < 1$, the strict inequality signs in the necessary and sufficient conditions are reversed.*[17]

Figure 5 illustrates a case in which men under MOSM and women under WOSM are strictly better off in the short run, but are strictly worse off in the long run due to the induced evolution of the distribution of traits. To figure out why such a configuration can occur, consider the case where $p^0 + q^0 > 1$ and $U_{aa} > U_{bb}$. As type-a individuals are initially in the majority, the cultural distribution converges to $(1, 1)$ under WOSM. Hence, in the long run, the utility of a man will be U_{aa}. On the contrary, since under MOSM all proposers are homophilic, the cultural distribution converges to a steady state (r, r) where cultural diversity is preserved. Hence, in the long run, there is a positive mass of bb couples such that, in expectation, the utility of a man will be lower than U_{aa}.

---

[17]When $p^0 + q^0 = 1$, the steady state under WOSM is $(p^w, q^w) = (p^0, q^0)$, so men's and women's average payoffs involve U_{ab}, U_{ba}, V_{ab}^1, V_{ab}^2, V_{ba}^1 and V_{ba}^2; the steady state under MOSM is (p^m, q^m) such that $p^m = q^m$, but there is no definitive relation between (p^m, q^m) and (p^0, q^0). All in all, when $p^0 + q^0 = 1$, there is no clean condition to characterize when a gender is strictly better off (or worse off) under MOSM than under WOSM in the long run.

4.3.2 Biased sex ratio

We demonstrate how a biased sex ratio influences equilibrium matching and, in turn, the cultural trans-
mission process. We assume that at the beginning of each time t, and before matching takes place, a mass
λ of adult males arrive in the society. Those incoming men do not have a well-defined culture. They
randomly pick a cultural model within the population of adult males already present in the society and
adopt the trait of this role model with probability one. Hence, after this arrival, there is a mass $p^t(1 + \lambda)$
(resp., $(1 - p^t)(1 + \lambda)$) of type-a men (resp., type-b men) in the society. We focus on the case in which
men have homophilic preferences ($h_{ma} = h_{mb} = 1$) and not all women have homophilic preferences
($(h_{wa}, h_{wb}) \in [0, 1)^2$).

   Ashlagi et al. (2017) show that even the slightest imbalance between the number of individuals on
either side of the market can yield a unique stable matching that is favorable to individuals on the short
side. We establish that this is indeed the case in our setting. More specifically, even for an arbitrarily
small λ, there exists a unique stable matching in which as many women as possible receive their preferred
matching. Hence, compared with the benchmark case ($\lambda = 0$), this matching is close to WOSM and might
be in sharp contrast to MOSM if the preferences of women are very different from those of men. Lemma
3 strikingly illustrates this feature in the special case in which all women are heterophilic.[18] In Appendix
D.1, we describe the unique stable matching for any $(h_{wa}, h_{wb}) \in [0, 1)^2$.

Lemma 3. *Define the following functions: $\phi_1(p) := 1 - (1 + \lambda)p$ and $\phi_2(p) := (1 + \lambda)(1 - p)$. In the market
with mass $p(1+\lambda)$ of type-a men, mass $(1-p)(1+\lambda)$ of type-b men, mass q of type-a women, and mass $(1-q)$
of type-b women, the unique stable matching $\mu = (\mu_{aa}, \mu_{ab}, \mu_{ba}, \mu_{bb})$ with homophilic men and heterophilic
women is*

$$
\mu = \begin{cases}
(0, p(1 + \lambda), q, 1 - q - p(1 + \lambda)) & \text{if} \quad q < \phi_1(p), \\
(0, 1 - q, q, 0) & \text{if} \quad q \in [\phi_1(p), \phi_2(p)], \\
(q - (1 + \lambda)(1 - p), 1 - q, (1 + \lambda)(1 - p), 0) & \text{if} \quad q > \phi_2(p).
\end{cases}
$$

   We now assess the consequences of gender imbalance on cultural evolution. For illustrative purposes,
we only consider the case of perfect vertical transmission. The results can be generalized to the case of
inelastic vertical transmission. From Lemma 3, we get[19]

$$
p^{t+1} = \begin{cases}
p^t \left[q^t + p^t(1 + \lambda) \right] & \text{if} \quad q^t < \phi_1(p^t), \\
p^t & \text{if} \quad q^t \in \left[\phi_1(p^t), \phi_2(p^t) \right], \\
p^t + (1 - p^t) \left[q^t - (1 + \lambda)(1 - p^t) \right] & \text{if} \quad q^t > \phi_2(p^t),
\end{cases}
\tag{13}
$$

and

$$
q^{t+1} = \begin{cases}
q^t \left[q^t + p^t(1 + \lambda) \right] & \text{if} \quad q^t < \phi_1(p^t), \\
q^t & \text{if} \quad q^t \in \left[\phi_1(p^t), \phi_2(p^t) \right], \\
q^t + (1 - q^t) \left[1 - (1 + \lambda)(1 - p^t) \right] & \text{if} \quad q^t > \phi_2(p^t).
\end{cases}
\tag{14}
$$

---

[18]According to Lemma 3, as gender imbalance vanishes ($\lambda \to 0$), $\mu \to (0, p, q, 1-p-q)$ when $q < 1-p$ and $\mu \to (p+q-1, 1-q, 1-p, 0)$ when $q > 1 - p$. Hence, the realized matching is very similar to the one obtained in the absence of gender imbalance when proposers are heterophilic (see Figure 2) even though, here, all men are homophilic and we consider MOSM.

[19]For the sake of clarity, in the main text we only present the cultural dynamics for the case in which all women are heterophilic ($h_{wa} = h_{wb} = 0$). In Appendix D.2 we present the dynamics for any $(h_{wa}, h_{wb}) \in [0, 1)^2$.
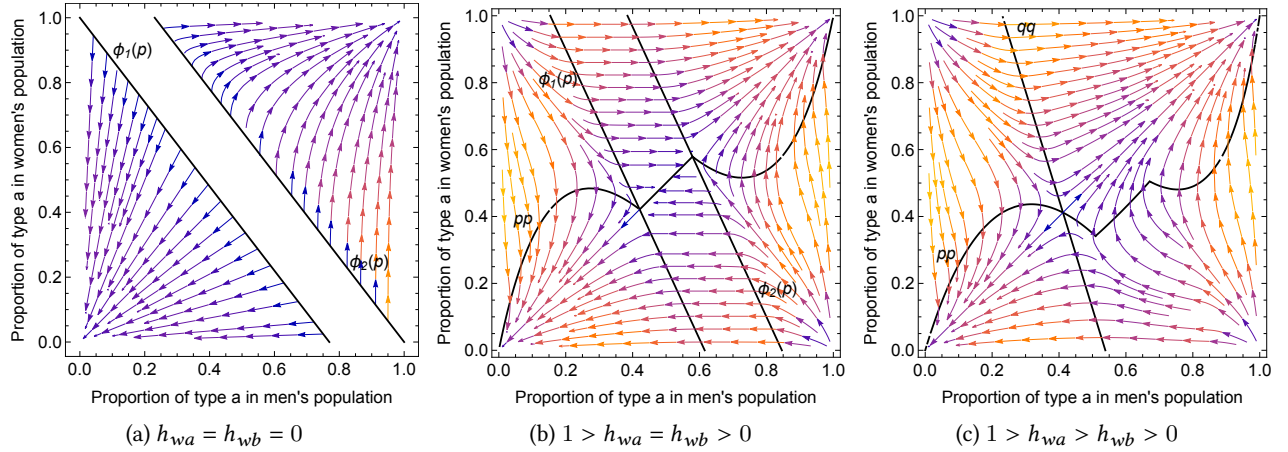
Figure 6: Evolution under perfect vertical transmission in homogamies and gender imbalance

Note. $h_{ma} = h_{mb} = 1$ in panel (a), $\lambda = 0.3$ and $h_{wa} = h_{wb} = 0.2$ in panel (b) and $h_{wa} = 0.3$ and $h_{wb} = 0.15$ in panel (c).

Cultural evolution in the case in which all women are heterophilic is summarized in Proposition 5a and illustrated in Figure 6a. When $q^t \in \left[\phi_1(p^t), \phi_2(p^t)\right]$, there are more type-$a$ men than type-b women and more type-b men than type-a women. Hence, all women can be matched with a man of the opposite type. As a consequence, there is no homogamous couple and the initial distribution of traits perpetuates over time. Outside this region, $(0, 0)$ and $(1, 1)$ are asymptotically stable. Specifically, for any (p^0, q^0) such that $q^0 < \phi_1(p^0)$ (resp., $q^0 > \phi_2(p^0)$), the distribution of cultural traits converges toward $(0, 0)$ (resp., $(1, 1)$), as in the case in which proposers are heterophilic (Section 4.1).

Proposition 5 (Gender imbalance). *Suppose there are more proposers than receivers ($\lambda > 0$) and all proposers are homophilic ($h_{ma} = h_{mb} = 1$). Also vertical transmission is perfect in homogamies.*

(a) *If $h_{wa} = h_{wb} = 0$ (receivers are heterophilic), then $\left\{(p^t, q^t) \in [0, 1]^2 : q^t \in \left(\phi_1(p^t), \phi_2(p^t)\right)\right\}$ is a stable set. Moreover, both $(0, 0)$ and $(1, 1)$ are asymptotically stable, and (p^t, q^t) converges to $(1, 1)$ if $q^0 < \phi_1(p^0)$ and $(0, 0)$ if $q^0 > \phi_2(p^0)$.*

(b) *If $1 > h_{wa} = h_{wb} > 0$, then $\left\{(p^t, q^t) \in [0, 1]^2 : q^t \in \left(\phi_1(p^t), \phi_2(p^t)\right) \text{ and } q^t = p^t\right\}$ is a stable set and the steady states $(0, 0)$ and $(1, 1)$ are attracting.*

(c) *If $1 > h_{wa} > h_{wb} > 0$, then there are three steady states: (r, r), where $r = \frac{1 - h_{wa}}{2(1 - h_{wa}) + \lambda} < 1/2$, which is unstable, and $(0, 0)$ and $(1, 1)$, which are attracting.*

Proposition 5 also summarizes the cases in which not all receivers are heterophilic. The dynamic system for the case in which $1 > h_{wa} = h_{wb} > 0$ (Proposition 5b) is illustrated in Figure 6b and the case in which $1 > h_{wa} > h_{wb} > 0$ (Proposition 5c) is illustrated in Figure 6c. In all three configurations, culturally homogeneous steady states $(0, 0)$ and $(1, 1)$ are attracting. Hence, even for arbitrarily small values of λ, cultural evolution is radically different from the $\lambda = 0$ case. In particular, under MOSM, when $\lambda = 0$ the steady states are characterized by cultural diversity (see Proposition 1); when λ is positive but very close

to 0, the stable steady states essentially correspond to the ones obtained under WOSM with heterophilic proposers (see Proposition 2) and are characterized by cultural homogeneity.

Grosjean and Khattar (2019) and Baranov et al. (2021) show that a male-biased sex ratio, which originated from the British policy of sending convicts to Australia, had persistent effects on the culture, and in particular on gender role attitudes or the extent of masculine norms, even though gender balance was restored after the transportation of convicts stopped. Gay (2019); Teso (2019); and Alix-Garcia et al. (2020) reach a similar conclusion with respect to the impact of a female-biased sex ratio caused by World War I in France, the transatlantic slave trade in Sub-Saharan Africa, and the War of the Triple Alliance in Paraguay, respectively. The introduction of a biased sex ratio in our model offers a new channel of persistence of historical gender imbalance on culture: Imbalanced sex ratio influences the matching pattern, which, in turn, impacts the intergenerational transmission process and finally the long-run distribution of cultural traits.

To see how temporary gender imbalance can lead to path-dependence phenomena, consider two countries that differ in sex ratio only. In both countries, men have homophilic preferences while women have mixed preferences (we assume $1 > h_{wa} > h_{wb} > 0$ but we could have assumed the opposite ordering of h_{wa} and h_{wb}) and MOSM is selected. There is no gender imbalance in Country 1 ($\lambda_1 = 0$), but there are more men than women in Country 2 ($\lambda_2 > 0$). The initial cultural distribution (p^0, q^0) is the same in both countries, and we assume, without loss of generality, that in Country 2, (p^0, q^0) belongs to the basin of attraction of $(1, 1)$ (see Figure 6c). Under these assumptions, in Country 1, the cultural distribution converges to a point on the first diagonal $(p_1^*, q_1^*) = (r^*, r^*)$ (cultural diversity); in Country 2, it converges to $(p_2^*, q_2^*) = (1, 1)$ (cultural homogeneity). If, later in time, gender imbalance disappears in Country 2 so that $\lambda_2 = \lambda_1 = 0$, since $(1, 1)$ is on the first diagonal, it would not have any consequence for the cultural composition of Country 2. In the end, even if initial states were the same in the two countries, temporary differences in sex ratio have a long-lasting impact on the cultural composition of each country. Figure 7 illustrates this phenomenon.

5   Imperfect vertical transmission in homogamies

In this section, we consider imperfect vertical transmission in homogamies that is culturally substitutable with oblique transmission and general marital preference distributions.

Recall that imperfect vertical transmission with cultural substitutability means that the probability of a direct transmission $d(r)$ is strictly decreasing in r, the share of the transmitted trait in the same gender population.

Given the transmission technology and equations (1) and (2), cultural evolution is characterized by rearrangement of equations (1) and (2):

$$p^{t+1} = p^t + \mu_{aa}^t d(p^t)(1 - p^t) - \mu_{bb}^t d(1 - p^t)p^t; \qquad (15)$$

$$q^{t+1} = q^t + \mu_{aa}^t d(q^t)(1 - q^t) - \mu_{bb}^t d(1 - q^t)q^t. \qquad (16)$$

An interpretation of the transition is as follows. Each man is subject to societal impact through oblique

(a) Evolution of p^t in both countries

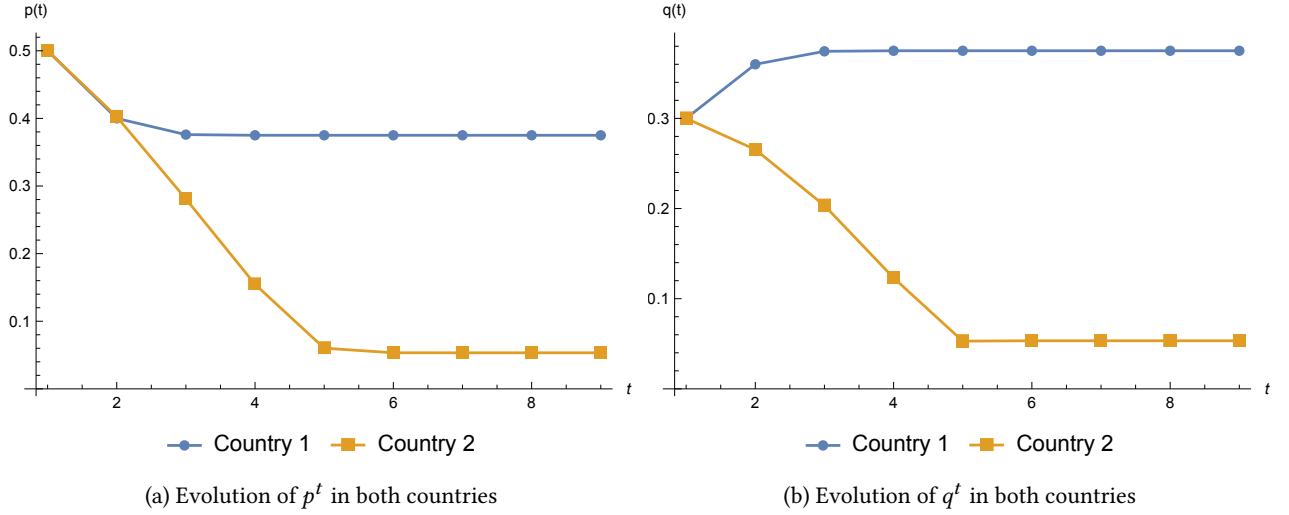(b) Evolution of q^t in both countries

Figure 7: Temporary biased sex ratio leads to path dependence

Note. For the illustrations, we use $h_{wa} = 0.3$, $h_{wb} = 0.15$, $p^0 = 0.5$, $q^0 = 0.3$ in both countries. In Country 1, $\lambda = 0$. In Country 2, $\lambda = 0.01$ before date $t = 4$ and $\lambda = 0$ for all $t \geqslant 4$.

transmission and possesses trait a with probability p^t, except that the probability increases by $d(p^t)(1-p^t)$ when vertical transmission (of trait a) is successful in aa homogamies, and the probability decreases by $d(1 - p^t)p^t$ when vertical transmission (of trait b) is successful in bb homogamies.

In the rest of the section, we will start with the cases with uniformly homophilic proposers or an entirely homophilic cultural group. We then consider the case in which each cultural group has the same proportion of homophilic agents, and the case in which different cultural groups differ in their proportion of homophily.

5.1   Homophilic proposers or cultural groups

Assume all proposers are homophilic. Suppose $p^t \geqslant q^t$. Using the properties of the stable matching (see Figure 1a), cultural evolution is characterized by

$$
\begin{aligned}
p^{t+1} - p^t &= q^t d(p^t)(1 - p^t) - (1 - p^t)d(1 - p^t)p^t; \\
q^{t+1} - q^t &= q^t d(q^t)(1 - q^t) - (1 - p^t)d(1 - q^t)q^t.
\end{aligned}
$$

We show that only (p^*, q^*) such that $p^* = q^* := r$ can be a steady state, and in addition, r satisfies

$$
r(1 - r)[d(r) - d(1 - r)] = 0.
$$

Hence, when $d(\cdot)$ is strictly decreasing, the solutions are 0, $1/2$, and 1. This equation also clarifies that when homogamies have inelastic vertical transmission, i.e., $d(r) = d$ for all r, any (r, r) is a steady state (Proposition 1). To establish the global asymptotical stability of $(1/2, 1/2)$, we use cultural substitutability
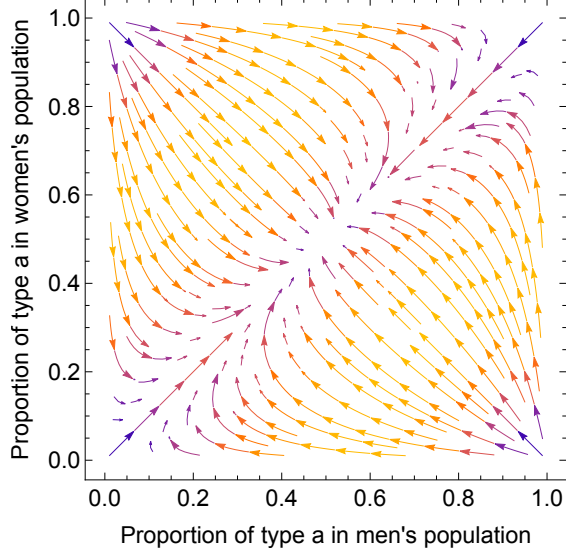
20

Figure 8: Cultural evolution with homophilic proposers under imperfect vertical transmission in homogamies

and find a Lyapunov function.

Proposition 6. *Suppose all proposers are homophilic or at least one cultural group is homophilic, and imperfect vertical transmission is culturally substitutable in homogamies. There is a unique globally asymptotically stable steady state* $(1/2, 1/2)$.

Cultural evolution in this case is illustrated in Figure 8. To explain this result, first note that, even under imperfect vertical transmission with cultural substitutability, homogamies have a more effective transmission technology than heterogamies. Hence, the tendency toward a balanced cultural distribution between sex is preserved (starting near the northwest or the southeast corner of Figure 8, the cultural distribution tends to the $q^t = p^t$ line). However, compared to the previous case, cultural substitutability creates an additional force in favor of cultural diversity. If p^t is close to q^t and there are fewer type-a individuals in the whole population ($p^t + q^t < 1$), the probability of a direct transmission is larger for aa families than for bb families. Hence, trait a spreads over time within both populations of men and women and (p^t, q^t) will converge to $(1/2, 1/2)$.

5.2 Cultural groups of the same proportion of homophily

Consider the configuration in which there is a mixture of homophilic and heterophilic individuals but the proportion of homophily is the same in each cultural group. This corresponds to the case in which $\tau = 1$ and ρ can take any value between 0 and 1 (i.e., $h_{ma} = h_{mb} = h_m$ and $h_{wa} = h_{wb} = h_w$). Recall that we define the *elasticity of cultural substitution*, $\varepsilon_d(r) = -rd'(r)/d(r)$, in Section 2.3. It measures the relative responsiveness of familial vertical transmission and societal oblique transmission to population change. A higher elasticity indicates that vertical transmission is less effective on the margin under a population

increase and more effective on the margin under a population decrease. In other words, higher elasticity of cultural transmission substitution implies that a minority population has a higher effectiveness with their vertical transmission. We will show that the elasticity of cultural substitution crucially determines whether the symmetric culturally heterogeneous steady state $(1/2, 1/2)$ is stable.

For subsequent results, we assume that the elasticity cannot take extremely high values to avoid the probability of a vertical transmission overreacting to small changes in the cultural composition of the society:

Assumption 2. $\frac{1}{\varepsilon_d(1/2)} > \frac{1/2 \cdot d(1/2)}{1 - 1/2 \cdot d(1/2)}$.

Assumption 2 prevents the dynamic system from exhibiting too much oscillation, which prevents it from convergence. Because $d(1/2) \leqslant 1$, the right-hand side of the inequality is at most 1. The inequality is easily satisfied. For example, it holds for $d(r) = (1-r)^\eta$ for any $\eta > 0$.

Proposition 7. *Suppose Assumption 2 is satisfied. Suppose mass $h_m \in (0,1)$ of proposers are homophilic and imperfect vertical transmission is culturally substitutable in homogamies. The steady state $(1/2, 1/2)$ is asymptotically stable if and only if the likelihood ratio of homophily is strictly higher than the inverse of the elasticity of cultural substitution at $1/2$:*

$$\frac{h_m}{1 - h_m} > \frac{1}{\varepsilon_d(1/2)}. \tag{17}$$

When there is mass $h_w \in (0,1]$ of homophilic receivers, only (r, r), where $0 < r < 1$, can be a steady state. When receivers are all heterophilic, $(0,0)$ and $(1,1)$ are asymptotically stable steady states.

We show that only states with equal masses of type-a men and women, (r, r), can be steady states. By symmetry, $(1-r, 1-r)$ is a steady state when so is (r, r), and they also share the stability property. Of those steady states, $(1/2, 1/2)$ can be thought of as a special case—the state in which there is neither a dominant trait nor a dominated trait. In other states, there is asymmetry that results in the identification of majority and minority groups. Two forces contribute to make $(1/2, 1/2)$ asymptotically stable: homophily and cultural substitutability. The role played by these two forces has already been discussed in Section 3.2, in which we show that, under imperfect cultural transmission, $(1/2, 1/2)$ is the unique long-run outcome when all proposers are homophilic. The inequality (17) in Proposition 7 refines this result, by showing that a full symmetry between the two cultural groups is likely to be a long-run outcome if the proportion of homophilic proposers is sufficiently large and/or cultural substitutability between oblique and vertical socialization is sufficiently strong.

To better illustrate the results of Proposition 7, it is useful to consider the case $d(r) = (1-r)^\eta$, for $\eta > 0$ such that the elasticity of cultural substitution is a constant.[20] In this case, a large proportion—higher than $1/(1+\eta)$—of homophilic proposers results in a unique stable steady state $(1/2, 1/2)$ (Figure 9a) and a small proportion—lower than $1/(1+\eta)$—of homophilic proposers results in two stable steady states (r^*, r^*)

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[20] As shown in Appendix A, $d(r) = (1-r)^\eta$, for $\eta > 0$, corresponds to the probability of vertical transmission in a socialization model a la Bisin and Verdier (2001) assuming a socialization cost function with constant elasticity.

and $(1 - r^*, 1 - r^*)$ with $r < 1/2$ (Figure 9b). Moreover, r^* is increasing in h_m.[21] Hence, the size of the minority group increases with the mass of homophilic proposers such that a society not very tolerant of intermarriage is likely to be more culturally diverse in the long run. Conversely, in a more tolerant society, intermarriage tends to accelerate the assimilation of the minority to the dominant culture.

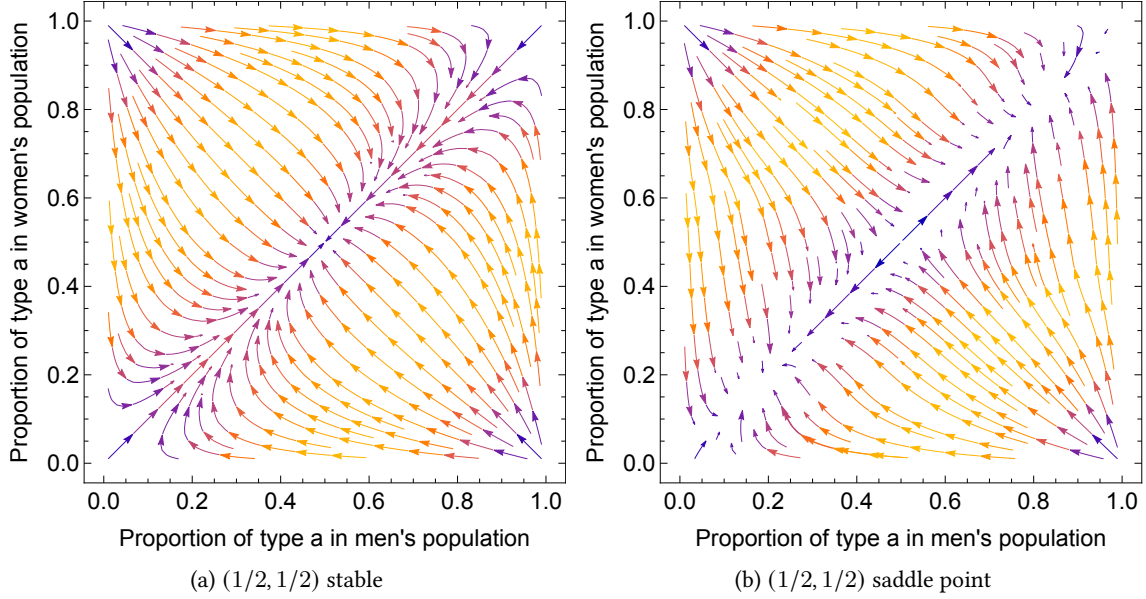(a) $(1/2, 1/2)$ stable

(b) $(1/2, 1/2)$ saddle point

Figure 9: Evolution with a mixture of homophilic and heterophilic proposers under imperfect vertical transmission in homogamies

Note. $h_m = h_w = 0.8$ and $d(r) = (1 - r)^{1/2}$ in panel (a); $h_m = h_w = 0.4$ and $d(r) = (1 - r)^{1/2}$ in panel (b).

Let us finally state that since the proportion of homophilic proposers plays a critical role, the choice of MOSM or WOSM might also have an impact when $h_m \neq h_w$. For instance, if $d(r) = (1 - r)^\eta$, for $\eta > 0$, and $h_m > \frac{1}{1+\eta} > h_w$, MOSM will induce convergence toward the steady state with symmetric cultural diversity $(1/2, 1/2)$, while under WOSM, the society will converge toward a situation with a minority and a majority group. Hence, all of the consequences of gender-imbalanced distributions, discussed in Section 4.3, generalize to the setting with imperfect vertical transmission in homogamies.

5.3 Cultural groups of different proportions of homophily

Suppose now that both cultural groups have heterophilic agents: $h_{ma} = h_{wa} = h_a \in [0, 1)$, $h_{mb} = h_{wb} = h_b \in [0, 1)$. Note that this is a special case of Section 4.2, such that there is no gender difference in preferences within each cultural group ($\rho = 1$) and culture a is less tolerant of intermarriage than culture b ($\tau \in (0, 1]$). The characterization of stable matching (into six cases) is relegated to Appendix E.4.

Proposition 8. *Suppose Assumption 2 is satisfied. Suppose both cultural groups are nonhomophilic and imperfect vertical transmission is culturally substitutable in homogamies. Only (r, r), where $r \in [0, 1]$, can*

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[21]When $\eta = 1$, this r^* exactly equals h_m. See Proposition 9 in Online Appendix OA.1.

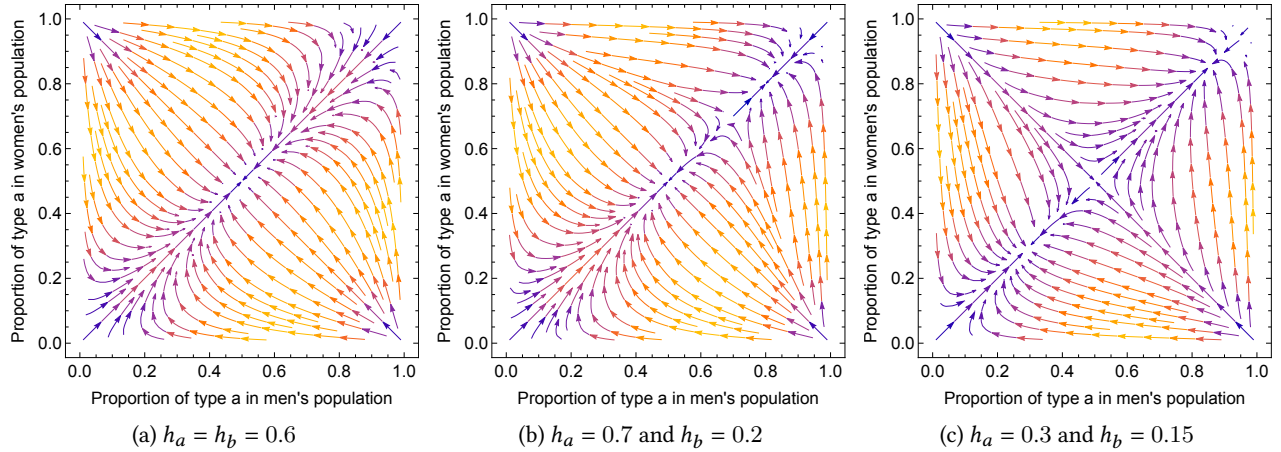(a) $h_a = h_b = 0.6$ (b) $h_a = 0.7$ and $h_b = 0.2$ (c) $h_a = 0.3$ and $h_b = 0.15$

Figure 10: Evolution under perfect vertical transmission in homogamies and gender imbalance

be a steady state. The steady state $(1/2, 1/2)$ is asymptotically stable if and only if

$$\frac{h_a}{1 - h_a} > \frac{1}{\varepsilon_d(1/2)}.$$

When type-b cultural group is heterophilic, $(1, 1)$ is asymptotically stable.

In common with Proposition 7, Proposition 8 highlights the combined role of homophily and cultural substitutability in the sustainability of a high level of cultural diversity. One substantial difference between the two propositions is that when the two cultural groups differ in their level of tolerance of intermarriage, the asymptotic stability of (r, r) does not necessarily ensure the existence of the symmetric equilibrium $(1 - r, 1 - r)$.

To illustrate this point, let us consider $d(r) = 1 - r$.[22] In this case, three configurations may arise and are depicted in Figure 10.[23] If homophilic individuals form the majority of the population ($h_a + h_b \geqslant 1$), $(1/2, 1/2)$ is the globally stable steady state such that symmetric diversity is always preserved in the long-run (Figure 10a). If homophilic individuals are in minority in the whole population ($h_a + h_b < 1$) but in the majority within one cultural group ($h_a > 1/2 \geqslant h_b$), the steady state with symmetric cultural diversity $(1/2, 1/2)$ (now locally stable) coexists with another locally stable steady state with asymmetric cultural diversity $(1 - h_b, 1 - h_b)$ in which the cultural group less tolerant of intermarriage (group a) is the dominant group (Figure 10b). If both h_a and h_b are lower than $1/2$, two locally stable steady states with asymmetric cultural diversity, (h_a, h_a) and $(1 - h_b, 1 - h_b)$ coexist (Figure 10c).

These results confirm that a sufficiently high marital homophily in the entire population allows for the preservation of cultural diversity. However, when marital preferences are also transmitted across generations, a high intolerance for intermarriage for one cultural group might be sufficient to maintain

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[22]As discussed in Appendix A, this transmission probability might be obtained under the usual assumption of quadratic socialization costs in the cultural transmission model proposed by Bisin and Verdier (2001).

[23]Formal results are stated and proven in Online Appendix OA.1 (Proposition 10).

a high level of diversity. In the configuration depicted in Figure 10b, the basin of attraction of the stable steady state with symmetric cultural diversity is increasing in h_a, the degree of homophily within the cultural group which is less tolerant to intermarriage.[24] Hence, such an increase would render more likely the convergence to $(1/2, 1/2)$. Moreover, when a group cannot transmit a high enough homophilic marital preference (due to environmental or feasibility constraints), as is the case for group b in this configuration, it may become a remnant minority group (at the stable steady state $(1-h_b, 1-h_b)$, the proportion of type-b individuals is increasing in h_b).

This intermediate configuration depicted in Figure 10b also illustrates the fact that for a given conditional distribution of marital preferences, the initial condition matters for the persistence of a high level of cultural diversity. Indeed, considering two countries that start from close, but different, initial conditions in terms of cultural distribution. One might converge to the $(1/2, 1/2)$ steady state characterized by symmetric cultural diversity and the other to $(1 - h_b, 1 - h_b)$, where we can clearly identify a minority and a majority culture.

5.4 Historical examples of cultural integration and preservation

The analysis in Section 5 demonstrates that being sufficiently homophilic is the key for the survival of a cultural group. This will help explain cultural preservation for Mongolians and Manchurians in China, Orthodox Jews, and the Chinese minority in Southeast Asian Muslim countries. Our results also imply that if a minority cultural group is sufficiently open to intermarriage, it can be assimilated by the majority. This explains the examples of the cultural integration of Xianbeis in China and Chinese minorities in Thailand. We elaborate on these below. One should keep in mind that, in reality, marital preferences are likely to be endogenous to socioeconomic and political contexts that are not taken into account by the current analysis.

5.4.1 Effects of government

In the last 2,000 years, for an extended time period, many northern nomadic groups—e.g., Xianbeis, Mongolians, and Manchurians—conquered and governed the heartland of China (*Zhongyuan*) inhabited by Han Chinese. Their population sizes were similarly small compared with the Han Chinese, but they differed in their governing policies toward ethnic intermarriage and integration. Xianbeis have been genetically and culturally integrated with the Han due to the intermarriage policies promoted and practiced by their governing bodies, and the Mongolians and Manchurians have preserved their cultural traditions and identities partly due to the governing dynasties' policies against intermarrying with Han Chinese.

Xianbeis' integration with the Han Chinese is an example of how cultural integration can be achieved with a small group of heterophilic elites. The Northern Wei dynasty established by the Tuoba clan of the Xianbei ethnic group, who were originally from northern Mongolia and Siberia, ruled northern China from 385 to 535 AD (Liu, 2020).[25] As the Northern Wei unified northern China around 439, the emperors'

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[24]Indeed, the two saddle paths converging to the saddle point (h_a, h_a) separate the state space between the basin of attraction of $(1/2, 1/2)$ and the basin of attraction of $(1 - h_b, 1 - h_b)$.

[25]*Mulan* is believed to be based on a Northern Wei Xianbei heroine who joined the army for her father.

desire for Han Chinese institutions and cultures grew. The Northern Wei started to arrange for local Han Chinese elites to marry daughters of the Xianbei Tuoba royal family in the 480s (Watson, 1991).[26] More than 50% of Tuoba Xianbei princesses of the Northern Wei were married to southern Han Chinese men from imperial families and aristocrats from southern China of the southern dynasties who defected and moved north to join the Northern Wei. The Sinicization was thorough: The royal families moved to central China and adopted Chinese surnames, all Xianbei officials were forced to speak and write Chinese, and Xianbei family and imperial traditions were abandoned for Chinese traditions. Other nomadic groups of the time—e.g., Qiang, Xiongnu (Huns), and Rouran—also joined the ethnic integration. With the rise of nomadic groups and the collapse of weak Han Chinese rule in northern China, due to politically encouraged ethnic intermarriage, this was one of the biggest—if not the biggest—periods of cultural integration in Chinese history. These groups no longer have separate cultural identities, but they infused the genetic makeup of northern Chinese (Dien and Knapp, 2020).

In contrast, Mongolian and Manchurian cultural and ethnic identities have been preserved by the politically motivated homophilic policies of their governing elites. Mongolians' Yuan dynasty (1271 to 1368) and Machurians' Qing dynasty (1644 to 1912) conquered China and governed from Beijing. Both dynasties adopted ethnic tier systems in which the Han Chinese were treated as inferior to the governing ethnicities in terms of political and economic rights (Franke and Twitchett, 1994; Peterson, 2002).[27] Intermarriage was not encouraged, if not completely banned. The governing body maintained their non-Han blood (though the Manchurians intermarried with Mongolians during the Qing dynasty). As a result, they are officially recognized ethnic minorities in modern China. Admittedly, many other factors have contributed to their cultural preservation (Mongolians have an independent nation-state and a large autonomous region within China, and Manchurians' governance is so recent that its longer-term cultural implications are still evolving). Nonetheless, intermarriage policies and political and economic rules that affected intermarrying incentives steered them away from Xianbei-style cultural integration.

5.4.2 Effects of religion

Religion is a frequent barrier to intermarriage and frequently serves as an important base for mate selection (Marcson, 1951). Profound values are attached to religious group membership, and such membership exercises strong control over marital behavior, which renders religious endogamy prescriptive. For example, Orthodox Judaism upholds historic Jewish attitudes toward intermarriage, which it discourages. Intermarriage is considered to be a deliberate rejection of Judaism, and consequently an intermarried person is often cut off from the Orthodox Jewish community; see Bisin and Verdier (2000) for a discussion. As a result, Orthodox Jews are able to preserve their distinctive culture.

Many parts of Southeast Asia have been continuously settled by Chinese for several centuries. In Indonesia and Malaysia, the practice of Islam has been an important expression of ethnic and national iden-

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[26]The subsequent royal families governing Sui and Tang dynasties (581 to 907 AD) were from this elite group (the so-called *Guanlong elites*, named after the region they governed from). Both the Sui dynasty's Yang royal family and the Tang dynasty's Li royal family had maternal Xianbei lineages.

[27]Yuan's priority order was Mongolians, ethnic groups in western China, northern Chinese, and southern Chinese. Qing politically and economically favored Manchurian and Mongolian elites (Eight Banners).

tity for natives and forms a strong obstacle to intermarriage between natives and Chinese (Silcock, 1963; Murray, 1968; Edmonds, 1968). As a result, Chinese still retain their names and languages and continue to identify, generation after generation, as Chinese in Indonesia and Malaysia. In contrast, in Thailand, Buddhism is the main religion, which is arguably more permissive and tolerant of intermarriage (Skinner, 2008). Chinese minorities have assimilated to the host culture by adopting the Thai language and names.

6 Conclusion

We demonstrate that the joint consideration of marital preferences, matching markets, and intergenerational transmission technologies is required for a more complete understanding of cultural evolution. Under perfect, in general, inelastic familial transmission in homogamies, with the presence of a small mass of heterophilic individuals, cultural homogeneity is the generic long-run outcome; cultural heterogeneity arises only when all proposers are homophilic or all members of a cultural group are homophilic. When familial transmission in homogamies is substitutable with oblique transmission, cultural heterogeneity arises, as demonstrated by Bisin and Verdier (2001). However, we demonstrate that it is sustained only when not all individuals are heterophilic; the resilience of cultural traits relies on both the socialization efforts of minority families to pass on their traits and the homophilic marital preferences of sufficiently many proposers or sufficiently many members of a cultural group. We apply our model to the potential long-lasting impact of a temporary gender imbalance on cultural evolution and the historical differences in cultural assimilation and preservation of minority groups due to government policies and religious practices.

Additional transmission technologies and matching mechanisms and more than two cultural groups can be considered within our framework, and may generate additional insights into the evolution of cultural traits, which we leave for future research. Furthermore, in this paper, we compare the outcomes under different exogenously given distributions of marital preferences and stable matching schemes. How matching schemes may endogenously evolve with culture warrants further investigation.

Appendix

A Microfoundation of cultural substitutability

Consider a husband and wife who both have trait i. They have the possibility to transmit this trait to their daughter (symmetric reasoning would apply for their son) with a probability τ_i^t. This probability corresponds to a socialization effort whose cost is given by a strictly increasing and convex function $c\left(\tau_i^t\right)$ with $c(0) = 0$ and $\partial c(0)/\partial \tau_i^t = 0$. We also assume a form of *cultural intolerance* (Bisin and Verdier, 2001): Parents prefer their child not to deviate from their own culture. To make things as simple as possible, we assume that parents derive a utility $v \in (0, 1)$ from having a child with trait i while this utility is normalized to 0 if their child adopts trait $j \neq i$. Then, given the cultural transmission process described in Section 2.3,

parents from an *aa* couple choose their socialization effort τ_a^t to maximize

$$\tau_a^t v + (1 - \tau_a^t) q^t v - c\left(\tau_a^t\right).$$

Hence, the optimal socialization effort (that also corresponds to the probability of direct transmission) must satisfy:

$$(1 - q^t) v = c'\left(\tau_a^t\right).$$

By the properties of $c(\cdot)$, it is clear that the optimal value of τ_a^t is decreasing in q^t and is 0 when $q^t = 1$. Symmetrically, τ_b^t (the socialization effort for a *bb* couple) is increasing in q^t and 0 when $q^t = 0$.

Note that, if we assume the cost function has a constant elasticity, $c\left(\tau_i^t\right) = \frac{\eta}{1+\eta}\left(\tau_i^t\right)^{\frac{1+\eta}{\eta}}$, for $\eta > 0$, the optimal socialization efforts are $\tau_a^t = v^\eta (1 - q^t)^\eta$ and $\tau_b^t = v^\eta (q^t)^\eta$. This tells us that the d function defined in Section 2.3 takes the form of $d(r) = v^\eta (1 - r)^\eta$ and the *elasticity of cultural substitution* is $\varepsilon_d(r) = \frac{\eta r}{1-r}$, which is increasing in η. Note that, the elasticity of the cost function is $\varepsilon_c(\tau) = 1 + 1/\eta$. Hence, the *elasticity of cultural substitution* is higher when the elasticity of the cost function is lower. Indeed, if the elasticity of the cost function is low, parents can increase the direct transmission probability without increasing the socialization costs too much. Hence, they can more easily adjust d in reaction to a change in r.

In this case, the condition stated in Proposition 7,

$$h_m > \frac{d(1/2)}{d(1/2) - d'(1/2)/2} = \frac{1}{1 + \varepsilon_d(1/2)}$$

is simply rewritten as $h_m > 1/(1 + \eta)$ where η is the elasticity of cultural substitution at the point $r = 1/2$.

B   Proofs in Section 3

B.1   Proof of Lemma 1

Proof of Lemma 1. Suppose $p^t \geqslant q^t$. Cultural evolution is characterized by equations (3) and (4). When $p^0 = q^0$, we have $p^t = q^t$ for any t. Consider (p^0, q^0) that satisfies $0 \leqslant q^0 < p^0 \leqslant 1$ with either the first or the last inequality being strict or both. By subtracting equation (4) from equation (3), we have $p^{t+1} - q^{t+1} = (p^t - q^t)^2$. Since $0 < p^0 - q^0 < 1$, we have $\lim_{t \to \infty} (p^t - q^t) = 0$. In other words, $\lim_{t \to \infty} p^t = \lim_{t \to \infty} q^t = r^*$, for some $r^* \in [0, 1]$.

By the same logic, we can prove similar results for the case $p^t < q^t$. For (p^0, q^0) that satisfies $0 \leqslant p^0 < q^0 \leqslant 1$ with either the first or the last inequality being strict or both, we have $\lim_{t \to \infty} p^t = \lim_{t \to \infty} q^t = r^*$, or some $r^* \in [0, 1]$.

The steady state $(1, 0)$ is unstable because if $(p^0, q^0) = (1, \varepsilon)$ for some arbitrarily small $\varepsilon > 0$, $\lim_{t \to \infty} p^t = \lim_{t \to \infty} q^t = 1$. Similarly, $(0, 1)$ is unstable.

Now consider (r^*, r^*) for some $r^* \in [0, 1]$. For any arbitrarily small $\varepsilon > 0$ and $\delta > 0$, if $(p^0, q^0) = (r^* + \varepsilon, r^* + \delta)$, then (p^0, q^0) cannot be either $(0, 1)$ or $(1, 0)$. According to the above analysis, $\lim_{t \to \infty} p^t = \lim_{t \to \infty} q^t$. Hence, $\{(r^*, r^*) | r^* \in [0, 1]\}$ constitutes a stable set of steady states. □

B.2 Proof of Proposition 1

Proof of Proposition 1. The cultural evolution when $p^t \geqslant q^t$ is characterized by the following dynamic system:

$$p^{t+1} = p^t + d(1 - p^t)(q^t - p^t); \tag{18}$$

$$q^{t+1} = q^t + dq^t(p^t - q^t). \tag{19}$$

When $p^0 = q^0$, we have $p^t = q^t$ for any t. Consider (p^0, q^0) that satisfies $0 \leqslant q^0 < p^0 \leqslant 1$ with either the first or the last inequality being strict or both. By subtracting equation (4) from equation (3), we have $p^{t+1} - q^{t+1} = (1-d)(p^t - q^t) + d(p^t - q^t)^2$. Since $0 < p^0 - q^0 < 1$ and $d > 0$, we have $\lim_{t \to \infty} (p^t - q^t) = 0$. In other words, $\lim_{t \to \infty} p^t = \lim_{t \to \infty} q^t = r^*$, for some $r^* \in [0, 1]$.

By the same logic, we can prove similar results for the case $p^t < q^t$. For (p^0, q^0) that satisfies $0 \leqslant p^0 < q^0 \leqslant 1$ with either the first or the last inequality being strict or both, we have $\lim_{t \to \infty} p^t = \lim_{t \to \infty} q^t = r^*$, or some $r^* \in [0, 1]$.

The steady state $(1, 0)$ is unstable because if $(p^0, q^0) = (1, \varepsilon)$ for some arbitrarily small $\varepsilon > 0$, $\lim_{t \to \infty} p^t = \lim_{t \to \infty} q^t = 1$. Similarly, $(0, 1)$ is unstable. □

C   Proofs and omitted details in Section 4

C.1   Proof of Proposition 2

Proof of Proposition 2. Let us consider the case where the probability of direct transmission in homogamies is a constant, and it equals $d \in (0, 1]$. It encompasses the perfect vertical transmission case $d = 1$ and the inelastic imperfect vertical transmission case $d \in (0, 1)$. The cultural evolution when $p^t + q^t > 1$ is characterized by the following dynamic system:

$$p^{t+1} = p^t + d(1 - p^t)(p^t + q^t - 1); \tag{20}$$

$$q^{t+1} = q^t + d(1 - q^t)(p^t + q^t - 1). \tag{21}$$

$d > 0$ implies that $p^{t+1} > p^t$, $q^{t+1} > q^t$. Consider the region T_ε in which $p^t + q^t \geqslant 1 - \varepsilon$, for some arbitrarily small $\varepsilon > 0$. T_ε is a compact set since it is closed and bounded. For any $(p^t, q^t) \in T_\varepsilon$, we have $|(p^t, q^t), (1, 1)| > |(p^{t+1}, q^{t+1}), (1, 1)|$. Hence, the dynamic system is a contraction mapping in the compact set T_ε, and by the contraction mapping theorem, (p^t, q^t) converges to $(1, 1)$ as time approaches infinity. Hence, $(1, 1)$ is attracting. Also, since the distance between (p^t, q^t) and $(1, 1)$ is monotonically decreasing in t, $(1, 1)$ must be stable. Therefore, it is asymptotically stable. Since ε is arbitrarily small, we can say that for any (p^0, q^0) such that $p^0 + q^0 > 1$, $\lim_{t \to \infty} p^t = \lim_{t \to \infty} q^t = 1$.

By using the same logic, we can prove that for any (p^0, q^0) that satisfies $p^0 + q^0 < 1$, $\lim_{t \to \infty} p^t = \lim_{t \to \infty} q^t = 0$. Also, $(0, 0)$ is asymptotically stable.

Now consider a steady state $(r^*, 1 - r^*)$ for some $r^* \in [0, 1]$. For any arbitrarily small $\varepsilon > 0$, if $(p^0, q^0) = (r^* + \varepsilon, 1 - r^*)$, $p^0 + q^0 > 1$. According to the above analysis, $\lim_{t \to \infty} p^t = \lim_{t \to \infty} q^t = 1$. Hence,

any steady state $(r^*, 1 - r^*)$ for $r^* \in [0, 1]$ is unstable.

Now consider imperfect transmission with cultural substitutability $d'(r) < 0$. When $p^t + q^t < 1$, the cultural evolution is characterized by

$$p^{t+1} - p^t = -\mu_{bb}^t d(1 - p^t)p^t = -(1 - p^t - q^t)d(1 - p^t)p^t < 0; \qquad (22)$$

$$q^{t+1} - q^t = -\mu_{bb}^t d(1 - q^t)q^t = -(1 - p^t - q^t)d(1 - q^t)q^t < 0. \qquad (23)$$

By equations (22) and (23), the system tends toward $(0, 0)$. Using the same argument of the contraction mapping theorem as in Proposition 2, we can say that for any (p^0, q^0) such that $p^0 + q^0 < 1$, $\lim_{t \to \infty} p^t = \lim_{t \to \infty} q^t = 0$. When $p^t + q^t > 1$, the cultural evolution is characterized by

$$p^{t+1} - p^t = \mu_{aa}^t d(p^t)(1 - p^t) = (p^t + q^t - 1)d(p^t)(1 - p^t) > 0; \qquad (24)$$

$$q^{t+1} - q^t = \mu_{aa}^t d(p^t)(1 - q^t) = (p^t + q^t - 1)d(p^t)(1 - q^t) > 0. \qquad (25)$$

By equations (24) and (25), the system tends toward $(1, 1)$. Using the same argument of the contraction mapping theorem as above, we can say that for any (p^0, q^0) such that $p^0 + q^0 > 1$, $\lim_{t \to \infty} p^t = \lim_{t \to \infty} q^t = 1$. And when $p^t + q^t = 1$, the cultural evolution is characterized by

$$p^{t+1} - p^t = 0;$$
$$q^{t+1} - q^t = 0.$$

In summary, the asymptotically stable steady states are $(0, 0)$ and $(1, 1)$, and any (p^*, q^*) such that $p^* + q^* = 1$ is an unstable steady state. $\qquad \square$

C.2 Stable matching with mixtures of homophilic and heterophilic proposers and receivers (Section 4.2)

Homophilic men and women of the same type, M_{θ_1} and W_{θ_1}, want to be matched together. Hence, at any stable matching the mass of a_1a_1 couples is $\min\{p^t h_{ma}, q^t h_{wa}\}$, i.e.,

$$\begin{cases} q^t h_{wa} & \text{if} \quad q^t < \left(\frac{h_{ma}}{h_{wa}}\right)p^t =: h_1(p^t), \\ p^t h_{ma} & \text{otherwise.} \end{cases}$$

Similarly, the mass of b_1b_1 couples is $\min\{(1 - p^t)h_{mb}, (1 - q^t)h_{wb}\}$, i.e.,

$$\begin{cases} (1 - p^t)h_{mb} & \text{if} \quad q^t < 1 - (1 - p^t)\left(\frac{h_{mb}}{h_{wb}}\right) =: h_2(p^t), \\ (1 - q^t)h_{wb} & \text{otherwise.} \end{cases}$$

Moreover, for $\theta \neq \theta'$, heterophilic men and women of the opposite types, M_{θ_2} and $W_{\theta_2'}$, want to be matched together. Hence, at any stable matching the mass of ab couples must be at least $\min\{p^t(1 -$

$h_{ma}), (1 - q^t)(1 - h_{wb})\}$, or equivalently

$$\begin{cases} p^t(1 - h_{ma}) & \text{if} \quad q^t < 1 - \left(\frac{1 - h_{ma}}{1 - h_{wb}}\right) p^t =: h_3(p^t), \\ (1 - q^t)(1 - h_{wb}) & \text{otherwise.} \end{cases}$$

and the mass of ba couples must be at least $\min\{(1 - p^t)(1 - h_{mb}), q^t(1 - h_{wa})\}$, or equivalently

$$\begin{cases} q^t(1 - h_{wa}) & \text{if} \quad q^t < \left(\frac{1 - h_{mb}}{1 - h_{wa}}\right)(1 - p^t) =: h_4(p^t), \\ (1 - p^t)(1 - h_{mb}) & \text{otherwise.} \end{cases}$$

(a) $\tau \leqslant \rho$ (b) $\tau > \rho$ (c) Matching-outcome state space partition
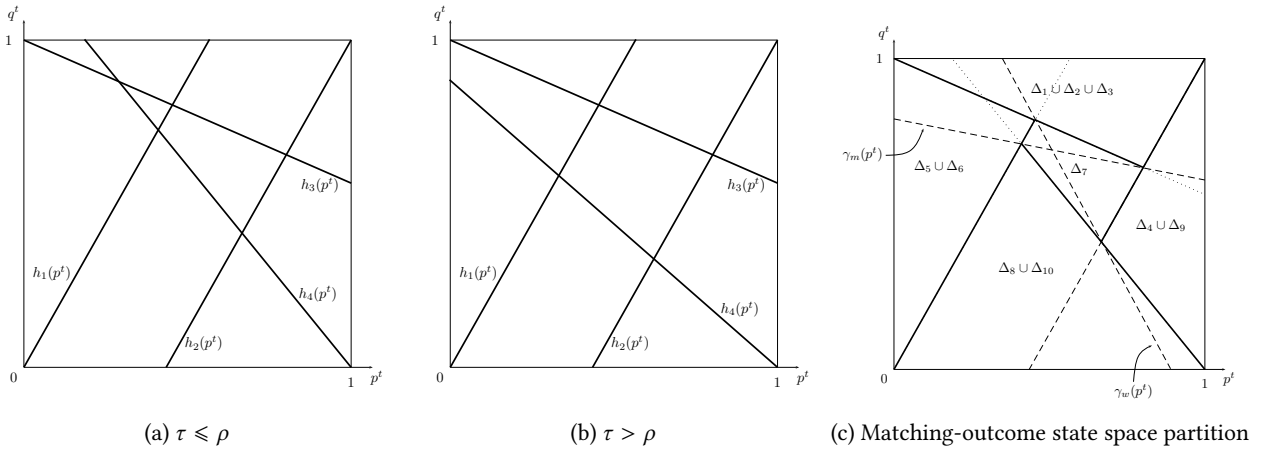
Figure 11: Partitioning regions for the characterization of stable matching.

Figure 11 depicts the functions $h_1(p^t)$, $h_1(p^t)$, $h_1(p^t)$ and $h_4(p^t)$. To derive this figure, we use the fact that $h_1(0) = 0$ and $h_1(1) = 1/\rho > 1$, $h_2(0) = 1 - 1/\rho < 0$ and $h_2(1) = 1$, $h_3(0) = 1$ and $h_3(1) = \frac{h_{ma}(1 - \rho\tau)}{1 - \rho\tau h_{ma}} \in [0, 1]$, $h_4(1) = 0$ and $h_4(0) = \frac{1 - \tau h_{ma}}{1 - \rho h_{ma}}$ which is lower than 1 iff $\tau > \rho$. Hence, when $\tau \leqslant \rho$ (Figure 11a), $h_3(p^t)$ crosses $h_4(p^t)$ once, while, when $\tau > \rho$ (Figure 11b), $h_3(p^t)$ is always above $h_4(p^t)$. Finally, we can verify that, when $\tau \leqslant \rho$, the crossing point between $h_3(p^t)$ and $h_4(p^t)$ is located above $h_1(p^t)$. To see this, we can note that the value of p^t such that $h_1(p^t) = h_3(p^t)$, $p_{13} = \frac{\rho(1 - \tau\rho h_{ma})}{1 + \rho(1 - h_{ma}(1 + \tau))}$, is higher than the value of p^t such that $h_1(p^t) = h_4(p^t)$, $p_{14} = \frac{\rho(1 - \tau h_{ma})}{1 + \rho(1 - h_{ma}(1 + \tau))}$. As illustrated in Figure 11, $h_1(p^t)$, $h_2(p^t)$, $h_3(p^t)$ and $h_4(p^t)$ partition the unit square in the following ten disjoint sets:[28]

$$\begin{aligned} \Delta_1 &:= \left\{ (p^t, q^t) \in [0, 1]^2 : h_3(p^t) > q^t \geqslant h_4(p^t) \right\} \\ \Delta_2 &:= \left\{ (p^t, q^t) \in [0, 1]^2 : q^t \geqslant \max\{h_1(p^t), h_3(p^t), h_4(p^t)\} \right\}; \\ \Delta_3 &:= \left\{ (p^t, q^t) \in [0, 1]^2 : h_1(p^t) > q^t \geqslant \max\{h_2(p^t), h_3(p^t)\} \right\}; \\ \Delta_4 &:= \left\{ (p^t, q^t) \in [0, 1]^2 : h_2(p^t) > q^t \geqslant h_3(p^t) \right\}; \\ \Delta_5 &:= \left\{ (p^t, q^t) \in [0, 1]^2 : \min\{h_3(p^t), h_4(p^t)\} > q^t \geqslant h_1(p^t) \right\}; \end{aligned}$$

---

[28] Note that the set Δ_1 is nonempty only when $\tau \leqslant \rho$ (see Figure 11a).

$$\Delta_6 \;:=\; \left\{(p^t,q^t) \in [0,1]^2 : h_3(p^t) > q^t \geqslant \max\{h_1(p^t), h_4(p^t)\}\right\};$$

$$\Delta_7 \;:=\; \left\{(p^t,q^t) \in [0,1]^2 : \min\{h_1(p^t), h_3(p^t)\} > q^t \geqslant \max\{h_2(p^t), h_4(p^t)\}\right\};$$

$$\Delta_8 \;:=\; \left\{(p^t,q^t) \in [0,1]^2 : \min\{h_1(p^t), h_4(p^t)\} > q^t \geqslant h_2(p^t)\right\};$$

$$\Delta_9 \;:=\; \left\{(p^t,q^t) \in [0,1]^2 : \min\{h_2(p^t), h_3(p^t)\} > q^t \geqslant h_4(p^t)\right\};$$

$$\Delta_{10} \;:=\; \left\{(p^t,q^t) \in [0,1]^2 : \min\{h_2(p^t), h_4(p^t)\} > q^t\right\}.$$

Below, we describe the stable matching in each region.

Region	μ_{aa}	μ_{ba}	μ_{bb}	μ_{ab}
Δ_1	$p - (1-q)(1-h_{wb})$	$1 - p - (1-q)h_{wb}$	$(1-q)h_{wb}$	$(1-q)(1-h_{wb})$
Δ_2	$p - (1-q)(1-h_{wb})$	$1 - p - (1-q)h_{wb}$	$(1-q)h_{wb}$	$(1-q)(1-h_{wb})$
Δ_3	$p - (1-q)(1-h_{wb})$	$1 - p - (1-q)h_{wb}$	$(1-q)h_{wb}$	$(1-q)(1-h_{wb})$
Δ_4	$q - (1-p)(1-h_{mb})$	$(1-p)(1-h_{mb})$	$(1-p)h_{mb}$	$1 - q - (1-p)h_{mb}$
Δ_5	ph_{ma}	$q - ph_{ma}$	$1 - q - p(1-h_{ma})$	$p(1-h_{ma})$
Δ_6	ph_{ma}	$q - ph_{ma}$	$1 - q - p(1-h_{ma})$	$p(1-h_{ma})$
Δ_8	qh_{wa}	$q(1-h_{wa})$	$1 - p - q(1-h_{wa})$	$p - qh_{wa}$
Δ_9	$q - (1-p)(1-h_{mb})$	$(1-p)(1-h_{mb})$	$(1-p)h_{mb}$	$1 - q - (1-p)h_{mb}$
Δ_{10}	qh_{wa}	$q(1-h_{wa})$	$1 - p - q(1-h_{wa})$	$p - qh_{wa}$

If $(p,q) \in \Delta_7$, MOSM and WOSM do not coincide. Define $\gamma_w(p) := \frac{1 - p^t - h_{wb}}{1 - h_{wa} - h_{wb}}$ and $\gamma_m(p) := 1 - h_{mb} - p^t(1 - h_{ma} - h_{mb})$. The stable matching in Δ_7 is described as follows.

	μ_{aa}	μ_{ba}	μ_{bb}	μ_{ab}
		WOSM		
$q < \gamma_w(p)$	qh_{wa}	$q(1-h_{wa})$	$1 - p - q(1-h_{wa})$	$p - qh_{wa}$
$q \geqslant \gamma_w(p)$	$p - (1-q)(1-h_{wb})$	$1 - p - (1-q)h_{wb}$	$(1-q)h_{wb}$	$(1-q)(1-h_{wb})$
		MOSM		
$q < \gamma_m(p)$	$q - (1-p)(1-h_{mb})$	$(1-p)(1-h_{mb})$	$(1-p)h_{mb}$	$1 - q - (1-p)h_{mb}$
$q \geqslant \gamma_m(p)$	ph_{ma}	$q - ph_{ma}$	$1 - q - p(1-h_{ma})$	$p(1-h_{ma})$

We can easily verify that, $\gamma_w(p^t)$ passes through the crossing point between $h_1(p^t)$ and $h_3(p^t)$ and the crossing point between $h_2(p^t)$ and $h_4(p^t)$ while $\gamma_m(p^t)$ passes through the crossing point between $h_1(p^t)$ and $h_4(p^t)$ and the crossing point between $h_2(p^t)$ and $h_3(p^t)$. The two functions are depicted in Figure 11c that also illustrates a new partition of the unit square, in which each region corresponds to a particular stable matching.

C.3 Proof of Proposition 3

Construction of the phase diagrams in Figure 4. Cultural evolution is driven by the following two dimensional dynamical system:

$$p^{t+1} \;=\; \mu_{aa}^t + (\mu_{ab}^t + \mu_{ba}^t)p^t;$$

$$q^{t+1} \;=\; \mu_{aa}^t + (\mu_{ab}^t + \mu_{ba}^t)q^t.$$

In the following, for each region partitioning the unit square in Figure 11c, we will replace the proportions μ_{aa}, μ_{ab} and μ_{ba} by those corresponding to the stable matching. Before that, let us define the following functions that will be useful in our analysis:

$$
\begin{aligned}
\kappa(x;h) &:= 1 - h - x(1 - 2h); \\
f(x;h,\lambda) &:= 1 - \frac{[x - \lambda(1-x)](1-x)}{1 - h - x(1-2h)}; \\
k(x;h,\lambda) &:= \frac{x[1 - (1+\lambda)x]}{h + x(1 - 2h)}; \\
g(x;h,\lambda) &:= \frac{1 - h - (1+\lambda)x}{1 - 2h} + \frac{\lambda(h - h_{wa})}{(1 - 2h)(h_{wb} - h_{wa})}.
\end{aligned}
$$

If $(p^t, q^t) \in \Delta_1 \cup \Delta_2 \cup \Delta_3$:

$$
\begin{aligned}
p^{t+1} &= p^t - (1 - q^t)(1 - h_{wb}) + \left[1 - p^t + (1 - q^t)(1 - 2h_{wb})\right] p^t; \\
q^{t+1} &= p^t - (1 - q^t)(1 - h_{wb}) + \left[1 - p^t + (1 - q^t)(1 - 2h_{wb})\right] q^t.
\end{aligned}
$$

Hence, $p^{t+1} > p^t$ iff $q^t > f(p^t; h_{wb}, 0)$. Given the properties of $f(p^t; h_{wb}, 0)$, $h_1(p^t)$ and $h_3(p^t)$, for all $(p^t, q^t) \in \Delta_1 \cup \Delta_2 \cup \Delta_3$, q^t is higher than $f(p^t; h_{wb}, 0)$ such that $p^{t+1} > p^t$. Moreover, $q^{t+1} > q^t$ iff $q^t > g(p^t; h_{wb}, 0)$ which is downward slopping (if $h_{wb} < 1/2$ with $g(0; h_{wb}, 0) = \frac{1 - h_{wb}}{1 - 2h_{wb}} > 1$, $g(1/2; h_{wb}, 0) = 1/2$ and $g(1; h_{wb}, 0) = \frac{-h_{wb}}{1 - 2h_{wb}} < 0$.

If $(p^t, q^t) \in \Delta_5 \cup \Delta_6$:

$$
\begin{aligned}
p^{t+1} &= p^t h_{ma} + \left[q^t + p^t(1 - 2h_{ma})\right] p^t; \\
q^{t+1} &= p^t h_{ma} + \left[q^t + p^t(1 - 2h_{ma})\right] q^t.
\end{aligned}
$$

Hence, $p^{t+1} > p^t$ iff $q^t > \kappa(p^t; h_{ma})$ which is downward slopping (if $h_{ma} < 1/2$) with $\kappa(0; h_{ma}) = 1 - h_{ma} \in (0, 1)$, $\kappa(1/2; h_{ma}) = 1/2$ and $\kappa(1; h_{ma}) = h_{ma} \in (0, 1)$. Moreover, $q^{t+1} > q^t$ iff, $p^t > k(q^t; h_{ma}, 0)$.

If $(p^t, q^t) \in \Delta_8 \cup \Delta_{10}$:

$$
\begin{aligned}
p^{t+1} &= q^t h_{wa} + \left[p^t + q^t(1 - 2h_{wa})\right] p^t; \\
q^{t+1} &= q^t h_{wa} + \left[p^t + q^t(1 - 2h_{wa})\right] q^t.
\end{aligned}
$$

Hence, $p^{t+1} > p^t$ iff $q^t > k(p^t; h_{wa}, 0)$. Moreover, $q^{t+1} > q^t$ iff $q^t > g(p^t; h_{wa}, 0)$ with $g(0; h_{wa}, 0) = \frac{1 - h_{wa}}{1 - 2h_{wa}} > 1$, $g(1/2; h_{wa}, 0) = 1/2$ and $g(1; h_{wa}, 0) = \frac{-h_b}{1 - 2h_{wa}} < 0$.

If $(p^t, q^t) \in \Delta_4 \cup \Delta_9$:

$$p^{t+1} = q^t - (1 - p^t)(1 - h_{mb}) + \left[1 - q^t + (1 - p^t)(1 - 2h_{mb})\right] p^t;$$
$$q^{t+1} = q^t - (1 - p^t)(1 - h_{mb}) + \left[1 - q^t + (1 - p^t)(1 - 2h_{mb})\right] q^t.$$

Hence, $p^{t+1} > p^t$ iff $q^t > \kappa(p^t; h_{mb})$ with $\kappa(0; h_{mb}) = 1 - h_{mb} \in (0, 1)$, $\kappa(1/2; h_{mb}) = 1/2$ and $\kappa(1; h_{mb}) = h_{mb} \in (0, 1)$. Moreover, $q^{t+1} > q^t$ iff $p^t > f(q^t; h_{mb}, 0)$, which is always true when $(p^t, q^t) \in \Delta_4 \cup \Delta_9$.

If $(p^t, q^t) \in \Delta_7$:

- Consider WOSM. If $q^t < \gamma_w(p^t)$, the dynamic system is the same as in the case $(p^t, q^t) \in \Delta_8 \cup \Delta_{10}$ such that $p^{t+1} > p^t$ iff $q^t > k(p^t; h_{wa}, 0)$ and $q^{t+1} < q^t$ iff $q^t < g(p^t; h_{wa}, 0)$. If $q^t \geqslant \gamma_w(p^t)$, the dynamic system is the same as in the case $(p^t, q^t) \in \Delta_1 \cup \Delta_2 \cup \Delta_3$ such that $p^{t+1} > p^t$ iff $q^t > f(p^t; h_{wb}, 0)$ and $q^{t+1} > q^t$ iff $q^t > g(p^t; h_{wb}, 0)$. Now, $g(p^t; h_{wb}, 0)$ is always below $\gamma_w(p^t)$, such that, if $q^t \geqslant \gamma_w(p^t)$, $q^{t+1} > q^t$. To sum-up, the line of equation $q^t = g(p^t; h_{wa}, 0)$—which is below the line $q^t = \gamma_w(p^t)$—corresponds to the locus of stationarity of q^t; while the curve of equation $q^t = k(p^t; h_{wa}, 0)$ if $q^t < \gamma_w(p^t)$ and $q^t = f(p^t; h_{wb}, 0)$ for $q^t \geqslant \gamma_w(p^t)$ corresponds to the stationnary locus of p^t.

- Consider MOSM. If $q^t < \gamma_m(p^t)$, the dynamic system is the same as in the case $(p^t, q^t) \in \Delta_4 \cup \Delta_9$ such that $q^{t+1} > q^t$ iff $p^t > f(q^t; h_{mb}, 0)$ and $p^{t+1} < p^t$ iff $q^t > \kappa(p^t; h_{mb})$. If $q^t \geqslant \gamma_m(p^t)$, the dynamic system is the same as in the case $(p^t, q^t) \in \Delta_5 \cup \Delta_6$ such that $q^{t+1} > q^t$ iff $p^t > k(q^t; h_{ma}, 0)$ and $p^{t+1} > p^t$ iff $q^t > \kappa(p^t; h_{ma})$. Now, $\kappa(p^t; h_{ma})$ is below $\gamma_m(p^t)$ such that, if $q^t \geqslant \gamma_m(p^t)$, $p^{t+1} > p^t$. To sum-up, the line of equation $q^t = \kappa(p^t; h_{mb})$, which is below the line $q^t = \gamma_m(p^t)$, corresponds to the stationnary locus of p^t, while the curve of equation $p^t = k(q^t; h_{ma}, 0)$ if $q^t \geqslant \gamma_m(p^t)$ and $p^t = f(q^t; h_{mb}, 0)$ if $q^t < \gamma_m(p^t)$ corresponds to the stationnary locus of q^t.

From the analysis above, we can draw the qq and pp curves correspond to the stationary locus of q^t and p^t respectively. Several configurations according to the regions the two locus belong to. However, in all possible instances and for both MOSM and WOSM, the qq locus always cross the pp locus only once, in $(1/2, 1/2)$ and from above. Moreover, qq and pp are continuous—since the proportions of each type of couple at stable matching are so—and the analysis above allows us to conclude that pp is stable while qq is unstable.

Proof of Proposition 3. We first show that $(0, 0)$ is asymptotically stable. Consider the region T_ε in which $p^t + q^t \leqslant 1 - \varepsilon$, for some arbitrarily small $\varepsilon > 0$. T_ε is a compact set since it is closed and bounded. For any $(p^t, q^t) \in T_\varepsilon$, we have

$$p^{t+1} + q^{t+1} = 2\mu_{aa}^t + (\mu_{ab}^t + \mu_{ba}^t)(p^t + q^t)$$
$$< \mu_{aa}^t + \mu_{ab}^t + \mu_{aa}^t + \mu_{ba}^t = p^t + q^t.$$

The inequality comes from the fact that, $p^t + q^t < 1$ (since $(p^t, q^t) \in T_\varepsilon$) while the last equality holds since,

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at any stable matching all type a men and all type a women must be matched, such that: $\mu_{aa}^t + \mu_{ab}^t = p^t$ and $\mu_{aa}^t + \mu_{ba}^t = q^t$.

This implies that for any $(p^t, q^t) \in T_\varepsilon$, $p^t + q^t > p^{t+1} + q^{t+1}$. Hence, $p^t + q^t$ converges to 0 as time approaches infinity by the monotone convergence theorem, implying that (p^t, q^t) converges to 0 as time approaches infinity as well. Hence, $(0,0)$ is attracting. Also, for any $\delta > 0$, for any (p^0, q^0) satisfies that $\sqrt{(p^0)^2 + (q^0)^2} < \frac{\sqrt{2}}{2}\delta$, we have $\sqrt{(p^t)^2 + (q^t)^2} \leqslant p^t + q^t < p^0 + q^0 < \delta$, given that $p^t + q^t$ is monotonically decreasing in t. Hence $(0,0)$ is stable. Therefore, it is asymptotically stable. Since ε is arbitrarily small, we can say that for any (p^0, q^0) such that $p^0 + q^0 < 1$, (p^t, q^t) converges to $(0,0)$ as time approaches infinity.

By applying the same logic, we can show that $(1,1)$ is asymptotically stable and for any (p^0, q^0) such that $p^0 + q^0 > 1$, (p^t, q^t) converges to $(1,1)$ as time approaches infinity.

To further check the stability properties of $(1/2, 1/2)$ we first have to note that, $(1/2, 1/2)$ may either belongs to Region Δ_7 if $h_4(1/2) < 1/2$ (i.e., $h_{wa} < h_{mb}$), or to Region $\Delta_8 \cup \Delta_{10}$ otherwise. Then, we have to check the Jacobian matrix of the dynamics evaluated at $(\frac{1}{2}, \frac{1}{2})$ in Region Δ_7 for MOSM and WOSM and in Region $\Delta_8 \cup \Delta_{10}$. Since, considering WOSM (resp., MOSM), the dynamics in Region Δ_7 is identical to the dynamics in Region $\Delta_8 \cup \Delta_{10}$ or $\Delta_1 \cup \Delta_2 \cup \Delta_3$ (resp., Region $\Delta_4 \cup \Delta_9$ or $\Delta_5 \cup \Delta_6$), the Jacobian matrix at $(\frac{1}{2}, \frac{1}{2})$ has only to be checked in these four regions.

In region $\Delta_1 \cup \Delta_2 \cup \Delta_3$, the Jacobian matrix evaluated at $(1/2, 1/2)$, is

$$\begin{bmatrix} 2(1-p) + (1-2h_{wb})(1-q) & 1 - h_{wb} - (1-2h_{wb})p \\ 1-q & 3(1-h_{wb}) - p - 2(1-2h_{wb})q \end{bmatrix}_{(p=\frac{1}{2}, q=\frac{1}{2})} = \begin{bmatrix} \frac{3}{2} - h_{wb} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} - h_{wb} \end{bmatrix}.$$

In region $\Delta_4 \cup \Delta_9$, the Jacobian matrix evaluated at $(1/2, 1/2)$, is

$$\begin{bmatrix} 3(1-h_{mb}) - q - 2(1-2h_{mb})p & 1-p \\ 1 - h_{mb} - (1-2h_{mb})q & 2(1-q) + (1-2h_{mb})(1-p) \end{bmatrix}_{(p=\frac{1}{2}, q=\frac{1}{2})} = \begin{bmatrix} \frac{3}{2} - h_{mb} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} - h_{mb} \end{bmatrix}.$$

In region $\Delta_5 \cup \Delta_6$, the Jacobian matrix evaluated at $(1/2, 1/2)$, is

$$\begin{bmatrix} h_{ma} + q - 2(2h_{ma} - 1)p & p \\ h_{ma} - (2h_{ma} - 1)q & 2q - (2h_{ma} - 1)p \end{bmatrix}_{(p=\frac{1}{2}, q=\frac{1}{2})} = \begin{bmatrix} \frac{3}{2} - h_{ma} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} - h_{ma} \end{bmatrix}.$$

In region $\Delta_8 \cup \Delta_{10}$, the Jacobian matrix evaluated at $(1/2, 1/2)$, is

$$\begin{bmatrix} 2p - (2h_{wa} - 1)q & h_{wa} - (2h_{wa} - 1)p \\ q & h_{wa} + p - 2(2h_{wa} - 1)q \end{bmatrix}_{(p=\frac{1}{2}, q=\frac{1}{2})} = \begin{bmatrix} \frac{3}{2} - h_{wa} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} - h_{wa} \end{bmatrix}.$$

In all four configurations, the eigenvalues are given by $1 - h < 1$ and $2 - h > 1$ (where h equals to h_{wb}, h_{mb}, h_{ma} or h_{wa}). Hence, $(\frac{1}{2}, \frac{1}{2})$ is a saddle point.

Finally, when $p^t + q^t = 1$ we must have $p^{t+1} + q^{t+1} = 1$. In this case, for $p^t = 1 - p^t > 1/2$, we have:

$$p^{t+1} > \mu_{aa}^t + \frac{1}{2}\left[\mu_{ab}^t + \mu_{ba}^t\right] = \frac{1}{2}\left[2\mu_{aa}^t + \mu_{ab}^t + \mu_{ba}^t\right] = \frac{p^t + q^t}{2} = \frac{1}{2}.$$

Moreover, since $p^t = \mu^t_{aa} + \mu^t_{ab}$ and $1 - q^t = \mu^t_{bb} + \mu^t_{ab}$, the condition $p^t = 1 - q^t$ implies $\mu^t_{aa} = \mu^t_{bb}$. Hence, in this case, $p^{t+1} = \mu^t_{aa} + p^t \left[1 - \mu^t_{aa} - \mu^t_{bb}\right]$ such that $p^{t+1} - p^t = \mu^t_{aa}(1 - 2p^t)$ wich is negative if $p^t > 1/2$. Then by the monotone convergence theorem, for any $p^0 = 1 - q^0$, as time goes to infinity, p^t converges to $\frac{1}{2}$, which automatically implies that q^t converges to $\frac{1}{2}$ as well.

Hence, the unique saddle path that converges toward $(1/2, 1/2)$ and that splits the state space between the basin of attraction of $(0,0)$ $(p^t + q^t < 1)$ and the basin of attraction of $(1,1)$ $(p^t + q^t > 1)$ exactly corresponds to the straight line $q^t = 1 - p^t$.

The above result is straightforwardly generalized to the case of inelastic vertical transmission in homogamies. To see this, consider $d(r) = d \in (0,1]$. The system (1)-(2), that characterizes the cultural dynamics, can be rewritten as

$$p^{t+1} = p^t + d\left[(1 - p^t)\mu^t_{aa} - p^t\mu^t_{bb}\right]; \tag{26}$$

$$q^{t+1} = q^t + d\left[(1 - q^t)\mu^t_{aa} - q^t\mu^t_{bb}\right]. \tag{27}$$

One can observe that, for both $d = 1$ (perfect vertical transmission) and $d \in (0,1)$ (inelastic vertical transmission), $p^{t+1} > p^t$ iff $(1 - p^t)\mu^t_{aa} > p^t\mu^t_{bb}$ and $q^{t+1} > q^t$ iff $(1 - q^t)\mu^t_{aa} > q^t\mu^t_{bb}$. Hence, the long-run distribution of cultural traits must be the same under perfect vertical transmission and inelastic vertical transmission. $\square$

C.4   Proof of Proposition 4

Proof of Proposition 4. Suppose $p^0 + q^0 > 1$. Under MOSM, the stable steady state is some $(p^*, q^*) \in (0,1)^2$ such that $p^* = q^*$ and the associate stable matching is $\mu^*_{aa} = p^*$, $\mu^*_{bb} = 1 - p^*$ and $\mu^*_{ab} = \mu^*_{ba} = 0$. Men's average payoff is $p^* U_{aa} + (1 - p^*)U_{bb}$ and women's average payoff is

$$p^*\left[h_{wa}V^1_{aa} + (1 - h_{wa})V^2_{aa}\right] + (1 - p^*)\left[h_{wb}V^1_{bb} + (1 - h_{wb})V^2_{bb}\right].$$

Under WOSM, the stable steady state is $(p^*, q^*) = (1,1)$ so that $\mu^*_{aa} = 1$ and $\mu^*_{bb} = \mu^*_{ab} = \mu^*_{ba} = 0$. Men's payoff is U_{aa} and women's average payoff is $h_{wa}V^1_{aa} + (1 - h_{wa})V^2_{aa}$.

When $U_{aa} > U_{bb}$ (resp., $U_{aa} < U_{bb}$), men are strictly better off (worse off) under MOSM than under WOSM in the long run. When $h_{wa}V^1_{aa} + (1 - h_{wa})V^2_{aa} > h_{wb}V^1_{bb} + (1 - h_{wb})V^2_{bb}$ (resp., $h_{wa}V^1_{aa} + (1 - h_{wa})V^2_{aa} < h_{wb}V^1_{bb} + (1 - h_{wb})V^2_{bb}$), women are strictly better (resp., worse off) under MOSM than under WOSM in the long run. $\square$

D   Omitted details with gender imbalance (Section 4.3.2)

We consider a market with a mass $p(1 + \lambda)$ of type-a men, a mass $(1 - p)(1 + \lambda)$ of type-b men, a mass q of type-a women and a mass $(1 - q)$ of type-b women. All men have homophilic preferences ($h_{ma} = h_{mb} = 1$) and not all women have homophilic preferences ($h_{wa} < 1$ or $h_{wb} < 1$).[29]

---

[29]For the sake of clarity, we present the results in the case $h_{wa} + h_{wb} < 1$ but they all generalize to any $(h_{wa}, h_{wb}) \in [0,1)^2$.

D.1 Stable matching with gender imbalance

The two following intermediary results are useful for characterizing the stable matching in the presence of gender imbalance.

Lemma 4. *At any stable matching, if there exists a positive mass of unmatched type-a men, all type-a_1 and b_2 women must be matched with a type-a man; and if there exists a positive mass of unmatched type-b men, all type-a_2 and b_1 women must be matched with a type-b man.*

Proof. Let us prove the first point, the proof of the second point would follow exactly the same logic. By contradiction, assume that there exists a positive mass of unmatched type-a men and a positive mass of either type-a_1 or type-b_2 women who are either unmatched or matched with a type-b man. Then these women will form blocking pairs with the unmatched type-a men. Hence, the matching cannot be stable. $\square$

Lemma 5. *At any stable matching, either all type-a_1 and b_2 women are matched with a type-a man, or all type-a_2 and b_1 women are matched with a type b-man (or both).*

Proof. There are more men than women such that, at any matching, some men (either of type-a, of type-b or of both types) will remain unmatched. The result of the lemma is directly derived from this simple fact associated with Lemma 4. $\square$

According to Lemma 5, we have three main configurations to consider. Define

$$\phi_1(p) := \frac{1 - h_{wb} - (1 + \lambda)p}{1 - h_{wb} - h_{wa}} \quad \text{and} \quad \phi_2(p) := \frac{1 - h_{wb} + \lambda - (1 + \lambda)p}{1 - h_{wb} - h_{wa}},$$

we can easily verify that, *i.* if $q < \phi_1(p)$, the number of type-a_1 and b_2 women exceeds the number of type-a men while the number of type-b men exceeds the number of type-a_2 and b_1 women; *ii.* if $q \geqslant \phi_2(p)$, the number of type-a men exceeds the number of type-a_1 and b_2 women while the number of type-a_2 and b_1 women exceeds the number of type-b men; and *iii.* if $q \in [\phi_1(p), \phi_2(p))$, the number of type-$a$ men exceeds the number of type-a_1 and b_2 women and the number of type-b men exceeds the number of type-a_2 and b_1 women. Let us characterizes the stable matching in each of those three configurations.

If $q < \phi_1(p)$ (case *i.*) we must have a mass $q(1 - h_{wa})$ of ba_1 couples and a mass $(1 - q)h_{wb}$ of bb_1 couples. Moreover, at any stable matching the mass of aa_1 couples is $\min\{(1 + \lambda)p, qh_{wa}\}$, i.e., $(1 + \lambda)p$ if $q \geqslant (1 + \lambda)p/h_{wa} =: \widetilde{\phi}_1(p)$ (note that both $\widetilde{\phi}_1(p)$ and $\phi_1(p)$ equal 1 for $p = h_{wa}/(1 + \lambda)$). In the latter case, all a men are matched with a_1 women such that the remaining a_1 women and all b_2 women will be matched with a b man: we will have a mass $qh_{wa} - (1 + \lambda)p$ of ba_1 couples and a mass $(1 - q)(1 - h_{wb})$ of bb_2 couples. In the case $q < \widetilde{\phi}_1(p)$, all the remaining a men will be matched with b_2 women and the residuals b_2 women with b men: we will have a mass $(1 + \lambda)p - qh_{wa}$ of ab_1 couples and a mass $(1 - q)(1 - h_{wb}) - (1 + \lambda)p + qh_{wa}$ of bb_2 couples.

If $q \geqslant \phi_2(p)$ (case *ii.*) we must have a mass qh_{wa} of aa_1 couples and a mass $(1 - q)(1 - h_{wb})$ of ab_2 couples. Moreover, at any stable matching the mass of bb_1 couples is $\min\{(1 + \lambda)(1 - p), (1 - q)h_{wb}\}$, i.e., $(1 + \lambda)(1 - p)$ if $q < 1 - (1 + \lambda)(1 - p)/h_{wb} =: \widetilde{\phi}_2(p)$ (note that both $\widetilde{\phi}_2(p)$ and $\phi_2(p)$ equal 0 for

$p = 1 - h_{wb}/(1 + \lambda))$. In the latter case, all b men are matched with b_1 women such that the remaining b_1 women and all a_2 women will be matched with a a man: we will have a mass $(1-q)h_{wb}-(1+\lambda)(1-p)$ of ab_1 couples and a mass $q(1-h_{wa})$ of aa_2 couples. In the case $q \geqslant \widetilde{\phi}_2(p)$, all the remaining b men will be matched with a_2 women and the residuals a_2 women with a men: we will have a mass $(1 + \lambda)(1 - p) - (1 - q)h_{wb}$ of ba_2 couples and a mass $a(1 - h_{wa}) - (1 + \lambda)(1 - p) + (1 - q)h_{wb}$ of aa_2 couples.

If $q \in [\phi_1(p), \phi_2(p))$ we must have a mass qh_{wa} of aa_1 couples, a mass $(1 - q)(1 - h_{wb})$ of ab_2, a mass $q(1 - h_{wa})$ of ba_1 couples and a mass $(1 - q)h_{wb}$ of bb_1 couples.
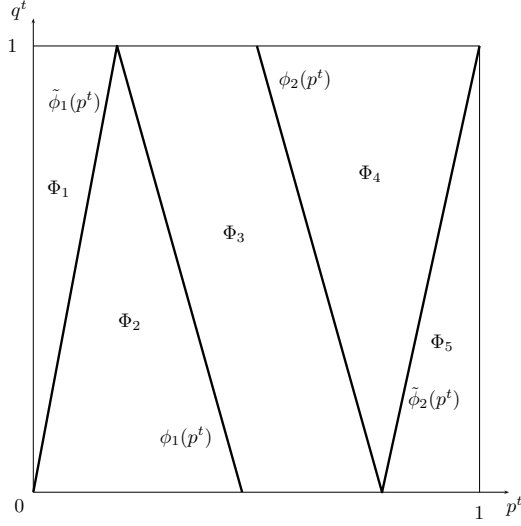

Figure 12: Matching-outcome state space partition

Hence, the unit square might be partitioned into the five following regions (see Figure 12)

$$\Phi_1 \quad := \quad \left\{(p, q) \in (0, 1)^2 : \widetilde{\phi}_1(p) \leqslant q\right\};$$

$$\Phi_2 \quad := \quad \left\{(p, q) \in (0, 1)^2 : q < \min\{\phi_1(p), \widetilde{\phi}_1(p)\}\right\};$$

$$\Phi_3 \quad := \quad \left\{(p, q) \in (0, 1)^2 : \phi_1(p) \leqslant q < \phi_2(p)\right\};$$

$$\Phi_4 \quad := \quad \left\{(p, q) \in (0, 1)^2 : \max\{\phi_2(p), \widetilde{\phi}_2(p)\} \leqslant q\right\};$$

$$\Phi_5 \quad := \quad \left\{(p, q) \in (0, 1)^2 : q < \widetilde{\phi}_2(p)\right\},$$

and the stable matching in each region is described in the following table.

Region	μ_{aa}	μ_{ba}	μ_{bb}	μ_{ab}
Φ_1	$p(1 + \lambda)$	$q - p(1 + \lambda)$	0	$1 - q$
Φ_2	qh_{wa}	$q(1 - h_{wa})$	$1 - q - p(1 + \lambda) + qh_{wa}$	$p(1 + \lambda) - qh_{wa}$
Φ_3	qh_{wa}	$q(1 - h_{wa})$	$(1 - q)h_{wb}$	$(1 - q)(1 - h_{wb})$
Φ_4	$q + (1 - q)h_{wb}$ $-(1 + \lambda)(1 - p)$	$(1 + \lambda)(1 - p)$ $-(1 - q)h_{wb}$	$(1 - q)h_{wb}$	$(1 - q)(1 - h_{wb})$
Φ_5	q	0	$(1 + \lambda)(1 - p)$	$1 - q - (1 + \lambda)(1 - p)$

Proof of Lemma 3. The results claimed in Lemma 3 can be directly deduced from the above description of the stable matching when we fix $h_{wa} = h_{wb} = 0$. Note that, in this case, we indeed have $\phi_1(p) = 1 - (1+\lambda)p$ and $\phi_2(p) = (1+\lambda)(1-p)$. □

D.2 Proof of Proposition 5

Construction of the phase diagrams in Figure 6. Cultural evolution is driven by the following two-dimensional dynamical system:

$$
\begin{aligned}
p^{t+1} &= \mu_{aa}^t + (\mu_{ab}^t + \mu_{ba}^t)p^t; \\
q^{t+1} &= \mu_{aa}^t + (\mu_{ab}^t + \mu_{ba}^t)q^t.
\end{aligned}
$$

In the following, for each region partitioning the unit square in Figure 12 we replace the proportions μ_{aa}, μ_{ab} and μ_{ba} by those corresponding to the stable matching (see Section D.1). In the following analysis, we will utilize the functions $f(x; h, \lambda)$, $k(x; h, \lambda)$ and $g(x; h, \lambda)$ defined in Appendix C.3.

If $(p^t, q^t) \in \Phi_1$:

$$
\begin{aligned}
p^{t+1} &= (1+\lambda)p^t + \left[q^t - (1+\lambda)p^t\right]p^t; \\
q^{t+1} &= (1+\lambda)p^t + \left[q^t - (1+\lambda)p^t\right]q^t.
\end{aligned}
$$

Since, in region Φ_1, $q^t \geqslant (1+\lambda)p^t/h_{wa} > (1+\lambda)p^t$, we must have $p^{t+1} > (1+\lambda)p^t > p^t$ such that p^t increases over time. Moreover, $q^{t+1} > q^t$ iff $(1+\lambda)p^t(1-p^t) > q^t(1-p^t)$ implying $(1+\lambda)p^t > q^t$ which contradicts the condition $q^t \geqslant (1+\lambda)p^t/h_{wa}$. Hence, q^t decreases over time.

If $(p^t, q^t) \in \Phi_2$:

$$
\begin{aligned}
p^{t+1} &= q^t h_{wa} + \left[(1+\lambda)p^t + q^t(1 - 2h_{wa})\right]p^t; \\
q^{t+1} &= q^t h_{wa} + \left[(1+\lambda)p^t + q^t(1 - 2h_{wa})\right]q^t.
\end{aligned}
$$

Straightforward algebra leads us to conclude that $p^{t+1} > p^t$ iff $q^t > k(p^t; h_{wa}, \lambda)$ and $q^{t+1} > q^t$ iff $q^t > g(p^t; h_{wa}, \lambda)$.

If $(p^t, q^t) \in \Phi_3$:

$$
\begin{aligned}
p^{t+1} &= q^t h_{wa} + \left[1 - h_{wb} + q^t(h_{wb} - h_{wa})\right]p^t; \\
q^{t+1} &= q^t h_{wa} + \left[1 - h_{wb} + q^t(h_{wb} - h_{wa})\right]q^t.
\end{aligned}
$$

Straightforward algebra leads us to conclude that $p^{t+1} > p^t$ iff

$$
q^t > \frac{p^t h_{wb}}{p^t h_{wb} + (1 - p^t)h_{wa}} =: \ell(p^t)
$$

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and $q^{t+1} > q^t$ iff $h_{wa} > h_{wb}$.

If $(p^t, q^t) \in \Phi_4$:

$$p^{t+1} = q^t + (1 - q^t)h_{wb} - (1 + \lambda)(1 - p^t) + \left[(1 + \lambda)p^t + (1 - q^t)(1 - 2h_{wb})\right] p^t;$$
$$q^{t+1} = q^t + (1 - q^t)h_{wb} - (1 + \lambda)(1 - p^t) + \left[(1 + \lambda)p^t + (1 - q^t)(1 - 2h_{wb})\right] q^t.$$

Straightforward algebras lead us to conclude that $p^{t+1} > p^t$ iff $q^t > f(p^t; h_{wb}, \lambda)$ and $q^{t+1} > q^t$ iff $q^t > g(p^t; h_{wb}, \lambda)$.

If $(p^t, q^t) \in \Phi_5$:

$$p^{t+1} = q^t + \left[1 - q^t - (1 + \lambda)(1 - p^t)\right] p^t;$$
$$q^{t+1} = q^t + \left[1 - q^t - (1 + \lambda)(1 - p^t)\right] q^t.$$

Since, in region Φ_5, $1 - q^t \geqslant (1 - q^t)h_{wb} > (1 + \lambda)(1 - p^t)$, we must have $q^{t+1} > q^t$ such that q^t increases over time. Moreover, $p^{t+1} > p^t$ iff $q^t > p^t\left[q^t + (1 + \lambda)(1 - p^t)\right]$ implying $1 - q^t < 1 - p^t(1 + \lambda) < (1 + \lambda)(1 - p^t)$ which contradicts the condition $1 - q^t > (1 + \lambda)(1 - p^t)/h_{wb}$. Hence, p^t decreases over time.

We are now in the position to draw the phase diagram associated with the joint dynamics of (p^t, q^t). To that end, let us first describe the locus of stationarity of q^t (qq locus) and p^t (pp locus) and discuss their stability in the three configuration listed in Proposition 5.

When $h_{wa} > h_{wb}$ the curve $q^t = g(p^t; h_{wa}, \lambda)$ belongs to region Φ_2 while $q^t > g(p^t; h_{wb}, \lambda)$ for all $(p^t, q^t) \in \Phi_4$. To see this, note that $g(p^t; h_{wa}, \lambda)$ reaches 1 at $p^t = h_{wa}/(1 + \lambda)$ exactly as $\phi_1(p^t)$ and $\widetilde{\phi}_1(p^t)$, and has a slope more negative than $\phi_1(p^t)$; while $g(p^t; h_{wb}, \lambda)$ reaches 0 at $p^t = 1 - h_{wb}/(1 + \lambda)$ exactly as $\phi_2(p^t)$ and $\widetilde{\phi}_2(p^t)$, and has a slope more negative than $\widetilde{\phi}_2(p^t)$. Hence, when $h_{wa} > h_{wb}$, the locus of stationarity of q^t (qq locus) corresponds to the curve $q^t = g(p^t; h_{wa}, \lambda)$ and is globally stable. When $h_{wa} = h_{wb}$, the curve $q^t = \phi_1(p^t)$ (resp., $q^t = \phi_2(p^t)$) exactly corresponds to the curve $q^t = g(p^t; h_{wa}, \lambda)$ (resp., $q^t = g(p^t; h_{wb}, \lambda)$) and $q^{t+1} = q^t$ for all $(q^t, p^t) \in \Phi_3$. Hence, the qq locus corresponds to the whole region Φ_3.

Regarding the pp locus—and in the case where both h_{wa} and h_{wb} are positive—note that $k(p^t; h_{wa}, \lambda)$ (resp., $f(p^t; h_{wb}, \lambda)$) is globally concave (resp., globally convex) for $p^t \in [0, 1]$. Moreover the slope of $k(p^t; h_{wa}, \lambda)$ (resp., $f(p^t; h_{wb}, \lambda)$) at $p^t = 0$ (resp., $p^t = 1$) equals $1/h_{wa}$ (resp., $1/h_{wb}$) which is positive and lower than the slope of $\widetilde{\phi}_1(p^t)$ (resp., $\widetilde{\phi}_2(p^t)$). Finally, $k(p^t; h_{wa}, \lambda)$ (resp., $f(p^t; h_{wb}, \lambda)$) equals 0 (resp., 1) when $p^t = 0$ (resp., $p^t = 1$) and $p^t = 1/(1 + \lambda)$ (resp., $p^t = \lambda/(1 + \lambda)$) while $\phi_1(p^t)$ (resp., $\phi_2(p^t)$) equals 0 (resp., 1) when $p^t = (1 - h_{wb})/(1 + \lambda) < 1/(1 + \lambda)$ (resp., $p^t = (h_{wa} + \lambda)/(1 + \lambda) > \lambda/(1 + \lambda)$). Thus $k(p^t; h_{wa}, \lambda)$ (resp., $f(p^t; h_{wb}, \lambda)$) crosses $\phi_1(p^t)$ (resp., $\phi_2(p^t)$) once. Hence, the pp locus is constituted by the curve $q^t = k(p^t; h_{wa}, \lambda)$ in region Φ_2, the curve $q^t = \ell(p^t)$ in region Φ_3 and the curve $q^t = f(p^t; h_{wb}, \lambda)$ in region Φ_4. The locus is continuous (since the proportions of each type of couple at the stable matching are continuous) and it is globally unstable. When $h_{wa} = h_{wb} = 0$, $k(p^t; 0, \lambda)$ (resp., $f(0; 0, \lambda)$) exactly corresponds to $\phi_1(p^t)$ (resp., $\phi_2(p^t)$). Hence p^t is decreasing (resp., increasing) when $q^t < \phi_1(p^t)$ (resp.,
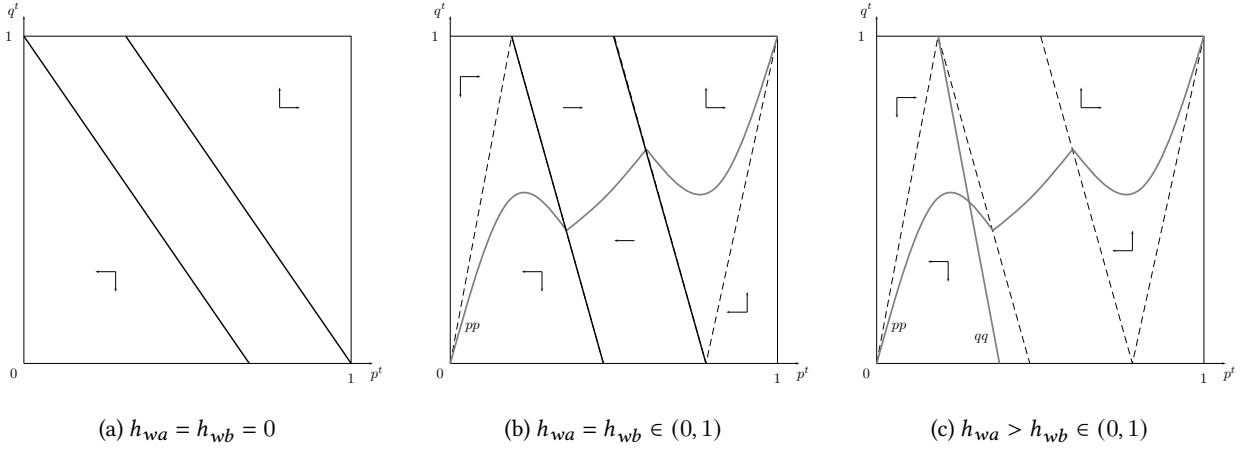
(a) $h_{wa} = h_{wb} = 0$ (b) $h_{wa} = h_{wb} \in (0,1)$ (c) $h_{wa} > h_{wb} \in (0,1)$

Figure 13: The cultural dynamics

$q^t \geqslant \phi_2(p^t)$). Moreover, $p^{t+1} = p^t$ for all $(q^t, p^t) \in \Phi_3$. Hence, the pp locus corresponds to the whole region Φ_3.

The phase diagrams in Figure 13 summarize these results. As shown on this Figure, when $h_{wa} > h_{wb}$: $(0,0)$, $(1,1)$ and the crossing point between the pp locus and the qq locus (r,r) with $r = \frac{1-h_{wa}}{2(1-h_{wa})+\lambda} < 1/2$ are steady states. In the case $h_{wa} = h_{wb} > 0$, the steady states $(0,0)$ and $(1,1)$ coexist with the stable set $\{(p,q) \in [0,1]^2 : q \in (\phi_1(p), \phi_2(p)) \text{ and } q = p\}$. Finally, when $h_{wa} = h_{wb} = 0$, the steady states $(0,0)$ and $(1,1)$ coexist with the stable set $\{(p,q) \in \Phi_3\}$.

Proof of Proposition 5. We successively address the three cases listed in Proposition 5.

1. $h_{wa} = h_{wb} = 0$. Consider the region T_ε in which $q^t \leqslant \phi_1(p^t) - \varepsilon$. For any $(p^t, q^t) \in T_\varepsilon$, $p^{t+1} < p^t$ and $q^{t+1} < q^t$. Hence, by the contraction mapping theorem (p^t, q^t) converges to $(0,0)$ as time approaches infinity and $(0,0)$ is asymptotically stable. Since ε is arbitrarily small, we can say that for any (p^0, q^0) such that $q^0 < \phi_1(p^0)$, (p^t, q^t) converges to $(0,0)$ as time approaches infinity. By applying the same logic, we can show that $(1,1)$ is asymptotically stable, and for any (p^0, q^0) such that $q^0 > \phi_2(p^0)$, (p^t, q^t) converges to $(1,1)$ as time approaches infinity. The set $\{(p^t, q^t) \in [0,1]^2 : q^t \in (\phi_1(p^t), \phi_2(p^t))\}$ is a stable set because any state is a steady state in this set and we can construct an open neighborhood $\mathcal{N}$, such that $\mathcal{N} \cap [0,1]^2$ is the set itself.

2. $h_{wa} = h_{wb} = h_w > 0$. We first check that $\{(p^t, q^t) \in [0,1]^2 : q^t \in (\phi_1(p^t), \phi_2(p^t)) \text{ and } q^t = p^t\}$ is a stable set. Consider $(p^t, q^t) \in \Phi_3$, we have $\Delta^{t+1} := p^{t+1} - q^{t+1} = (1 - h_w)(p^t - q^t) = (1 - h_w)\Delta^t$. Hence, Δ^t converges to 0. Moreover, when $p^t \in \left(\frac{h_w}{1+\lambda}, \frac{h_w+\lambda}{1+\lambda}\right)$, the straight line $p^t = q^t$ belongs to Φ_3. Hence, for any $(p^0, q^0) \in \left\{(p,q) \in \Phi_3 : p \in \left(\frac{h_w}{1+\lambda}, \frac{h_w+\lambda}{1+\lambda}\right)\right\}$ we have $\lim_{t\to\infty} p^t = \lim_{t\to\infty} q^t = q^0$. Let us now verify that $(0,0)$ and $(1,1)$ are attracting. Consider the neighborhood of $(0,0)$: $T_\varepsilon := \{(p,q) \in [0,\varepsilon]^2\}$. Clearly, there exists ε sufficiently low such that, when (p^0, q^0) belongs to T_ε and is above the pp curve—$(p^0, q^0) \in \{(p,q) \in [0,\varepsilon]^2 : q^t \geqslant k(p^t; h_{wa}, \lambda)\}$—at some t, (p^t, q^t) will enter

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the region below the pp curve. Then, it will converge to $(0, 0)$. We can use similar arguments to show that $(1, 1)$ is attracting.

3. $h_{wa} > h_{wb} \in (0, 1)$. The arguments developed in the previous point allow us to show that $(0, 0)$ and $(1, 1)$ are attracting. We now have to check the local stability of the interior steady state (r, r) with $r = \frac{1 - h_{wa}}{2(1 - h_{wa}) + \lambda}$. We know that this steady state belongs to region Φ_2 then we check the Jacobian matrix of the dynamics in region Φ_2 evaluated at (r, r), which is

$$
\mathcal{J}|_{p=r, q=r} = \begin{bmatrix} \frac{(1 - h_{wa})(3 + 2\lambda - 2h_{wa})}{2(1 - h_{wa}) + \lambda} & \frac{1 - h_{wa}(1 - \lambda)}{2(1 - h_{wa}) + \lambda} \\ \frac{(1 - h_{wa})(1 + \lambda)}{2(1 - h_{wa}) + \lambda} & \frac{3 + \lambda - h_{wa}(5 - 2h_{wa})}{2(1 - h_{wa}) + \lambda} \end{bmatrix}.
$$

The eigenvalues are given by $2 - h_{wa} > 1$ and $1 - h_{wa} < 1$. Hence, (r, r) is a saddle point.

$\square$

E   Proofs and omitted details in Section 5

E.1   Proof of Proposition 6

Proof of Proposition 6. We look for steady states p^* and q^*. We first show that there is no steady state such that $p^* \neq q^*$. Suppose there is. Without loss of generality, by symmetry, suppose $p^* > q^*$; by cultural substitutability, $d(p^*) < d(q^*)$ and $d(1 - p^*) > d(1 - q^*)$. The cultural evolution equations at the steady state become

$$
\begin{aligned}
0 &= q^* d(p^*)(1 - p^*) - (1 - p^*)d(1 - p^*)p^*; \\
0 &= q^* d(q^*)(1 - q^*) - (1 - p^*)d(1 - q^*)q^*.
\end{aligned}
$$

Equating the two equations yields

$$
q^*[d(p^*)(1 - p^*) - d(q^*)(1 - q^*)] = (1 - p^*)[d(1 - p^*)p^* - d(1 - q^*)q^*].
$$

By $p^* > q^*$, because $d(p^*) < d(q^*)$, the term in the square bracket on the left-hand side of the equation is negative, and because $d(1 - p^*) > d(1 - q^*)$, the term in the square bracket on the right-hand side of the equation is positive. Hence, the equation cannot hold, and we cannot have a steady state such that $p^* > q^*$ (or $p^* < q^*$ by symmetry).

We now consider the gender-symmetric steady states such that $p^* = q^* := r$. The steady state must satisfy

$$
r(1 - r)[d(r) - d(1 - r)] = 0.
$$

The equation holds when $r = 0$, $r = 1$, or $r = 1/2$. First, we show that $(0, 0)$ and $(1, 1)$ are unstable. Take any $p^t = q^t := r^0$. The system of equations becomes

$$
p^{t+1} - p^t = r^0(1 - r^0)[d(r^0) - d(1 - r^0)];
$$

$$q^{t+1} - q^t = r^0(1 - r^0)[d(r^0) - d(1 - r^0)].$$

Both are positive if $r^0 < 1/2$ and negative if $r^0 > 1/2$. Hence, $(0,0)$ and $(1,1)$ cannot be stable. Next, we prove that $(1/2, 1/2)$ is stable. Maintain the assumption $p^t \geqslant q^t$. When $p^t > 1/2$,

$$p^{t+1} - p^t = (1 - p^t)[q^t d(p^t) - p^t d(1 - p^t)] < 0,$$

where the inequality is derived from $d(p^t) < d(1 - p^t)$ and $q^t \leqslant p^t$. Similarly, when $q^t < 1/2$,

$$q^{t+1} - q^t = q^t[d(q^t)(1 - q^t) - (1 - p^t)d(1 - q^t)] > 0.$$

When $p^t < 1/2$, $q^t d(p^t) - p^t d(1 - p^t) > 0$ when $p^t = q^t$, so by continuity of $d(\cdot)$, $p^{t+1} - p^t$ is positive for (p^t, q^t) sufficiently close to $(1/2, 1/2)$. Similarly, when $q^t > 1/2$, $q^{t+1} - q^t$ is negative for (p^t, q^t) sufficiently close to $(1/2, 1/2)$, because $d(q^t)(1 - q^t) - (1 - p^t)d(1 - q^t) < 0$ when $p^t = q^t$.

We can analogously derive results on the sign of $p^{t+1} - p^t$ and $q^{t+1} - q^t$ for the case $p^t \leqslant q^t$.

The Lyapunov function that helps prove global stability is

$$\mathcal{V}(p, q) = \begin{cases} (q - 1/2)^2 & \text{if } p \geqslant q \text{ and } p + q < 1 \\ (p - 1/2)^2 & \text{if } p \geqslant q \text{ and } p + q \geqslant 1 \\ (q - 1/2)^2 & \text{if } p < q \text{ and } p + q \geqslant 1 \\ (p - 1/2)^2 & \text{if } p < q \text{ and } p + q < 1 \end{cases}$$

The function satisfies: (i) $\mathcal{V}(1/2, 1/2) = 0$, (ii) $\mathcal{V}(p^{t+1}, q^{t+1}) < \mathcal{V}(p^t, q^t)$ for all $(p^t, q^t) \neq (1/2, 1/2)$, and (iii) $\mathcal{V}(p^t, q^t) > 0$ for all (p^t, q^t), and (iv) $\|(p, q)\| \to \infty, \mathcal{V}(p, q) \to \infty$. In addition, the dynamic system is Lipschitz continuous, because $d(\cdot)$ is Lipschitz by assumption ($d(\cdot)$ being differentiable and bounded on the closed interval $[0, 1]$ implies a bounded first derivative, which implies Lipshitz continuity). By Theorem 1.4 of Bof et al. (2018), the existence of such a Lyapunov function implies global asymptotic stability.   □

E.2   Stable matching with mixtures of homophilic and heterophilic proposers and receivers when $h_{ma} = h_{mb} = h_m$ and $h_{wa} = h_{wb} = h_w$

The following characterization is needed for the Proof of Proposition 7. Consider the stable matching with mass p of type-a men and mass q of type-b women. Homophilic men and women of the same type, M_{θ_1} and W_{θ_1}, want to be matched together. Hence, at any stable matching the mass of $a_1 a_1$ couples is $\min\{p^t h_m, q^t h_w\}$, i.e., $p^t h_m$ if $q > h_m p/h_w =: h_1(p)$, and $q^t h_w$ otherwise. The mass of $b_1 b_1$ couples is $\min\{(1 - p^t)h_m, (1 - q^t)h_w\}$, i.e., $(1 - p^t)h_m$ if $q^t < 1 - h_m(1 - p^t)/h_w =: h_2(p^t)$, and $(1 - q^t)h_w$ otherwise.

Moreover, for $\theta \neq \theta'$, heterophilic men and women of the opposite types, M_{θ_2} and $W_{\theta'_2}$, want to be matched together. Hence, at any stable matching the mass of ab couples must be at least

$$\begin{cases} p^t(1 - h_m) & \text{if } q^t < 1 - \left(\frac{1 - h_m}{1 - h_w}\right)p^t =: h_3(p^t), \\ (1 - q^t)(1 - h_w) & \text{otherwise.} \end{cases}$$
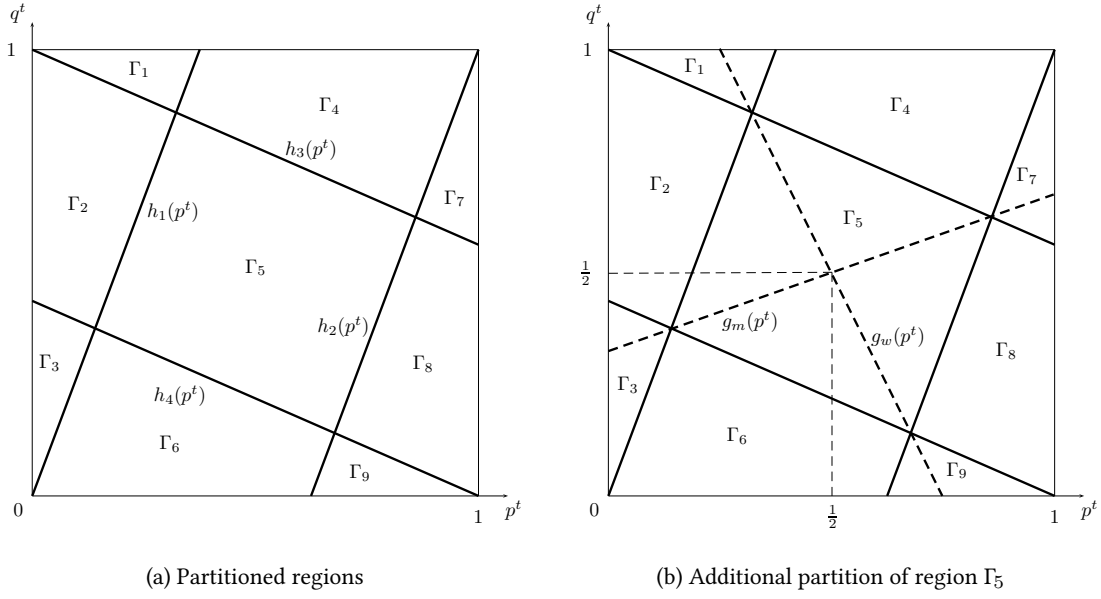
(a) Partitioned regions   (b) Additional partition of region Γ_5

Figure 14: Partitioning regions for characterization of stable matching

and the mass of bb couples must be at least

$$
\begin{cases}
(1-p^t)(1-h_m) & \text{if} \quad q^t > \left(\frac{1-h_m}{1-h_w}\right)(1-p^t) =: h_4(p^t), \\
q^t(1-h_w) & \text{otherwise}.
\end{cases}
$$

As Figure 14a depicts, the unit square can be partitioned into nine disjoint sets according to the position of (p^t, q^t) with respect to the four functions $h_1(p^t)$, $h_2(p^t)$, $h_3(p^t)$ and $h_4(p^t)$.

Below, we describe the stable matching in each region. Except region Γ_5, there is a unique stable matching, summarized in the table below.

Region	μ_{aa}	μ_{ba}	μ_{bb}	μ_{ab}
Γ_1	$p-(1-q)(1-h_w)$	$1-p-(1-q)h_w$	$(1-q)h_w$	$(1-q)(1-h_w)$
Γ_2	ph_m	$q-ph_m$	$1-q-p(1-h_m)$	$p(1-h_m)$
Γ_3	ph_m	$q-ph_m$	$1-q-p(1-h_m)$	$p(1-h_m)$
Γ_4	$p-(1-q)(1-h_w)$	$1-p-(1-q)h_w$	$(1-q)h_w$	$(1-q)(1-h_w)$
Γ_6	qh_w	$q(1-h_w)$	$1-p-q(1-h_w)$	$p-qh_w$
Γ_7	$q-(1-p)(1-h_m)$	$(1-p)(1-h_m)$	$(1-p)h_m$	$1-q-(1-p)h_m$
Γ_8	$q-(1-p)(1-h_m)$	$(1-p)(1-h_m)$	$(1-p)h_m$	$1-q-(1-p)h_m$
Γ_9	qh_w	$q(1-h_w)$	$1-p-q(1-h_w)$	$p-qh_w$

If $(p,q) \in \Gamma_5$, MOSM and WOSM do not coincide. Define $g_w(p) := (1-h_w-p)/(1-2h_w)$ and $g_m(p) := 1-h_m-p(1-2h_m)$. The stable matching in Γ_5 is described as follows.

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	μ_{aa}	μ_{ba}	μ_{bb}	μ_{ab}
		WOSM		
$q < g_w(p)$	qh_w	$q(1-h_w)$	$1-p-q(1-h_w)$	$p-qh_w$
$q \geqslant g_w(p)$	$p-(1-q)(1-h_w)$	$1-p-(1-q)h_w$	$(1-q)h_w$	$(1-q)(1-h_w)$
		MOSM		
$q < g_m(p)$	$q-(1-p)(1-h_m)$	$(1-p)(1-h_m)$	$(1-p)h_m$	$1-q-(1-p)h_m$
$q \geqslant g_m(p)$	ph_m	$q-ph_m$	$1-q-p(1-h_m)$	$p(1-h_m)$

As depicted in Figure 14b, $g_w(p^t)$ passes through the crossing point between $h_1(p^t)$ and $h_3(p^t)$ and the crossing point between $h_2(p^t)$ and $h_4(p^t)$ while $g_m(p^t)$ passes through the crossing point between $h_1(p^t)$ and $h_4(p^t)$ and the crossing point between $h_2(p^t)$ and $h_3(p^t)$. Moreover, $g_w(p^t)$ and $g_m(p^t)$ intersect at $(1/2, 1/2)$.

E.3 Proof of Proposition 7

Proof of Proposition 7. Suppose there are homophilic receivers ($0 < hw \leqslant 1$) in addition to a strict mixture of homophilic and heterophilic proposers.

Rearrange the steady state equations (15) and (16):

$$\frac{\mu_{aa}^*}{\mu_{bb}^*} = \frac{d(1-p^*)}{d(p^*)}\frac{p^*}{1-p^*};$$
$$\frac{\mu_{aa}^*}{\mu_{bb}^*} = \frac{d(1-q^*)}{d(q^*)}\frac{q^*}{1-q^*}.$$

Since the right-hand sides of the equations are strictly increasing, $p^* = q^*$ must hold in a steady state. According to Figure 14, the steady states must lie in regions Γ_4, Γ_5, and/or Γ_6.

Consider the stable matching characterized by Figure 14. When $(p^t, q^t) \in \Gamma_5 \cap \{q^t \geqslant g_m(p^t)\}$, the evolution is

$$p^{t+1} - p^t = p^t h_m d(p^t)(1-p^t) - [1-q^t-p^t(1-h_m)]d(1-p^t)p^t; \qquad (28)$$
$$q^{t+1} - q^t = p^t h_m d(q^t)(1-q^t) - [1-q^t-p^t(1-h_m)]d(1-q^t)q^t. \qquad (29)$$

. When $(p^t, q^t) \in \Gamma_5 \cap \{q^t < g_m(p^t)\}$, the evolution is

$$p^{t+1} - p^t = [q^t-(1-p^t)(1-h_m)]d(p^t)(1-p^t) - (1-p^t)h_m d(1-p^t)p^t;$$
$$q^{t+1} - q^t = [q^t-(1-p^t)(1-h_m)]d(q^t)(1-q^t) - (1-p^t)h_m d(1-q^t)q^t.$$

The Jacobian matrix near $(1/2, 1/2)$ is continuous and evaluated at $(1/2, 1/2)$ is

$$\mathcal{J}|_{p=1/2, q=1/2} = \begin{bmatrix} 1 + \frac{1}{2}h_m d'(\frac{1}{2}) + (\frac{1}{2}-h_m)d(\frac{1}{2}) & \frac{1}{2}d(\frac{1}{2}) \\ \frac{1}{2}d(\frac{1}{2}) & 1 + \frac{1}{2}h_m d'(\frac{1}{2}) + (\frac{1}{2}-h_m)d(\frac{1}{2}) \end{bmatrix}. \qquad (30)$$

The two eignenvalues are $1 - h_m(d(1/2) - d'(1/2)/2)$ and $1 - h_m(d(1/2) - d'(1/2)/2) + d(1/2)$. Because

$d' < 0$, the first eigenvalue is less than 1. Assumption 2 guarantees that the first eigenvalue is larger than -1 regardless of the value of h_m. If the second eigenvalue is also less than 1 (it must be larger than -1 given Assumption 2), then $(1/2, 1/2)$ is asymptotically stable; otherwise, it is a saddle point. The second eigenvalue is less than 1 if

$$h_m > \frac{d(1/2)}{d(1/2) - d'(1/2)/2},$$

which is a modification of the specified condition stated in the proposition for $(1/2, 1/2)$ to be stable.

Suppose all receivers are heterophilic (and there is a strict mixture of homophilic and heterophilic proposers). Suppose $q^t < (1 - p^t)(1 - h_m)$. The evolution becomes

$$
\begin{aligned}
p^{t+1} - p^t &= -(1 - p^t - q^t)d(1 - p^t)p^t < 0; \\
q^{t+1} - q^t &= -(1 - p^t - q^t)d(1 - q^t)q^t < 0.
\end{aligned}
$$

Hence, $(0, 0)$ is an asymptotically stable steady state by the contraction mapping theorem. Similarly, when receivers are all heterophilic, $(1, 1)$ is also an asymptotically stable steady state. $\qquad \square$

E.4 Stable matching for nonhomophilic cultural groups

Homophilic men and women of the same type, M_{θ_1} and W_{θ_1}, want to be matched together. Hence, at any stable matching the mass of $a_1 a_1$ couples is $\min\{p^t h_a, q^t h_a\}$, i.e., $p^t h_a$ if $q^t \geqslant p^t$ and $q^t h_a$ otherwise. The mass of $b_1 b_1$ couples is $\min\{(1 - p^t)h_b, (1 - q^t)h_b\}$, i.e., $(1 - p^t)h_b$ if $q^t < p^t$, and $(1 - q^t)h_b$ otherwise.

Moreover, for $\theta \neq \theta'$, heterophilic men and women of the opposite types, M_{θ_2} and $W_{\theta_2'}$, want to be matched together. Hence, at any stable matching the mass of ab couples must be at least $\min\{p^t(1 - h_a), (1 - q^t)(1 - h_b)\}$, or equivalently

$$
\begin{cases}
p^t(1 - h_a) & \text{if} \quad q^t < 1 - \left(\frac{1 - h_a}{1 - h_b}\right)p^t =: \widetilde{h}(p^t), \\
(1 - q^t)(1 - h_b) & \text{otherwise.}
\end{cases}
$$

and the mass of ba couples must be at least $\min\{(1 - p^t)(1 - h_b), q^t(1 - h_a)\}$, or equivalently

$$
\begin{cases}
(1 - p^t)(1 - h_b) & \text{if} \quad q^t \geqslant \left(\frac{1 - h_b}{1 - h_a}\right)(1 - p^t) =: \widehat{h}(p^t), \\
q^t(1 - h_a) & \text{otherwise.}
\end{cases}
$$

The unit square can be partitioned in six disjoint sets according to the position of q^t with respect to p^t, $\widehat{h}(p^t)$ and $\widetilde{h}(p^t)$:

$$
\begin{aligned}
\Theta_1 &:= \left\{(p^t, q^t) \in [0,1]^2 : q^t \geqslant p^t \text{ and } q^t \geqslant \widehat{h}(p^t)\right\}; \\
\Theta_2 &:= \left\{(p^t, q^t) \in [0,1]^2 : q^t \geqslant p^t \text{ and } q^t \in \left[\widetilde{h}(p^t), \widehat{h}(p^t)\right)\right\}; \\
\Theta_3 &:= \left\{(p^t, q^t) \in [0,1]^2 : q^t \geqslant p^t \text{ and } q^t < \widetilde{h}(p^t)\right\}; \\
\Theta_4 &:= \left\{(p^t, q^t) \in [0,1]^2 : q^t < p^t \text{ and } q^t \geqslant \widetilde{h}(p^t)\right\};
\end{aligned}
$$

$$\Theta_5 \;:=\; \left\{ (p^t, q^t) \in [0,1]^2 : q^t < p^t \text{ and } q^t \in \left[\widehat{h}(p^t), \widetilde{h}(p^t)\right) \right\};$$

$$\Theta_6 \;:=\; \left\{ (p^t, q^t) \in [0,1]^2 : q^t < p^t \text{ and } q^t < \widehat{h}(p^t) \right\}.$$

Below, we describe the stable matching in each region.

Region	μ_{aa}	μ_{ba}	μ_{bb}	μ_{ab}
Θ_1	$p - (1-q)(1-h_b)$	$1 - p - (1-q)h_b$	$(1-q)h_b$	$(1-q)(1-h_b)$
Θ_2	$p - (1-q)(1-h_b)$	$1 - p - (1-q)h_b$	$(1-q)h_b$	$(1-q)(1-h_b)$
Θ_3	ph_a	$q - ph_a$	$1 - q - p(1-h_a)$	$p(1-h_a)$
Θ_4	$q - (1-p)(1-h_b)$	$(1-p)(1-h_b)$	$(1-p)h_b$	$1 - q - (1-p)h_b$
Θ_5	$q - (1-p)(1-h_b)$	$(1-p)(1-h_b)$	$(1-p)h_b$	$1 - q - (1-p)h_b$
Θ_6	qh_a	$q(1-h_a)$	$1 - p - q(1-h_a)$	$p - qh_a$

Note that, the stable matching is identical in regions Θ_1 and Θ_2, on the one hand, and Θ_4 and Θ_5, on the other hand.

E.5 Proof of Proposition 8

Proof of Proposition 8. First, let us look at region $\Theta_1 \cup \Theta_2$, in which $\mu_{aa}^t = p^t - (1 - q^t)(1 - h_b)$ and $\mu_{bb}^t = (1 - q^t)h_b$. A steady state (p, q) must satisfy:

$$\big(p - (1-q)(1-h_b)\big)d(p)(1-p) - (1-q)h_b d(1-p)p = 0;$$

$$\big(p - (1-q)(1-h_b)\big)d(q)(1-q) - (1-q)h_b d(1-q)q = 0.$$

Suppose $q > p$, then we have $d(p)(1-p) > d(q)(1-q)$ and $d(1-p)p < d(1-q)q$, making the two equations impossible to hold at the same time. Hence, it must be the case that $p = q$. Note that $p = q = 1$ is always a steady state.

Next, let us look at region Θ_3, in which $\mu_{aa}^t = p^t h_a$ and $\mu_{bb}^t = 1 - q^t - p^t(1 - h_a)$. A steady (p, q) must satisfy:

$$ph_a d(p)(1-p) - \big((1 - q - p(1 - h_a)\big)d(1-p)p = 0;$$

$$ph_a d(q)(1-q) - \big((1 - q - p(1 - h_a)\big)d(1-q)q = 0.$$

Similarly, we can show that $p = q$. Note that $p = q = 1/2$ and $p = q = 0$ are always steady states.

The analysis for region $\Theta_4 \cup \Theta_5$ is similar to that for region $\Theta_1 \cup \Theta_2$, and the analysis for region Θ_6 is similar to that for region Θ_3.

Now, we examine the stability of $p = q = 1/2$. The Jacobian matrix near $(1/2, 1/2)$ is continuous and evaluated at $(1/2, 1/2)$ is

$$\mathcal{J}|_{p=1/2, q=1/2} = \begin{bmatrix} 1 + \frac{1}{2}h_a d'(\frac{1}{2}) + (\frac{1}{2} - h_a)d(\frac{1}{2}) & \frac{1}{2}d(\frac{1}{2}) \\ \frac{1}{2}d(\frac{1}{2}) & 1 + \frac{1}{2}h_a d'(\frac{1}{2}) + (\frac{1}{2} - h_a)d(\frac{1}{2}) \end{bmatrix}. \tag{31}$$

Note that the Jacobian Matrix is identical to the one for Proposition 7 in Section 5.2 by replacing h_m with h_a.

The two eigenvalues are $1 - h_a(d(1/2) - d'(1/2)/2)$ and $1 - h_a(d(1/2) - d'(1/2)/2) + d(1/2)$. Because $d' < 0$, the first eigenvalue is less than 1. Assumption 2 guarantees that the first eigenvalue is larger than -1 whatever the value of h_a. If the second eigenvalue is also less than 1 (it must be larger than -1 given Assumption 2), then $(1/2, 1/2)$ is asymptotically stable; otherwise, it is a saddle point. The second eigenvalue is less than 1 if

$$h_a > \frac{d(1/2)}{d(1/2) - d'(1/2)/2},$$

which is a modification of the specified condition stated in the proposition for $(1/2, 1/2)$ to be stable.

Suppose type-b agents are all heterophilic: $h_b = 0$. In region $\Theta_1 \cup \Theta_2$ near $(1, 1)$, the evolution satisfies

$$
\begin{aligned}
p^{t+1} - p^t &= (p^t + q^t - 1)d(p^t)(1 - p^t) > 0; \\
q^{t+1} - q^t &= (p^t + q^t - 1)d(q^t)(1 - q^t) > 0.
\end{aligned}
$$

Similar conditions hold for region $\Theta_4 \cup \Theta_5$. Hence, $(1, 1)$ is an asymptotically stable steady state by the contraction mapping theorem. □

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Online appendices: omitted details and proofs

OA.1: Quadratic socialization costs

Here we consider the case in which the probability of direct transmission within homogamies is linear: $d(r) = 1 - r$. As discussed in Appendix A, this would be the case in a standard cultural socialization model *á la* Bisin and Verdier (2001) under the usual assumption of quadratic socialization costs: $c(\tau_t^i) = (\tau_t^i)^2/2$. Below, we characterize the stable steady states of the cultural dynamics in this case when $h_{ma} = h_{mb} = h_m$ and $h_{wa} = h_{wb} = h_w$ (Proposition 9) and when $h_{ma} = h_{wa} = h_a$ and $h_{mb} = h_{wb} = h_b$ (Proposition 10).

Proposition 9. *Assume $h_{ma} = h_{mb} = h_m \in (0, 1)$, $h_{wa} = h_{wb} = h_w$ and men are proposers. If $h_w = 1$, $(0, 0)$ and $(1, 1)$ are asymptotically stable steady states. If $h_w < 1$:*

- *If $h_m \geqslant 1/2$: $(1/2, 1/2)$ is the unique asymptotically stable steady state.*

- *If $h_m < 1/2$: $(1/2, 1/2)$ is unstable and (h_m, h_m) and $(1 - h_m, 1 - h_m)$ are asymptotically stable steady states.*

Proof. We already know, form the proof of Proposition 7, that $p^* = q^*$ must hold in steady state. Moreover, a steady state (r, r), with $r \leqslant 1/2$ must be such that:

$$r h_m d(r)(1 - r) = (1 - r(2 - h_m))d(1 - r)r.$$

After simple algebra, this equation reduces to $r(h_m - r)(2r - 1) = 0$ so that 0, $1/2$ and h_m are solutions. In the case $r > 1/2$, a steady state (r, r) must be such that:

$$[r - (1 - r)(1 - h_m)]\, d(r)(1 - r) = (1 - r)h_m d(1 - r)r$$

After simple algebra, this equation reduces to $(1 - r)(1 - h_m - r)(2r - 1) = 0$ so that 1, $1/2$ and $1 - h_m$ are solutions.

The Jacobian matrix evaluated at (h_m, h_m) and $(1 - h_m, 1 - h_m)$ is

$$\mathcal{J}|_{p=1-h_m q=1-h_m} = \mathcal{J}|_{p=h_m q=h_m} = \begin{bmatrix} 1 - h_m(1 - h_m) & h_m^2 \\ h_m(1 - h_m) & 1 - h_m(2 - 3h_m) \end{bmatrix}. \tag{A.1}$$

The two eigenvalues are $\lambda_1 = 1 - 2h_m(1 - h_m) \in (0, 1)$ and $\lambda_2 = 1 - 2h_m(1/2 - h_m)$ which is lower than one when $h_m < 1/2$. Hence, in this case, both (h_m, h_m) and $(1 - h_m, 1 - h_m)$ are stable.

The Jacobian matrix evaluated at $(1/2, 1/2)$ is

$$\mathcal{J}|_{p=1/2 q=1/2} = \begin{bmatrix} 5/4 - h_m & 1/4 \\ 1/4 & 5/4 - h_m \end{bmatrix}. \tag{A.2}$$

The two eigenvalues are $\lambda_1 = 1 - h_m \in (0, 1)$ and $\lambda_2 = (3 - 2h_m)/2$ which is lower than one iff $h_m \geqslant 1/2$. Hence, in this case, $(1/2, 1/2)$ is stable, otherwise it is a saddle.

Finally, the Jacobian matrix evaluated at $(0,0)$ and $(1,1)$ is

$$\mathcal{J}|_{p=0 q=0} = \mathcal{J}|_{p=1 q=1} = \begin{bmatrix} 1 + h_m & 0 \\ h_m & 1 \end{bmatrix}. \tag{A.3}$$

The two eigenvalues are $\lambda_1 = 1$ and $\lambda_2 = 1 + h_m$. Hence, both $(0,0)$ and $(1,1)$ are unstable. $\quad\square$

Proposition 10. *Assume $h_{ma} = h_{wa} = h_a \in (0,1)$, $h_{mb} = h_{wb} = h_b \in (0,1)$ and $h_a \geqslant h_b$. If $h_a + h_b \geqslant 1$, $(1/2, 1/2)$ is the unique asymptotically stable steady state. If $h_a + h_b < 1$, then,*

- *when $h_a \geqslant 1/2 \geqslant h_b$, (h_a, h_a) is unstable and $(1/2, 1/2)$ and $(1 - h_b, 1 - h_b)$ are asymptotically stable steady states;*

- *when $1/2 > h_a \geqslant h_b$, $(1/2, 1/2)$ is unstable and (h_a, h_a) and $(1 - h_b, 1 - h_b)$ are asymptotically stable steady states.*

Proof. We already know, from the proof of Proposition 8, that $p^* = q^*$ must hold in steady state. Moreover, a steady state (r, r), with $r < \widehat{r} := \frac{1 - h_b}{2 - h_a - h_b}$ ($\widehat{r}$ is calculated such that $(\widehat{r}, \widehat{r})$ corresponds to the crossing point between $\tilde{h}(p^t)$ and $\widehat{h}(p^t)$) must be such that

$$r h_a d(r)(1 - r) = (1 - r(2 - h_a))d(1 - r)r.$$

We already know that this equation has three solutions $(0,0)$, $(1/2, 1/2)$ and (h_a, h_a) but (h_a, h_a) might be a solution iff $h_a \leqslant \widehat{r}$ which is true iff $h_a < 1 - h_b$. Otherwise $(h_a + h_b \geqslant 1)$ only $(0,0)$ and $(1,1)$ are solutions.

In the case $r > \widehat{r}$, the steady state (r, r) must be such that

$$[r - (1 - r)(1 - h_b)] d(r)(1 - r) = (1 - r)h_b d(1 - r)r.$$

We already know that this equation admits three solutions $(1,1)$, $(1/2, 1/2)$ and $(1 - h_b, 1 - h_b)$. However, since $\widehat{r} \geqslant 1/2$ (because $h_a \geqslant h_b$), $(1/2, 1/2)$ is not an admissible solution (if $h_a = h_b$, $(1/2, 1/2)$ is an admissible solution but it is confounded with $(1 - h_b, 1 - h_b)$). Moreover, $(1 - h_b, 1 - h_b)$ might be a solution if and only if $1 - h_b > \widehat{r}$ wich is true iff $h_a < 1 - h_b$.

Tu sum-up, if $h_a + h_b \geqslant 1$, the dynamics admit three steady-states: $(0,0)$, $(1/2, 1/2)$ and $(1,1)$; if $h_a + h_b < 1$, the dynamics admit five steady-states: $(0,0)$, $(1/2, 1/2)$, $(1,1)$, (h_a, h_a) and $(1 - h_b, 1 - h_b)$.

Finally, we know, from the stability analysis of the different steady states in the proof of Proposition 9, that $(0,0)$ and $(1,1)$ are unstable, $(1/2, 1/2)$ is stable iff $h_a \geqslant 1/2$ otherwise it is a saddle, (h_a, h_a) is stable iff $h_a < 1/2$ otherwise it is a saddle; $(1 - h_b, 1 - h_b)$ is stable iff $h_b < 1/2$. $\quad\square$